SOLUTIONS

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics

STAT-319-Term063-Quiz3

Name: ID: Serial:

Question1. (6-points)

Errors in an experimental transmission channel are found when the transmission is checked by a certifier that detects missing pulses. The number of errors found in an eight bit byte is a random variable with the following distribution: $F(x)=\begin{cases}0 & ,x<1\\ 0.7 & ,1\le x<4\\ 0.9 & ,4\le x<7\\ 1 & ,x\ge 7\end{cases}$ find the following:

a. $P(x\le 2)$ **The probability distribution of X is**

X	1	4	7
f(x)	0.7	0.2	0.1

$\Rightarrow P(x\le 2)=f(1)=0.7$ **(2-Points)**

b. $P(x>4)\Rightarrow P(x>4)=f(7)=0.1$ **(1-Point)**

c. If $Y=(2X+1)$ find σ_Y^2

$$\sigma_Y^2=E(Y-\mu_Y)^2$$

$$\mu_Y=\sum(2x+1)f(x)=(3)(0.7)+(9)(0.2)+(15)(0.1)=5.4$$

$$\Rightarrow \sigma_Y^2=\sum(2x+1-5.4)^2 f(x)=\sum(2x-4.4)^2 f(x)$$

$$=(2-4.4)^2(0.7)+(8-4.4)^2(0.2)+(14-4.4)^2(0.1)=15.84$$

(3-Points)

Question2. (4-Points)

The probability density function of the time to failure of an electronic component in a copier (in hours) is $f(x)=\begin{cases}\dfrac{e^{-x/1000}}{1000} & ,x>0\\ 0 & ,ow\end{cases}$ Find the following

a. The probability that a component fails before 1000 hours.

$$\Rightarrow P(x\le 1000)=\int_0^{1000}\frac{e^{-x/1000}}{1000}dx=-e^{-x/1000}\Big|_0^{1000}=-\left(e^{-1}-e^{0}\right)=1-0.3679=0.6321$$ **(2-Points)**

b. Determine the number of hours at which 10% of all components have failed.

We need to find *k* such that $P(x\le k)=0.1$

$$\Rightarrow P(x\le k)=\int_0^{k}\frac{e^{-x/1000}}{1000}dx=-e^{-k/1000}\Big|_0^{1000}=-\left(e^{-k/1000}-e^{0}\right)=1-e^{-k/1000}=0.10$$ **(2-Points)**

$$\Rightarrow e^{-k/1000}=0.9\Rightarrow -\frac{k}{1000}=\ln(0.9)\Rightarrow k=105.36\,hours$$