

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT 212: BUSINESS STATISTICS II

Semester 063

First Exam *SOLUTIONS*

Saturday July 24, 2007

7:00 - 8:15PM

Please **circle** your:

Instructor's name

& section number

Mohammad F. Saleh

Sec 1: (9:20 – 10:20)

Sec 2 : (10:30 – 11:30)

Marwan M. Almomani

Sec 3 : (10:30 – 11:30)

Name:

Student ID#:

Serial #:

Directions:

- 1) You must **show all work** to obtain full credit for questions on this exam.
- 2) DO NOT round your answers at each step. Round answers only if necessary at **your final step to 4 decimal places.**

Question No	Full Marks	Marks Obtained
<i>Part One</i>	<i>6</i>	
<i>Part Two</i>	<i>6</i>	
<i>Q1</i>	<i>12</i>	
<i>Q2</i>	<i>10</i>	
<i>Q3</i>	<i>13</i>	
<i>Q4</i>	<i>9</i>	
<i>Q5</i>	<i>11</i>	
<i>Q6</i>	<i>13</i>	
<i>Total</i>	<i>80</i>	

Part One (6 pts, 1 for each)

Indicate which of the following TRUE or FALSE

- T** 1. Whenever possible, in establishing the null and alternative hypotheses, the research hypothesis should be made the alternative hypothesis. **True**
- T** 2. If a decision maker wishes to reduce the chance of making a Type II error, one option is to increase the sample size. **True**
- F** 3. After performing a one tail test and reject H_0 , you realize you should have done a two tailed test, at the same significance level. You will also reject H_0 for the test. **False**
- T** 4. Testing the differences between tow means with dependent samples becomes a one sample test once you compute the differences of the paired observations. **True**
- T** 5. The p – value is the largest significance at level which you wouldn't reject the null hypothesis. **True**
- F** 6. In a null hypothesis test involving two population variances, if the null hypothesis states that the two variances are strictly equal, then the test statistic is a chi-square statistic. **False**

Part Two (6 pts 1 for each)

Circle the correct answer:

1. In a one tail test using z distribution as the test statistic and the 0.01 significance level, the critical value either

- a. -1.96 or +1.96
- b. -1.645 or +1.645
- c. -2.575 or +2.575
- d. .0 or 1

$$Z_{\alpha} = Z_{0.01} = 2.33$$

- ☒ e. None of these is correct.

2. A type II error is committed if we:

- a. Reject a true null hypothesis.
- b. Don't reject a true alternative hypothesis.
- ☒ c. Reject a true alternative hypothesis.
- d. Don't reject both null and alternative hypotheses at the same time.
- e. None of these is correct.

3. The F distribution:

- ☒ a. Cannot be negative.
- b. Cannot be positive.
- c. Is the same as the t distribution.
- d. Is the same as the z distribution.
- e. None of these is correct.

4. As the sample size increases, the t distribution approaches will be:

- ☒ a. The standard normal distribution.
- b. The chi distribution.
- c. The F distribution.
- d. Zero.
- e. None of these is correct.

5. Airline A and Airline B boast successful baggage routing rates of 95 and 98 percent, respectively. From this information we can determine.
- a. Airline A has better baggage service.
 - b. Airline B has better baggage service
 - c. The baggage services are equally accurate.
 - (d.) Nothing, we need more information to decide.**
 - e. None of these is correct.
6. For which of the following is a two sample test not appropriate?
- a. Seeing whether the proportions of childless couples with children who buy sports cars different.
 - b. Seeing whether the mean tea consumption is higher in KSA than in UAE.
 - (c.) Testing whether there are more men than women in Al – Dammam.**
 - d. Deciding whether average attendance at major league football games is the same in Al - Riyadh and Al – Dammam.
 - e. None of these is correct.

(c) → It can be tested using:

$$H_0: P_{men} \leq 0.5 \quad \text{vs} \quad H_A: P_{men} > 0.5$$

Which is a one-sample test for single proportion.

Part ThreeQuestion One (2+3+1+1+2+1+1+1 = 12 pts):

From old records, it is believed that the average speeds of the cars at Al – Khobar Street is 99 km per hour. A random sample of 25 cars was selected and the speed for each car registered. The sample gave an average of 107.92 km per hour and a standard deviation of 14.474 km per hour. Do the data support the old records? Use the 2% significance level.

- a. The test hypotheses are:

$$H_0: \mu = 99$$

$$\text{vs. } H_A: \mu \neq 99$$

② points

- b. The assumptions are:

1. Small sample size
2. σ is unknown
3. population is normal

③ points

- c. The critical values are:

$$t_{\alpha/2, n-1} = t_{0.01, 24} = 2.4922$$

① point

- d. The decision rule using the test statistic approach is:

$$\text{Reject } H_0 \text{ if } |t_c| > t_{\alpha/2, n-1}$$

① point

- e. The test statistic is:

$$t_c = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{107.92 - 99}{14.474/\sqrt{25}} = \frac{8.92}{2.8948} = 3.0814$$

② points

- f. The decision is:

$$\text{Since } |t_c| = 3.0814 > t_{\alpha/2, n-1} = 2.4922$$

Reject H_0

① point

- g. Your conclusion is:

The sample information do NOT support the old records which is believed that the average speed is 99 km/hr.

① point

- h. Based on your decision, what type of error you might have committed?

Type I error (Because H_0 was rejected)

① point

Question Two (2+2+1+1+2+1+1 = 10 pts):

A PC company uses two suppliers for rechargeable batteries for their notebook computers. Two factors are important quality features of the batteries: mean use time and variation. It is desirable that the mean use time be high and the variability be low. Recently, the PC maker conducted a test on batteries from the two suppliers. In the test, 9 randomly selected batteries from Supplier 1 were tested and 12 randomly selected batteries from Supplier 2 were tested. The following results were observed:

Supplier 1	Supplier 2
$n_1 = 9$	$n_2 = 12$
$\bar{x}_1 = 67.25 \text{ min.}$	$\bar{x}_2 = 72.40 \text{ min.}$
$s_1 = 11.2 \text{ min.}$	$s_2 = 9.9 \text{ min.}$

Based on these sample results, can the PC maker conclude that a difference exists between the two batteries with respect to the population standard deviations? Test using a 0.10 level of significance.

- a. The test hypotheses are:

$$H_0: \sigma_1^2 - \sigma_2^2 = 0 \quad \text{vs.} \quad H_A: \sigma_1^2 - \sigma_2^2 \neq 0 \quad \} \quad (2\text{-pts})$$

- b. The assumptions are

1. The two populations are normally distributed
2. Samples Variances are independent.

- c. The critical value(s) is(are):

$$F_{\alpha/2, n_1-1, n_2-1} = F_{0.05, 8, 11} = 2.948 \quad \} \quad (1\text{-pt})$$

- d. The decision rule is:

$$\text{Reject } H_0 \text{ if } F_c > F_{\alpha/2, n_1-1, n_2-1} \quad \} \quad (1\text{-pt})$$

- e. The test statistic is:

$$F_c = \frac{S_1^2}{S_2^2} = \frac{(11.2)^2}{(9.9)^2} = \frac{125.4}{98.01} = 1.2799 \quad \} \quad (2\text{-pts})$$

- f. The decision is:

$$\text{Since } F_c = 1.2799 \not> F_{\alpha/2, n_1-1, n_2-1} = 2.948 \quad \} \quad (1\text{pt})$$

\therefore Do NOT reject H_0

- g. The conclusion is:

Based on Samples information, there is NO difference between the two population standard deviations. } (1pt)

Question Three (2+1+1+5+1+1+2 = 13pts):

In the previous question (question 2). Can the PC maker conclude that a difference exists between the two batteries with respect to the population mean use time? Test using a 0.05 level of significance. Assuming that both samples selected randomly from normal populations

- a. The test hypotheses are:

$$H_0: \mu_1 - \mu_2 = 0 \quad \text{vs.} \quad H_A: \mu_1 - \mu_2 \neq 0 \quad \left. \vphantom{H_0} \right\} \text{2 pts}$$

- b. The critical value(s) is (are):

$$t_{\alpha/2, n_1+n_2-2} = t_{0.025, 19} = 2.0930 \quad \left. \vphantom{t_{\alpha/2}} \right\} \text{1 pt}$$

- c. The decision rule using the test statistic approach is:

$$\text{Reject } H_0 \text{ if } |t_c| > t_{\alpha/2, n_1+n_2-2} \quad \left. \vphantom{\text{Reject}} \right\} \text{1 pt}$$

- d. The test statistic is:

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(8)(11.2)^2 + (11)(9.9)^2}{9+12-2}} = 10.4671 \quad \left. \vphantom{S_p} \right\} \text{2 pts}$$

$$t_c = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{67.25 - 72.40 - 0}{10.4671 \sqrt{\frac{1}{9} + \frac{1}{12}}} = -1.1158 \quad \left. \vphantom{t_c} \right\} \text{3 pts}$$

- e. The decision is:

$$\text{Since } |t_c| = 1.11581 \not> t_{\alpha/2, n_1+n_2-2} = 2.0930 \quad \left. \vphantom{\text{Since}} \right\} \text{1 pt}$$

∴ Do NOT reject H_0

- f. The conclusion is:

Based on the samples information, there is NO difference between the two populations mean use time. } 1 pt

- g. What is other assumption required to perform the test in part (a)? is it satisfied? Explain.

I. The other assumption is: σ_1^2, σ_2^2 are unknown, but equal. } 1 pt

II. Yes, because $H_0: \sigma_1^2 - \sigma_2^2 = 0$ was not rejected in the previous question. } 1 pt

Question Four (2+1+1+1+2+1+1 = 9 pts):

An automatic filling machine is used to fill bottles with liquid detergent. A random sample of 20 bottles results in a sample variance of fill volume of 0.0153 (fluid ounces)². If the variance of fill volume exceeds 0.01 (fluid ounces)², an unacceptable proportion of bottles will be underfilled or overfilled. Is there evidence in the sample data to suggest that the manufacturer has a problem with underfilled or overfilled bottles? Use 0.05 level of significant, and assume that fill volume has a normal distribution.

- a. The test hypotheses are:

$$H_0: \sigma^2 \leq 0.01 \quad \text{vs.} \quad H_A: \sigma^2 > 0.01 \quad \} \text{ (2 pts)}$$

- b. The assumption is:

The population is normally distributed. } (1 pt)

- c. The critical value is:

$$\chi^2_{\alpha, n-1} = \chi^2_{0.05, 19} = 30.1435 \quad \} \text{ (1 pt)}$$

- d. The decision rule using test statistic approach is:

Reject H_0 if $\chi^2_c > \chi^2_{\alpha, n-1}$. } (1 pt)

- e. The test statistic is:

$$\chi^2_c = \frac{(n-1)S^2}{\sigma_0^2} = \frac{(19)(0.0153)}{0.01} = 29.07 \quad \} \text{ (2 pts)}$$

- f. The decision is:

Since $\chi^2_c = 29.07 \not> \chi^2_{\alpha, n-1} = 30.1435$ } (1 pt)

Do NOT reject H_0

- g. The conclusion is:

Based on the sample information, the manufacturer has NO problem with under filled or over filled bottles } (1 pt)

Question Five (2+1+3+3+1+1 = 11 pts):

A manager believed that at most 18 percent of the company's employees work overtime every week. If the observed proportion this week is 23 percent in a sample of 250 of the 2,500 employees, can you accept his belief as reasonable or must you conclude that other values is more appropriate?? Use 0.025 level of significant

- a. The test hypotheses are:

$$H_0: p \leq 0.18 \quad \text{vs.} \quad H_A: p > 0.18 \quad \left. \vphantom{H_0} \right\} \text{2 pts}$$

$$n = 250, \quad \bar{p} = 0.23$$

- b. The assumptions are:

$$\left. \begin{array}{l} 1. n p_0 \geq 5 \Rightarrow (250)(0.18) = 45 \geq 5 \\ 2. n(1-p_0) \geq 5 \Rightarrow (250)(1-0.18) = 205 \geq 5 \end{array} \right\} \text{1 pt}$$

- c. The test statistic is:

$$Z_c = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.23 - 0.18}{\sqrt{\frac{(0.18)(1-0.18)}{250}}} = 2.06 \quad \left. \vphantom{Z_c} \right\} \text{3 pts}$$

- d. The p-value is:

$$\begin{aligned} p\text{-value} &= P(Z > Z_c) \\ &= P(Z > 2.06) \\ &= 0.5 - P(0 < Z < 2.06) \\ &= 0.5 - 0.4803 = 0.0197 \end{aligned} \quad \left. \vphantom{p\text{-value}} \right\} \text{3 pts}$$

- e. The decision is:

$$\begin{aligned} &\text{Reject } H_0 \text{ if the } p\text{-value} < \alpha \\ &0.0197 < 0.025 \\ &\therefore \text{Reject } H_0. \end{aligned} \quad \left. \vphantom{\text{Reject } H_0} \right\} \text{1 pt}$$

- f. Your conclusion is:

Based on the sample information, there is NO evidence to conclude that at most 18% of the company's employees work overtime every week. 1 pt

Question Six (2+2+1+1+5+1+1 = 13 pts):

A random sample of 500 adult residents of Al-Dammam city found that 385 were in favor of increasing the highway speed limit to 75 mph, while another sample of 400 adult residents of Al-Riyadh city found that 267 were in favor of the increased speed limit. Do these data indicate that there is a difference in the support for increasing the speed limit between the residents of the two counties? Use $\alpha = 0.05$.

Let Sample 1 = Al-Dammam
Sample 2 = Al-Riyadh.

a. The test hypotheses are:

$$H_0: p_1 - p_2 = 0$$

vs.

$$H_A: p_1 - p_2 \neq 0$$

$$\bar{p}_1 = \frac{x_1}{n_1} = \frac{385}{500} = 0.77 \quad \text{(3 pts)}$$

$$\bar{p}_2 = \frac{x_2}{n_2} = \frac{267}{400} = 0.6675$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{385 + 267}{500 + 400} = 0.7244$$

(2 pts)

b. The assumptions are:

$$1. n_1 \bar{p}_1 = 500(0.77) = 385 \geq 5, \quad n_1(1 - \bar{p}_1) = 500(0.23) = 115 \geq 5 \quad \checkmark$$

$$2. n_2 \bar{p}_2 = 400(0.6675) = 267 \geq 5, \quad n_2(1 - \bar{p}_2) = 400(0.3325) = 133 \geq 5 \quad \checkmark \quad \text{(2 pts)}$$

c. The critical values are:

$$Z_{\alpha/2} = Z_{0.025} = 1.96 \quad \text{(1 pt)}$$

d. The decision rule using test statistic approach is:

$$\text{Reject } H_0 \text{ if } |Z_c| > Z_{\alpha/2} = 1.96 \quad \text{(1 pt)}$$

e. The test statistic is:

$$Z_c = \frac{\bar{p}_1 - \bar{p}_2 - 0}{\sqrt{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.77 - 0.6675 - 0}{\sqrt{(0.7244)(1 - 0.7244)\left(\frac{1}{500} + \frac{1}{400}\right)}} \quad \text{(2 pts)}$$

$$= \frac{0.1025}{0.029973336} = 3.42$$

f. The decision is:

$$\text{since } |Z_c| = 3.421 > Z_{\alpha/2} = 1.96 \quad \text{(1 pt)}$$

$$\therefore \text{Reject } H_0.$$

g. The conclusion is:

Based on samples information, there is a difference in the support for increasing the speed limit between the residents of the two counties. (1 pt)