

Name: _____

Key

I.D.# _____

Serial # _____

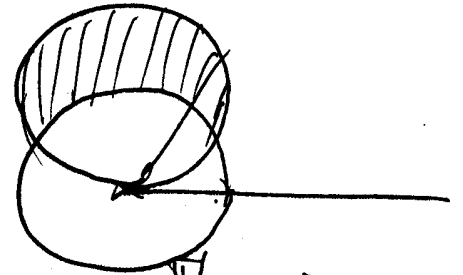
Q1: Find the area inside the polar curve $r = 2 \sin \theta$ and outside the curve

$$r = 1 \quad 2 \sin \theta = 1 \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$A = \frac{1}{2} \int_{\pi/6}^{5\pi/6} [4 \sin^2 \theta - 1] d\theta$$

$$= \int_{\pi/6}^{\pi/2} [2 - 2 \cos 2\theta - 1] d\theta$$

$$= \int_{\pi/6}^{\pi/2} (1 - 2 \cos 2\theta) d\theta = \left[\theta - \sin 2\theta \right]_{\pi/6}^{\pi/2} = \frac{\pi}{3} - 0 + \frac{\sqrt{3}}{2}$$



Q2: Find equation of the sphere that is centered at $(2, 1, 0)$, and tangent to the z -axis.

tangent to z -axis $\Rightarrow r = \sqrt{4 + 1} = \sqrt{5}$

$$(x-2)^2 + (y-1)^2 + z^2 = 5$$

Q3: Find the magnitude of $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$, where $\mathbf{u} = \langle 1, 2, -1 \rangle$ and $\mathbf{v} = \langle -2, 1, 0 \rangle$.

$$\vec{u} + \vec{v} = \langle -1, 3, -1 \rangle, \quad \vec{u} - \vec{v} = \langle 3, 1, -1 \rangle$$

$$\|\vec{u} + \vec{v}\| = \sqrt{1 + 9 + 1} = \sqrt{11}$$

$$\|\vec{u} - \vec{v}\| = \sqrt{9 + 1 + 1} = \sqrt{11}$$