

King Fahd University of Petroleum and Minerals  
Department of Mathematics  
Math 101 Fall 2003

Final Exam **A**

Tuesday 13 / 1 / 2004

Time  $2\frac{1}{2}$  hours

Name: \_\_\_\_\_ I.D.#: \_\_\_\_\_ Serial #: \_\_\_\_\_

Section #:

15

17

Answer all the question

For the multiple choice questions put your choice in this table

Question #	a	b	c	d	e
1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e
13	a	b	c	d	e
14	a	b	c	d	e
15	a	b	c	d	e
16	a	b	c	d	e

Question #	17	18	19	20	21	Total
Grade	/5	/4	/5	/4	/4	/22

1.  $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - 1}}{3x - 2}$

a.  $\frac{2}{3}$

b.  $\frac{4}{3}$

c.  $\frac{4}{2}$

d.

e.  $\frac{2}{3}$

2. For the function  $f(x) = x^4 - 2x^3 - 3$ , then only one of the following is TRUE:

a. At the point  $\left(\frac{3}{2}, \frac{21}{16}\right)$ ,  $f(x)$  has relative minimum

b. At the point  $(0, 3)$ ,  $f(x)$  has relative maximum

c. At the point  $\left(\frac{3}{2}, \frac{21}{16}\right)$ ,  $f(x)$  has relative maximum

d. At the point  $(0, 3)$ ,  $f(x)$  has relative minimum

e. At the point  $\left(\frac{3}{2}, \frac{21}{16}\right)$ ,  $f(x)$  has no relative extrema.

3. Let  $f(x) = \frac{x^2}{x^3 - 3}$ . If  $f^{-1}(x) = 2$ , then  $x$  is equal:

a.  $\frac{12}{15}$

b. 1

c.  $\frac{8}{5}$

d.  $\frac{4}{11}$

e.  $\frac{4}{15}$

4. Let the function  $f(x) = \begin{cases} \frac{\sin kx}{\sin 3x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$ . The value of  $k$  so that  $f(x)$  is continuous at  $x = 0$  is :

a. 2

b. 4

c. 6

d. 1

e. 3

5. Let  $f(x) = x^3 - 2x^2 - 4x + 2$ . Then the equation of the tangent to the graph of  $f(x)$  at  $x = 1$  is :

a.  $y - 3x + 3 = 0$

b.  $y - 3x + 2 = 0$

c.  $y - 3x + 2 = 0$

d.  $y - 3x + 4 = 0$

e.  $y - 3x + 3 = 0$

6.  $\lim_{x \rightarrow 0} \frac{3x - x \sin 5x}{x^2 - 2x - 3}$

a. 0

b. 3

c. does not exist

d. 1

e.

7.  $\lim_{x \rightarrow 1} \frac{3^x - 3}{x - 1}$

a.  $\ln 3$

b.  $3^3$

c.  $\ln 3^2$

d. 3

e.  $3 \ln 3$

8. Let  $f(x) = \begin{cases} ax^2 + b & \text{if } x < 1 \\ 4x & \text{if } x \geq 1 \end{cases}$ , for  $f(x)$  to be differentiable at  $x = 1$ , the values of  $a$  and  $b$  satisfy that  $a - 2b$  is equal to :

a. 2

b. 5

c. 3

d. 6

e. 4

9. Let  $f(x) = ke^x$ , then the value of  $k$  so that the line  $y = ex$  is tangent to  $f(x)$  is:

a. 2

b. 3

c. 1

d. 2

e. 1

10. Given  $f(x) = \frac{3}{x^2 - 1}$ , and  $g(x) = \sqrt{2x - 3}$ . Let  $F(x) = f(g(x))$ , then  $\frac{dF(x)}{dx}$  :

a.  $\frac{2}{x - 2\sqrt{2x - 3}}$

b.  $\frac{3}{2x - 4\sqrt{2x - 3}}$

c.  $\frac{6}{2x - 3\sqrt{2x - 3}}$

d.  $\frac{1}{x - 3\sqrt{2x - 3}}$

e.  $\frac{1}{2x - 1\sqrt{2x - 3}}$

11. A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 4 ft/s. Then the rate at which the area enclosed by the ripple is increasing at the end of 8 seconds is:

a. 256 .

- b. 128 .
- c. 16 .
- d. 32 .
- e. 512 .

12. The radius  $r$  of a sphere is measured with a percentage error of  $0.03\%$  . Then the estimated percentage error in the calculated surface area  $S$  of the sphere is : (where  $S = 4\pi r^2$ )

- a.  $0.09\%$
- b.  $0.06\%$
- c.  $0.03\%$
- d.  $0.6\%$
- e.  $0.3\%$

13. Let  $f(x) = 3x^3 - 3x + 4$ , then  $\frac{df^{-1}(2)}{dx}$  :

- a.  $\frac{1}{2}$
- b.  $\frac{1}{26}$
- c.  $\frac{1}{12}$
- d.  $\frac{1}{39}$
- e.  $\frac{1}{16}$

14. If  $y = \sec^{-1}(2x^3)$ , then  $\frac{dy}{dx}$  is equal to :

- a.  $\frac{1}{|2x^3| \sqrt{4x^6 - 1}}$
- b.  $\frac{1}{|2x^3| \sqrt{1 - 4x^6}}$
- c.  $\frac{3}{|x| \sqrt{4x^6 - 1}}$
- d.  $\frac{1}{|x| \sqrt{4x^6 - 1}}$

e.  $\frac{3}{\sqrt{1-4x^6}}$ .

15.  $\lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{e^x - 1}$  :

a. does not exist

b. 1

c.  $\frac{1}{2}$

d. 2

e. 1

16. Let  $f(x) = x^{\frac{2}{3}} - 6x$ ,  $x \in [1, 8]$ , then :

a. The absolute minimum of  $f(x)$  is  $-8$ .

b. The absolute maximum of  $f(x)$  is  $4\sqrt[3]{2}$ .

c. The absolute minimum of  $f(x)$  is  $0$ .

d. The absolute minimum of  $f(x)$  is  $-21$ .

e. The absolute maximum of  $f(x)$  is  $6$ .

### SOLVING PART

17. Find the dimensions of the rectangle of maximum area that can be inscribed in a semicircle of radius 10. One side of the rectangle is to sit on the diameter of the semicircle.

18. Use the mean value theorem to show that  $\tan x > x$ , for  $0 < x < \frac{\pi}{2}$

19. graph the function showing all necessary details, where

$$f(x) = \frac{x^2 - 1}{x^3}, \text{ and if } f(x) = 0 \text{ then } x = 1 \text{ or } -1,$$

$$f(x) = \frac{3x^2}{x^4}, \text{ and if } f(x) = 0 \text{ then } x = \sqrt{3} \text{ or } -\sqrt{3}$$

$$f(x) = \frac{2x^2 - 6}{x^5}, \text{ and if } f(x) = 0 \text{ then } x = \sqrt{6} \text{ or } -\sqrt{6}$$

$$f(\sqrt{3}) = 0.4, f(-\sqrt{3}) = 0.4, f(\sqrt{6}) = 0.34, f(-\sqrt{6}) = 0.34$$

20. Let  $f(x, y) = f(x) + f(y) - 3xy$ , and  $\lim_{h \rightarrow 0} \frac{f(h)}{h} = 7$ . Find  $f(x)$ .

21.  $\lim_{x \rightarrow 0} \sin x^{\frac{2}{\ln x}}$