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Noether Symmetries Versus Killing Vectors and Isometries of Spacetimes

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Symmetries of spacetime manifolds which are given by Killing vectors are compared with the symmetries of the Lagrangians of the respective spacetimes. We find the point generators of the one parameter Lie groups of transformations that leave invariant the action integral corresponding to the Lagrangian (Noether symmetries). In the examples considered, it is shown that the Noether symmetries obtained by considering the Larangians provide additional symmetries which are not provided by the Killing vectors. It is conjectured that these symmetries would always provide a larger Lie algebra of which the KV symmetres will form a subalgebra.

KEY WORDS: Noether symmetries; isometries of spacetimes; Lie algebras.
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1. INTRODUCTION

The Einstein field equations which govern the general theory of relativity (GR) are described in terms of the 4-Lorentzian metric g_{ab} and are highly non-linear equations. It has therefore been one of the fundamental problems in GR to find and understand solutions of the Einstein field equations through the symmetries they possess, see Meisner *et al*. (1973). These symmetries are given by Killing vectors (KVs): a KV is the one along which the Lie derivative of the metric is zero. Since these symmetries are pivotal to understand the physics of the gravitational fields, they have been throughly investigated and by now a large body of literature is available on them (Petrov, 1969). As fas as the KVs are concerned, they form a finite dimensional Lie group for the spacetime metric being non-degenerate. On the one hand the metric conservation laws are pivotal to study the symmetry groups admitted by them, there are other tensors of more physical interest whose

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