Algebraic computations for spinors in general relativity(*)

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Summary. — To be able to deal with spinors in general relativity an algebraic computational package to compute the complex spinor basis, the components of the position vector, metric, Riemann, Ricci and Weyl spinors, first scalar invariant and spin coefficients (in terms of null tetrad) is presented.

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1. – Introduction

Asghar *et al.* [1] at the Quaid-i-Azam University (QAU) have developed an algebraic package for the calculation of several tensorial quantities used in relativity. This package, written in MATHEMATICA [2], computes the components of Christoffel symbols, Riemann, Ricci, Weyl, and Einstein tensors and scalar invariants in the usual tensorial form, for any space of dimension ≥ 2 . Here in this paper we extend this work to deal with 2-component spinors. With this package one can compute the complex spinor basis, the components of the position vector, metric, Riemann, Ricci and Weyl spinors, first scalar invariant and spin coefficients (in terms of null tetrad). For comparison and analysis, the Riemann and Ricci tensors and Ricci scalar are also calculated in the usual four-vector formalism. This programme can be implemented on any personal computer with MATHEMATICA installed.

In general relativity the formalism most commonly used for the mathematical treatment of manifolds and their metrics is the tensor calculus, whereas in the specific case of four dimensions with a Lorentzian metric there is another formalism: the formalism of 2-component spinors. Although the general 2-component spinor calculus was first introduced some seventy years back by Cartan, people are not too familiar with it. Keeping

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