

A Complete Classification of Curvature Collineations of Cylindrically Symmetric Static Metrics

Ashfaque H. Bokhari,^{1,2} Abdul R. Kashif,¹ and Asghar Qadir^{1,2}

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Curvature collineations are symmetry directions for the Riemann tensor, as isometries are for the metric tensor and Ricci collineations are for the Ricci tensor. Complete listings of many metrics possessing some minimal symmetry have been given for a number of symmetry groups for the latter two symmetries. It is shown that a claimed complete listing of cylindrically symmetric static metrics by their curvature collineations [1] was actually incomplete and is completed here. It turns out that in this complete list, unlike the previous claim, there are curvature collineations that are distinct from the set of isometries and of Ricci collineations. The physical interpretation of some of the metrics obtained is given.

KEY WORDS: Cylindrically symmetric spacetime; curvature collineation.

In General Relativity one normally solves the highly non-linear Einstein equations for specific non-homogeneous (source) terms, to obtain physical consequences of relevant matter-energy distributions. An alternative approach is to specify the symmetries obeyed by the matter-energy distribution and try to look for all spacetimes possessing that symmetry [2]. If a complete listing can be found, we can use the Einstein equations to define the stress-energy tensor, and find the appropriate metric in the list. The geometric symmetries of a spacetime are given by the isometries, or *Killing vectors* (KVs), \mathbf{k} , satisfying the Killing equations for the metric coefficients, g_{ab} ,

$$L_k g_{ab} = g_{ab,c}k^c + g_{ac}k_{,b}^c + g_{cb}k_{,a}^c = 0.$$

¹Department of Mathematics, Quaid-i-Azam University Islamabad, Pakistan; e-mail: kashmology@yahoo.com

²Department of Mathematical Science, King Fahd University of Petroleum and Minerals, Dhahran, 31261, Saudi Arabia.