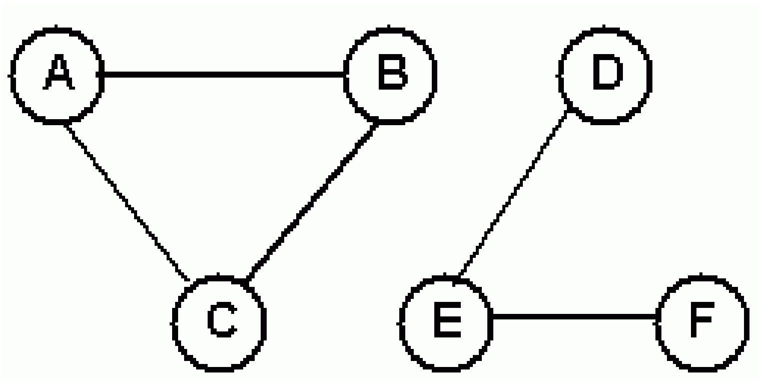
Testing for Connectedness and Cycles

- Connectedness of an Undirected Graph
- Implementation of Connectedness detection Algorithm.
- Implementation of Strong Connectedness Algorithm.
- Cycles in a Directed Graph.
- Implementation of a Cycle detection Algorithm.
- Review Questions.

Connectedness of an Undirected Graph

- An undirected graph G = (V, E) is connected if there is a path between every pair of vertices.
- Although the figure below appears to be two graphs, it is actually a single graph.
- Clearly, G is not connected. e.g. no path between A and D.
- G consists of two unconnected parts, each of which is a connected sub-graph --- connected components.



V = {A, B, C, D, E, F}

E = {{A, B}, {A, C}, {B, C}, {D, E}, {E, F}}

Implementation of Connectedness Algorithm

- A simple way to test for connectedness in an undirected graph is to use either depth-first or breadth-first traversal - Only if all the vertices are visited is the graph connected. The algorithm uses the following visitor:

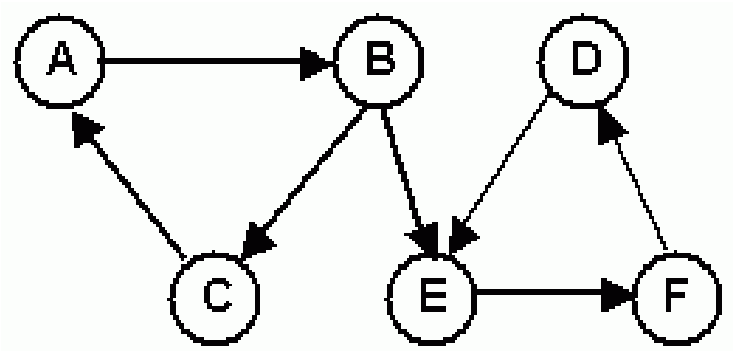
```
public class CountingVisitor extends AbstractVisitor {
   protected int count;
   public int getCount(){ return count;}
   public void visit(Object obj) {count++;}
}
```

- Using the CountingVisitor, the isConnected method is implemented as follows:

```
public boolean isConnected() {
   CountingVisitor visitor = new CountingVisitor();
   Iterator i = getVertices();
   Vertex start = (Vertex) i.next();
   breadthFirstTraversal(visitor, start);
   return visitor.getCount() == numberOfVertices;
}
```

Connectedness of a Directed Graph

- A directed graph G = (V, E) is strongly connected if there is a directed path between every pair of vertices.
- Is the directed graph below connected?
  - G is not strongly connected. No path between any of the vertices in {D, E, F}
  - However, G is weakly connected since the underlying undirected graph is connected.



V = {A, B, C, D, E, F}

E = {(A, B), (B, C), (C, A), (B, E), (D, E), (E, F), (F, D)

Implementation of Strong Connectedness Algorithm

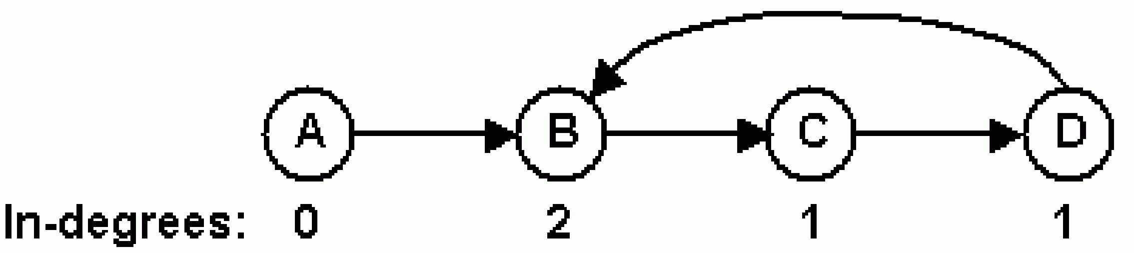
- A simple way to test for strong connectedness is to use |V| traversals - The graph is strongly connected if all the vertices are visited in each traversal.

```
public boolean isStronglyConnected() {
   if (!this.isDirected())
      throw new InvalidOperationException(
                          "Invalid for Undirected Graph");
   Iterator it = getVertices();
   while(it.hasNext()) {
      CountingVisitor visitor = new CountingVisitor();
      breadthFirstTraversal(visitor, (Vertex) it.next());
      if(visitor.getCount() != numberOfVertices)
         return false;
   }
   return true;
}
```

- Implementation of weak connectedness is done in the Lab.

Cycles in a Directed Graph

- An easy way to detect the presence of cycles in a directed graph is to attempt a topological order traversal.
  - This algorithm visits all the vertices of a directed graph if the graph has no cycles.
- In the following graph, after A is visited and removed, all the remaining vertices have in-degree of one.
- Thus, a topological order traversal cannot complete. This is because of the presence of the cycle { B, C, D, B}.




```
public boolean isCyclic()  {
   CountingVisitor visitor = new CountingVisitor();
   topologicalOrderTraversal(visitor);
   return visitor.getCount() != numberOfVertices;
}
```

Review Questions

1. Every tree is a directed, acyclic graph (DAG), but there exist DAGs that are not trees.

   a) How can we tell whether a given DAG is a tree?

   b) Devise an algorithm to test whether a given DAG is a tree.

2. Consider an acyclic, connected, undirected graph G that has n vertices. How many edges does G have?

3. In general, an undirected graph contains one or more connected components.

   a) Devise an algorithm that counts the number of connected components in a graph.

   b) Devise an algorithm that labels the vertices of a graph in such a way that all the vertices in a given connected component get the same label and vertices in different connected components get different labels.

4. Devise an algorithm that takes as input a graph, and a pair of vertices, v and w, and determines whether w is reachable from v.