AVL Search Trees

- Inserting in an AVL tree

- Insertion implementation

- Deleting from an AVL tree

Insertion

- Insert using a BST insertion algorithm.
- Rebalance the tree if an imbalance occurs.
- An imbalance occurs if a node's balance factor changes from -1 to -2 or from+1 to +2.
- Rebalancing is done at the deepest or lowest unbalanced ancestor of the inserted node.

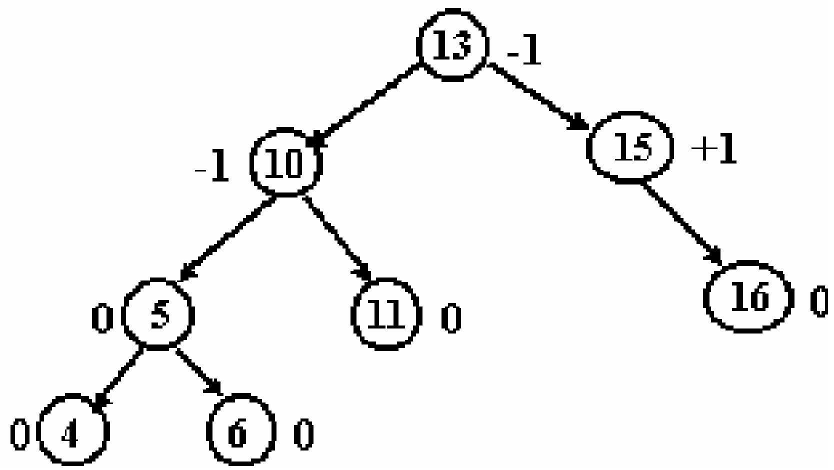
- There are three insertion cases:
  1. Insertion that does not cause an imbalance.

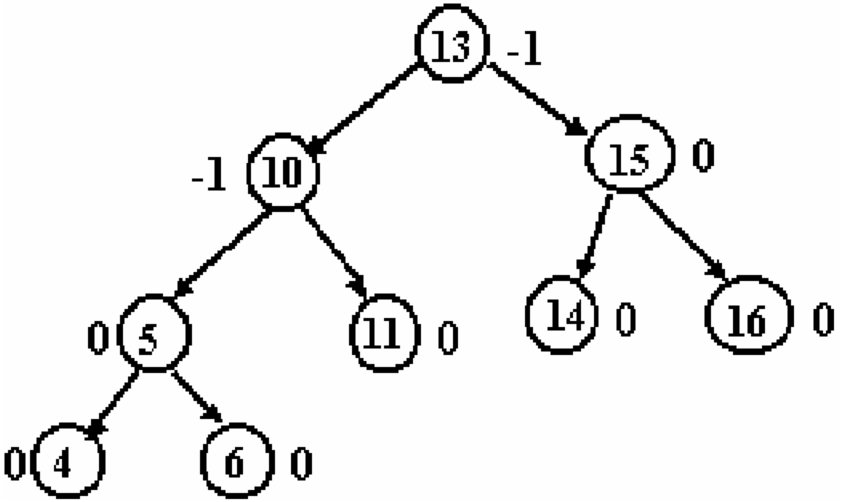
  2. Same side (left-left or right-right) insertion that causes an imbalance.
     - Requires a single rotation to rebalance.

  3. Opposite side (left-right or right-left) insertion that causes an imbalance.
     - Requires a double rotation to rebalance.

Insertion: case 1

- Example: An insertion that does not cause an imbalance.

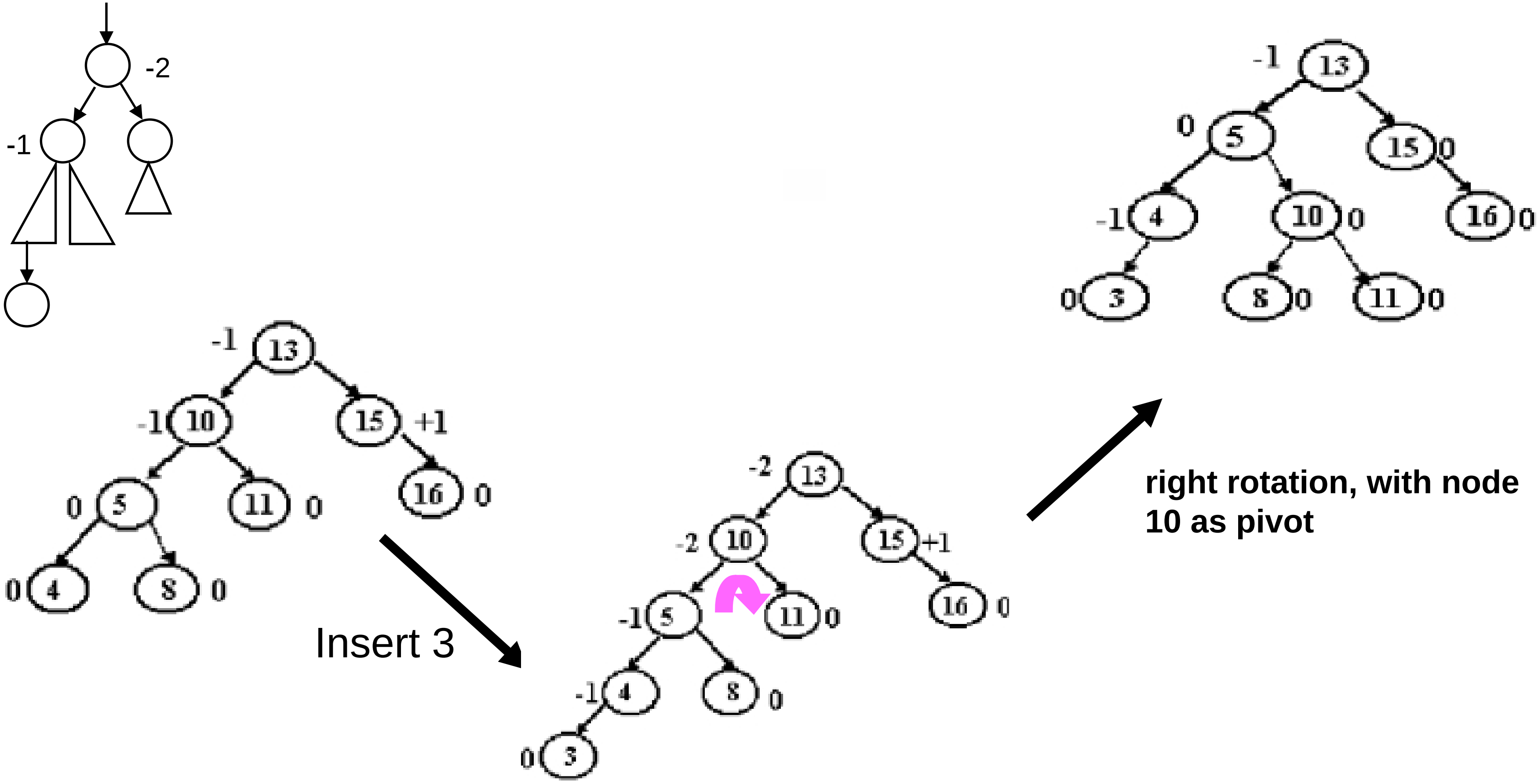




Insert 14

Insertion: case 2

- Case 2a: The lowest node (with a balance factor of -2) had a taller left-subtree and the insertion was on the left-subtree of its left child.
- Requires single right rotation to rebalance.

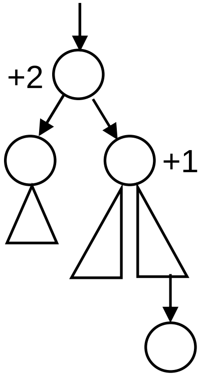


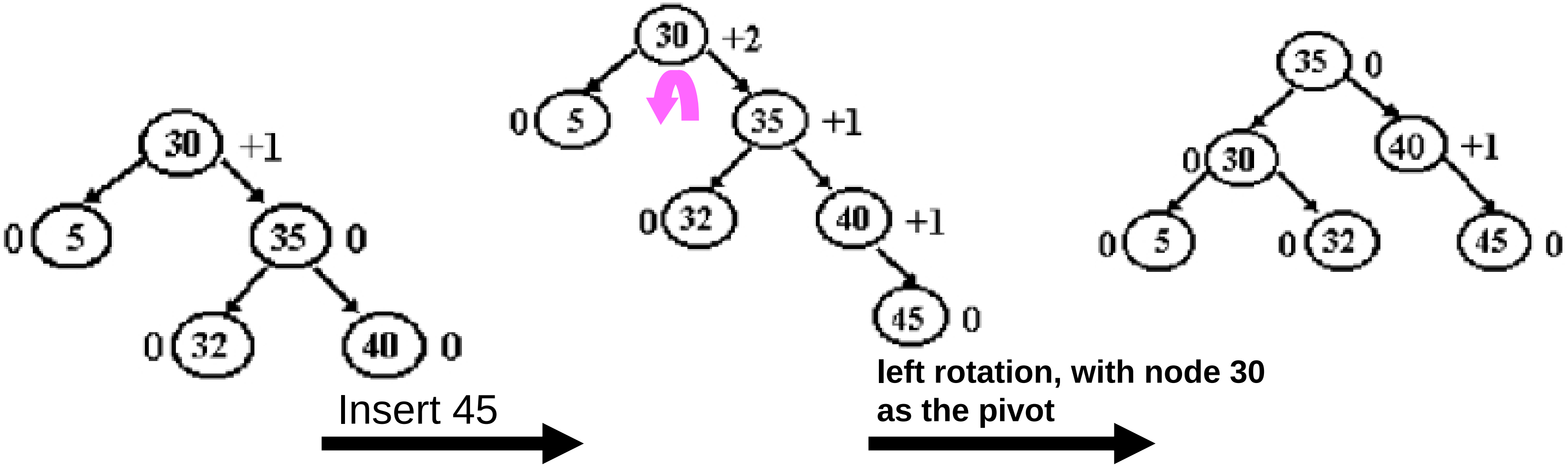
Insert 3

right rotation, with node 10 as pivot

Insertion: case 2 (contd)

- Case 2b: The lowest node (with a balance factor of +2) had a taller right-subtree and the insertion was on the right-subtree of its right child.
- Requires single left rotation to rebalance.



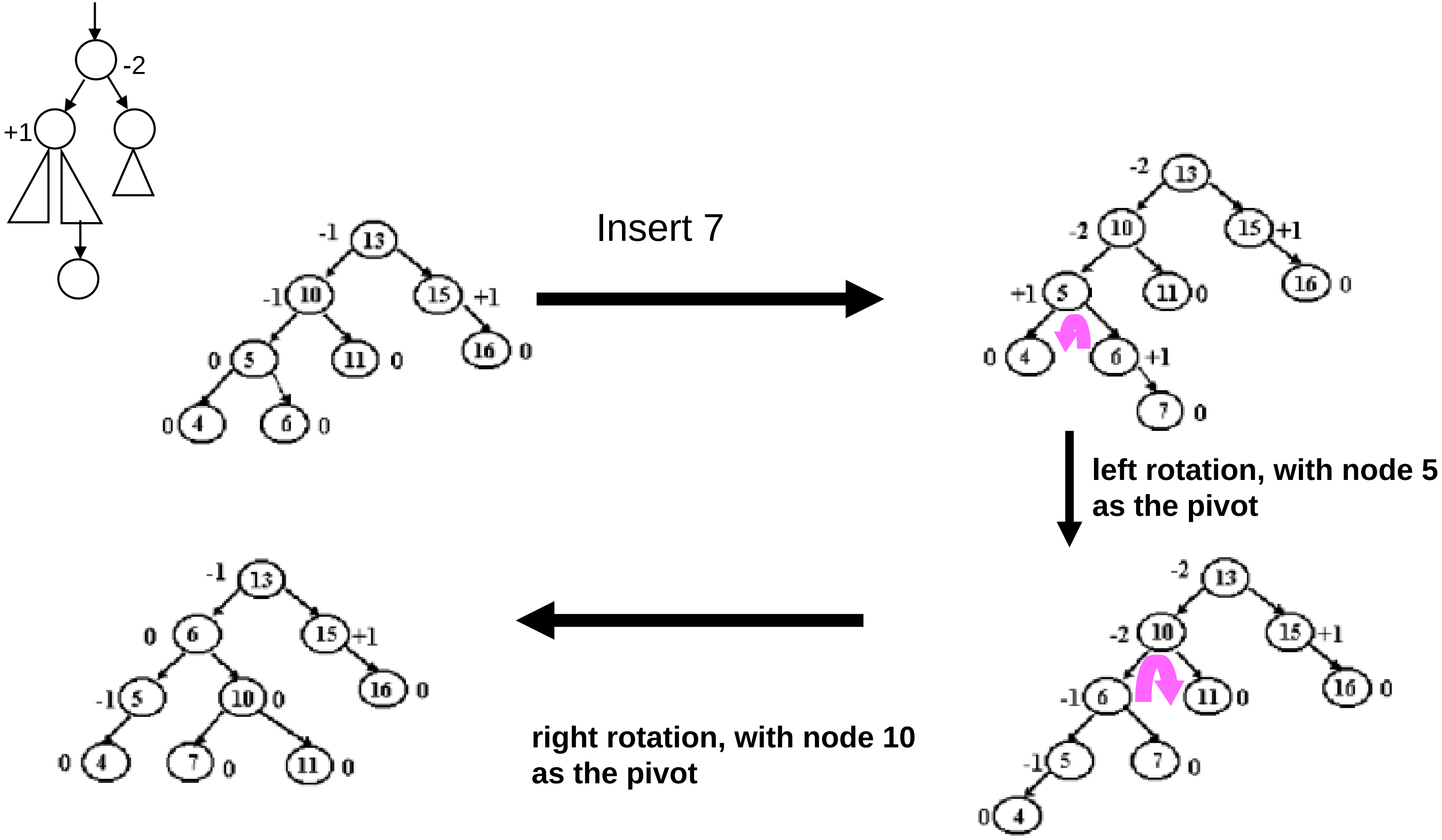


Insert 45

left rotation, with node 30 as the pivot

Insertion: case 3

- Case 3a: The lowest node (with a balance factor of -2) had a taller left-subtree and the insertion was on the right-subtree of its left child.
- Requires a double left-right rotation to rebalance.



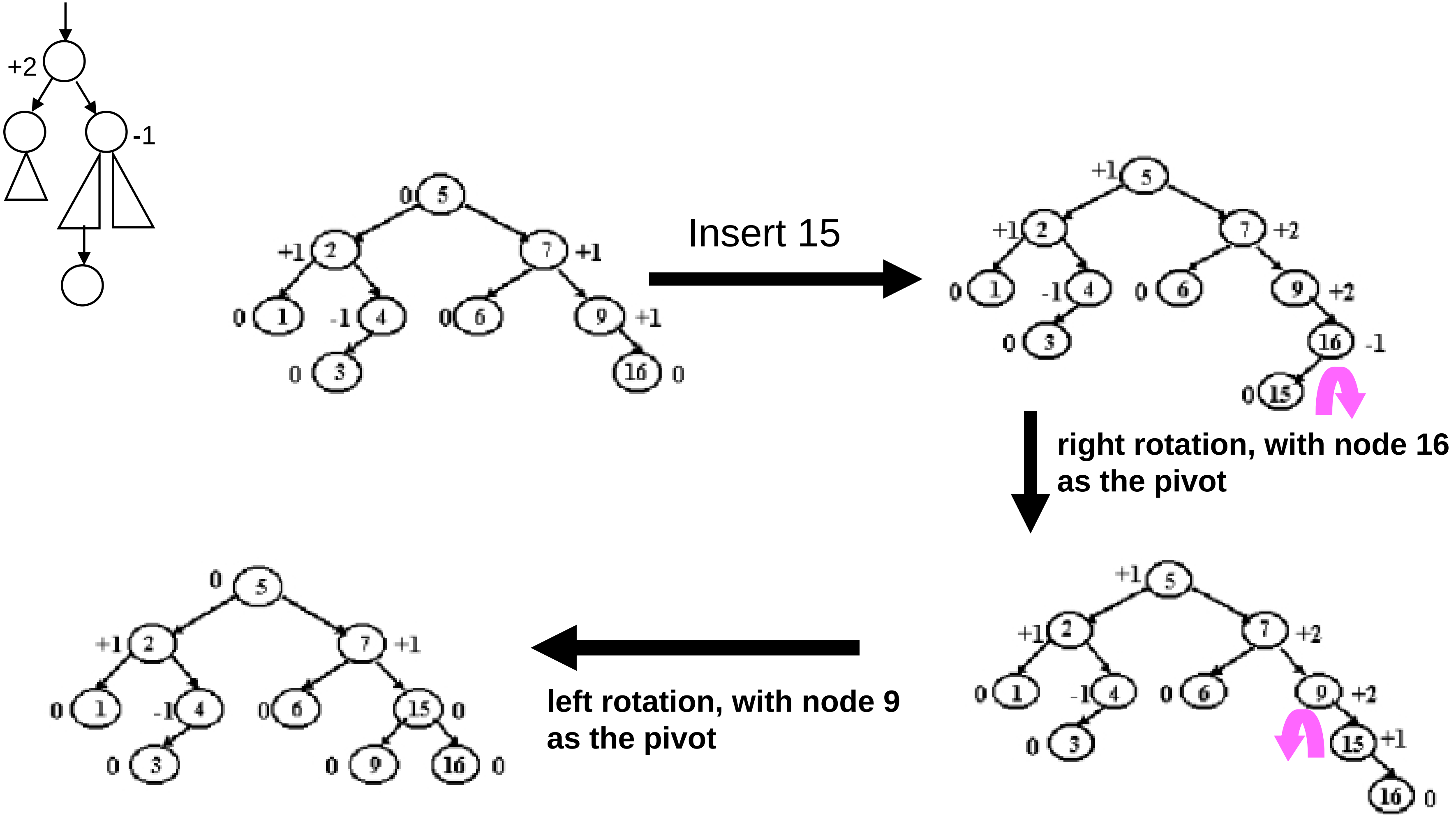
Insert 7

left rotation, with node 5 as the pivot

right rotation, with node 10 as the pivot

Insertion: case 3 (contd)

- Case 3b: The lowest node (with a balance factor of +2) had a taller right-subtree and the insertion was on the left-subtree of its right child.
- Requires a double right-left rotation to rebalance.

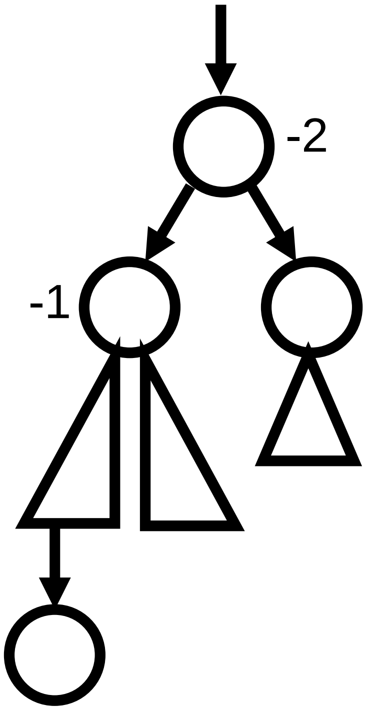


Insert 15

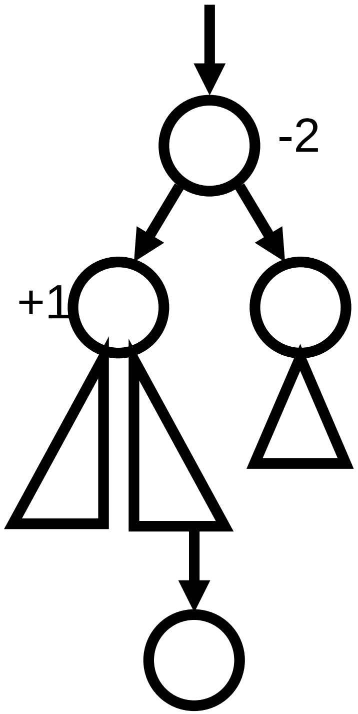
right rotation, with node 16 as the pivot

left rotation, with node 9 as the pivot

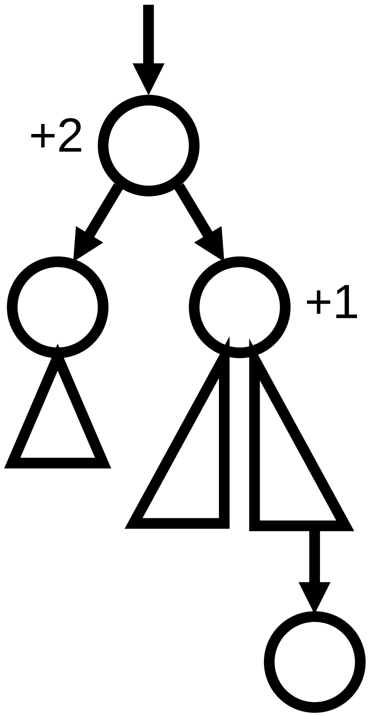
AVL Rotation Summary



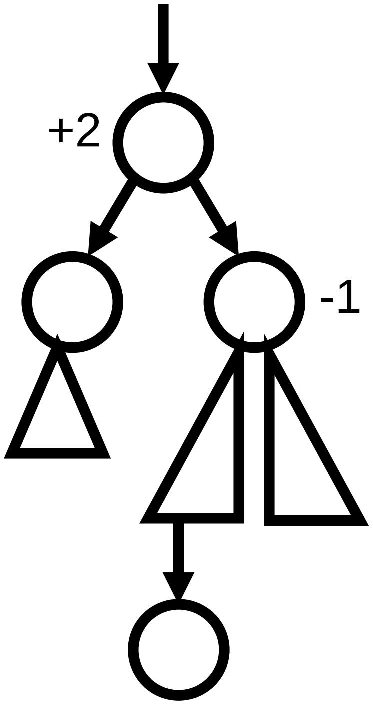
Single
right
rotation



Double
left-right
rotation



Single
left
rotation



Double
right-left
rotation

Insertion Implementation

- The insert method of the AVLTree class is:

```
public void insert(Comparable comparable){
       super.insert(comparable);
       balance();
}
```

Recall that the insert method of the BinarySearchTree class is:

```
public void insert(Comparable comparable){
  if(isEmpty()) attachKey(comparable);
  else {
     Comparable key = (Comparable) getKey();
     if(comparable.compareTo(key)==0)
       throw new IllegalArgumentException("duplicate key");
     else if (comparable.compareTo(key)<0)
       getLeftBST().insert(comparable);
     else
       getRightBST().insert(comparable);
    }
}
```

Insertion Implementation (contd)

- The AVLTree class overrides the attachKey method of the BinarySearchTree class:

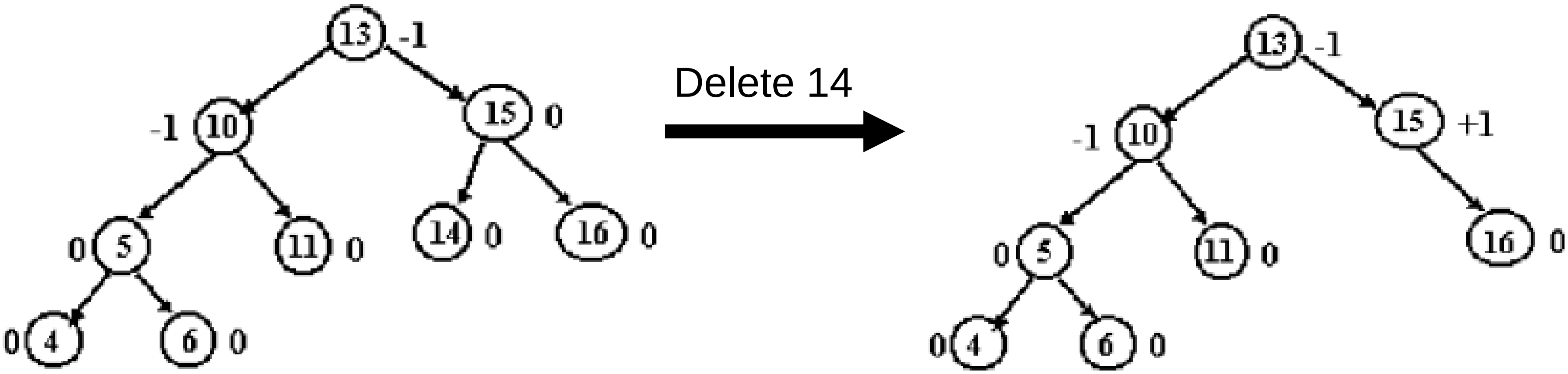
```
    public void attachKey(Object obj)
    {
        if(!isEmpty())
            throw new InvalidOperationException();
        else
        {
            key = obj;
            left = new AVLTree();
            right = new AVLTree();
            height = 0;
        }
    }
```

Insertion Implementation (contd)

```
protected void balance(){
    adjustHeight();
    int balanceFactor = getBalanceFactor();
    if(balanceFactor == -2){
        if(getLeftAVL().getBalanceFactor() < 0)
            rotateRight();
        else
            rotateLeftRight();
    }
    else if(balanceFactor == 2){
        if(getRightAVL().getBalanceFactor() > 0)
            rotateLeft();
        else
            rotateRightLeft();
    }
}
```

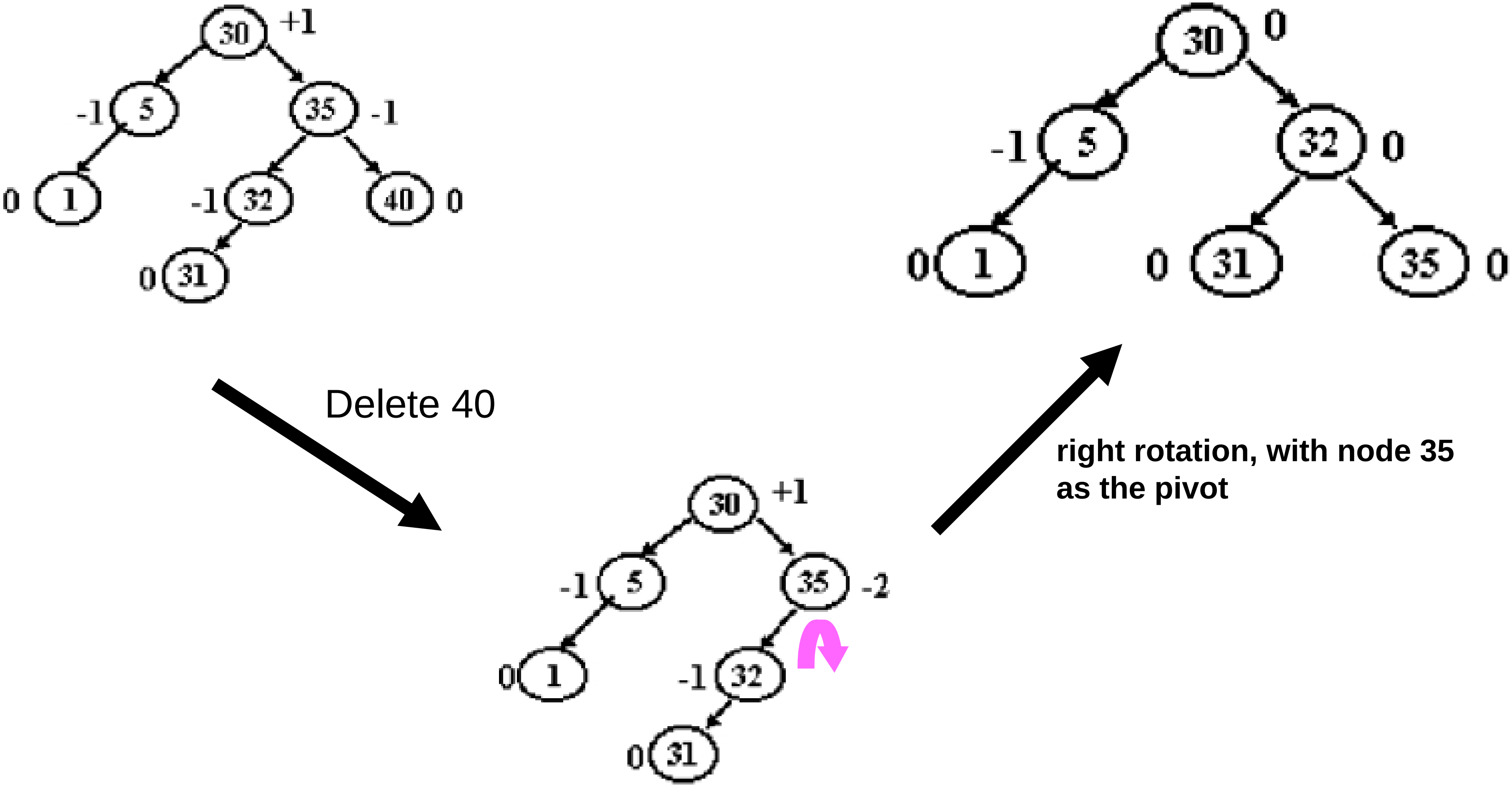
Deletion

- Delete by a BST deletion by copying algorithm.
- Rebalance the tree if an imbalance occurs.
- There are three deletion cases:
  1. Deletion that does not cause an imbalance.
  2. Deletion that requires a single rotation to rebalance.
  3. Deletion that requires two or more rotations to rebalance.
- Deletion case 1 example:



Delete 14

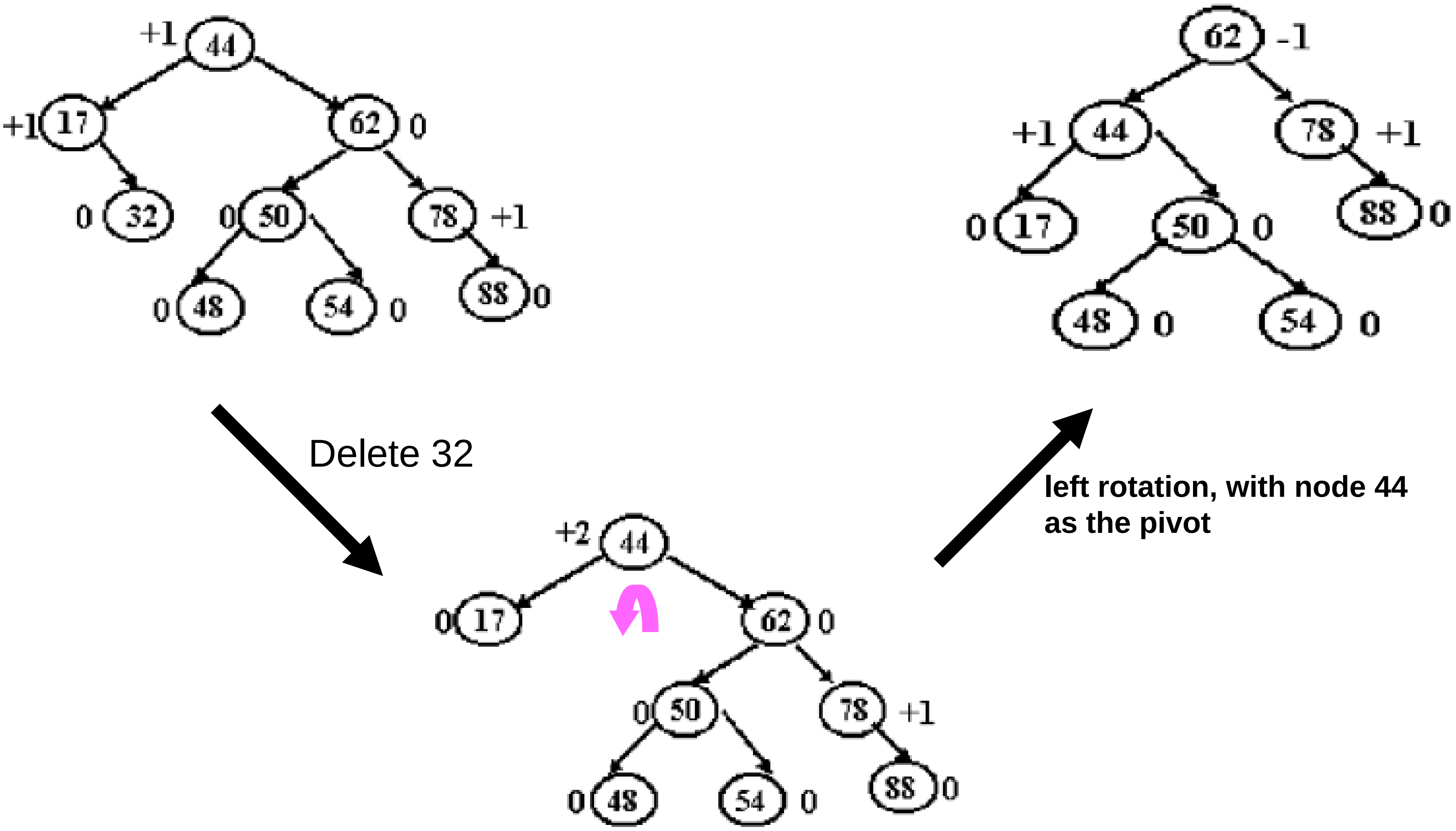
Deletion: case 2 examples



Delete 40

right rotation, with node 35 as the pivot

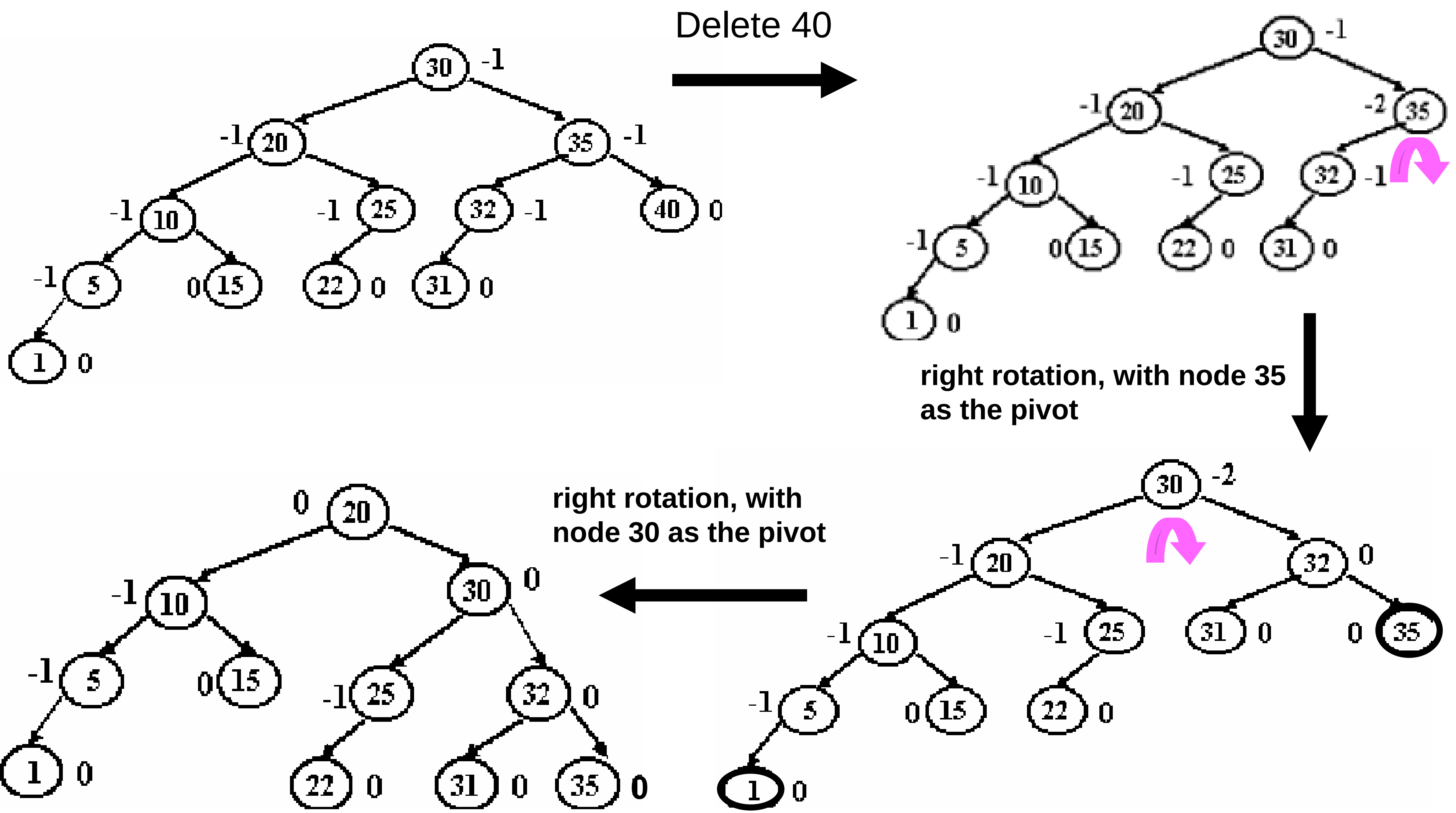
Deletion: case 2 examples (contd)



Delete 32

left rotation, with node 44 as the pivot

Deletion: case 3 examples



Delete 40

right rotation, with node 35 as the pivot

right rotation, with node 30 as the pivot