Efficient Convex-Elastic Net Algorithm to Solve the Euclidean Traveling Salesman Problem

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***Abstract*—This paper describes a hybrid algorithm that combines an adaptive-type neural network algorithm and a nondeterministic iterative algorithm to solve the Euclidean traveling salesman problem (E-TSP). It begins with a brief introduction to the TSP and the E-TSP. Then, it presents the proposed algorithm with its two major components: the convex-elastic net (CEN) algorithm and the nondeterministic iterative improvement (NII) algorithm. These two algorithms are combined into the efficient convex-elastic net (ECEN) algorithm. The CEN algorithm integrates the convex-Hull property and elastic net algorithm to generate an initial tour for the E-TSP. The NII algorithm uses two rearrangement operators to improve the initial tour given by the CEN algorithm. The paper presents simulation results for two instances of E-TSP: randomly generated tours and tours for well-known problems in the literature. Experimental results are given to show that the proposed algorithm can find the nearly optimal solution for the E-TSP that outperform many similar algorithms reported in the literature. The paper concludes with the advantages of the new algorithm and possible extensions.**

***Index Terms*—Neural network, optimization problems, traveling salesman problem (TSP).**

I. Introduction

There has been some interest in recent years in using hybrid algorithms that combine neural-network and operations research approaches to solve optimization problems [1]. The motivation for using these approaches is to improve the performance of the neural-network model and to generate feasible solutions, since neural-network approaches may not guarantee the generation of feasible solutions.

The traveling salesman problem (TSP) is a classical optimization problem which can be defined as follows. Let $G = (V, A)$ be a graph where $\boldsymbol{V}$ is a set of vertices and $\boldsymbol{A}$ is a set of arcs between vertices and each arc is associated with a non negative cost. The TSP consists of finding the tour of minimum length that passes through every vertex exactly once [2]. A special case of the TSP with triangular inequality is the Euclidean traveling salesman problem (E-TSP).

The E-TSP is to find a closed tour of minimum length through points that are given in two-dimensional space where the distances are computed according to the Euclidean metric. It can be defined formally as follows. Let $G = (V, A)$ be a graph where $\boldsymbol{V}$ is a set of $\boldsymbol{N}$ points in the plane, $\boldsymbol{A}$ is a set of edges between these points, and the Euclidean distance for each edge between point $\boldsymbol{i}$ and $\boldsymbol{j}$ is $\boldsymbol{C_{ij}}$. The E-TSP consists of determining a minimum distance tour passing through each point once [3].

Both the TSP and the E-TSP are NP-hard problems. In an instance with $\boldsymbol{N}$ cities, there are $(N!/2N)$ distinct tours [4]. Thus, it is unlikely that we can find a polynomial time algorithm for solving this problem exactly. Finding an efficient algorithm for the E-TSP that approaches the global minimum solution is a challenging task, since E-TSP contains several local minima that can mislead most algorithms. This paper is an attempt in that direction.

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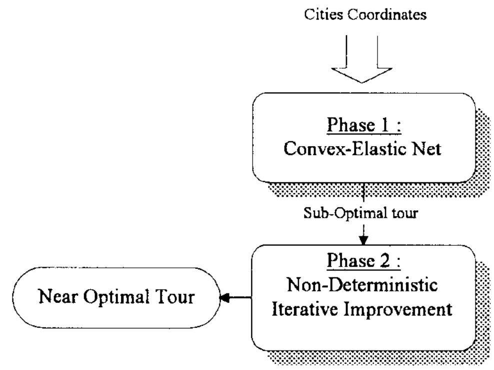

Fig. 1. The ECEN algorithm.

TABLE I
Average Tour Length Comparison for ECEN with Other Algorithms

	10 Cities	30 Cities	50 Cities	100 Cities
Optimal Tour	2.42-3.68	4.19-4.92	5.41-5.98	7.65-8.05
ECEN	2.47	4.32	5.57	8.04
Space Filing Curve	3.14	5.45	7.03	9.56
Hopfield and Tank	2.92	5.12	6.64	--
Guilty Net	2.78	4.81	6.21	8.81
Simulated Annealing	2.74	4.75	6.13	8.40
Elastic Net	2.45	4.51	5.58	8.23

II. The Efficient Convex-Elastic Net Algorithm

Many local search algorithms have been developed to solve the E-TSP. However, local search algorithms often get stuck at a local minima. To reduce the effect of local minima, we introduced an algorithm that consists of two phases: the first phase consists of the convex-elastic net (CEN) algorithm, and the second phase consists of the nondeterministic iterative improvement (NII) algorithm as shown in Fig. 1. The CEN is a global search algorithm that generates an initial tour for the second phase (local search) by enforcing the E-TSP constrains instead of transforming them into penalties. The NII is a local search algorithm that improves the tour found over time and local minimas are escaped through the noise added to the cost function. The two introduced algorithms are combined into a new one called the efficient convex-elastic net (ECEN) algorithm.

A. The Convex-Elastic Net Algorithm

In 1956, Merril Flood showed a general property for any optimal tour of the E-TSP [2]. This property is that, in any optimal solution for E-TSP, vertices located on the convex hull are visited in the order in which they appear on the convex-hull boundary. We propose the CEN algorithm that combines this property with the elastic net approach [5]. The proposed approach creates an initial tour as a rubber band that is originally shaped as the convex-hull of $\boldsymbol{N}$ cities. In each iteration the influence of all cities that are not on the rubber band on the nodes of the rubber band is computed and the nodes are displaced accordingly. A force, $\boldsymbol{F1}$, that keeps the rubber band nodes together is applied on each node and another force, $\boldsymbol{F2}$, pulls each node toward a city, such that the tour is stretched and more cities will be introduced to the rubber band. Once all cities are included in the rubber band, the tour found is a suboptimal solution for the given instance of the E-TSP as shown in Figs. 2 and 3.

TABLE II
SIMULATION RESULTS FOR SEVERAL PROBLEMS FROM THE OR LITERATURE

			Enhanced Convex-Elastic Net	
Problem	**Number of Cities**	**Optimal Tour Length**	**Tour Length**	**% from Optimal**
GRID100	100	100	100.8	0.8 %
KAR100	100	21,282	21,622.9	1.6 %
LIN318	318	44,169	46,231.7	4.7 %
GR96	96	55,209	57,634.1	4.4 %
GR137	137	69,853	72,150.4	3.2 %

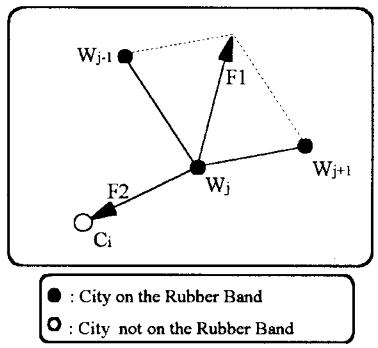

Fig. 2. Forces on the rubber band.

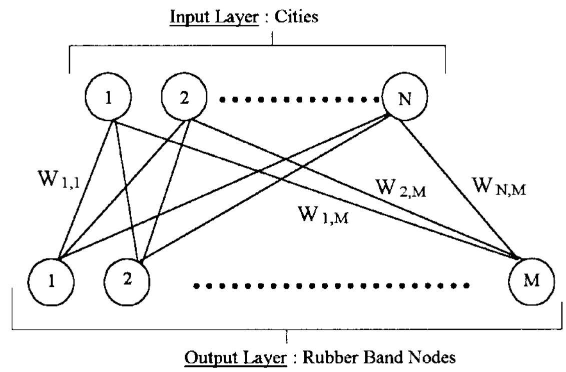

Fig. 4. The structure of the CEN algorithm.

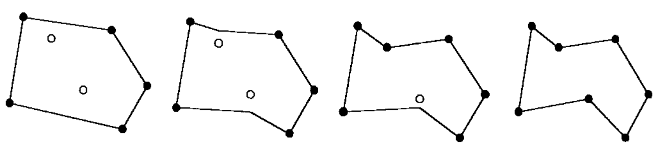

Fig. 3. Evolution of the CEN over time.

The CEN algorithm is an adaptive neural-based technique that is derived from the Kohonen's self-organizing feature maps approach. In this technique, the two-dimensional coordinates of cities are the input pattern and the similarity criteria is the Euclidean nearness. At each iteration, cities not on the rubber band are presented to the CEN two-layer network shown in Fig. 4. This figure shows that the CEN algorithm has N neurons and $M * N$ connections, where M is the maximum number of rubber band nodes and $M < 2N$. Therefore, the number of connections is $M * N \leq 2N^2$. When cities coordinates are presented to this network, a competition learning phase is initiated where the nearest prototype (rubber band node) to a particular pattern (city) is selected as the wining neuron.

B. The NII

The NII is developed to enhance the tour produced by the CEN algorithm. It uses a set of rearrangement operators to enhance a given tour. Each tour can be represented by a permutation list of numbers from 1 to N, where N is the number of cities. NII algorithm uses two types of powerful rearrangement operators: an operator for removing loops from a tour and an operator for changing the cities positions in a tour. These operators are simple rearrangement operators that are derived from two known heuristics, namely *2-optimal* (2-opt) [6] and the *point heuristic* [7]. These operators are used here to remove the intersecting paths from the tour produced by the *CEN algorithm.*

The NII algorithm starts with a given tour which is taken as the current tour. Then, one of the two rearrangement operators is applied to the current tour to get N new tours. Next, one of the new tours is selected based on its probability of selection. After that, the selected tour becomes the current tour. This process is continued until no further improvements can be found on the current tour.

The NII algorithm is developed to enhance the tour produced by the CEN algorithm. This is done by alternatively removing loops from the tour (reversing subtours) and moving a city from a position in the tour to another that reduces the tour length.

III. SIMULATION RESULTS

The proposed algorithm has been implemented using C language and coded on a Pentium 75 MHz PC. Since worst case analysis for E-TSP is as hard as finding the optimum, we performed probabilistic analysis only. The performance of our algorithm is tested against an instance of E-TSP where the cities coordinates are randomly generated within one unit square. Several test problems in this study are taken from the literature in order to compare our algorithm to the experimental results of other researchers.

In comparing our results to the optimal tour length, the expected length of the optimal tour which is found by Stein in 1977 is used. The *Stein* formula for N cities distributed uniformly over a unit square is given by

$$0.765 * \sqrt{N} \leq \text{Optimal tour length} \leq (0.765 + 4/N)^* \sqrt{N}.$$

In this study, $0.765 * \sqrt{N}$ was taken as the presumed lower bound optimal solution, and $(0.765 + 4/N) * \sqrt{N}$ was taken as the presumed upper bound optimal value [3].

Table I shows a comparison for the average tour length of several methods [8] with the ECEN algorithm for up to 100 cities, where 100 is the largest number of cities reported in the literature for neural network solutions of E-TSP. It is clear, from Table I, that the proposed ECEN algorithm produces better solutions than all of the other algorithms except the elastic net with ten cities, and the ECEN solutions are within the optimal solution bounds given by the Stein formula.

The proposed ECEN algorithm has been used to solve several problems from the operations research literature ranging in size from

96–318 cities and with known optimum solutions. These problems are GRID100 [9], KAR100 [7], LIN318 [6], GR96, and GR137 [10]. The ECEN algorithm found a tour for each one of these problems with a tour length that was never more than 5% longer than the optimum tour length. Table II shows the simulation results that compares the optimal tour length with tour length found by ECEN algorithm for these problems.

IV. Conclusion

In this paper, we have proposed the ECEN algorithm for solving the E-TSP. The ECEN algorithm has the following advantages. First, unlike the *Hopfield model*, it doesn't produce unfeasible solutions and there is no need to set the energy function parameters for each problem size. Second, it generates a suboptimal solutions that are better than the results generated by the neural-based techniques reported in literature such as *Hopfield model*, *guilty net*, and *elastic net.* Third, it scales well with problem size. Finally, it can escape from many local minima by exploring a larger number of solutions. This is similar to the *simulated annealing* technique in that it allows a local modification that increases the length of the tour with some probability of acceptance.

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A Visual Neural Classifier

Chester Ornes and Jack Sklansky

***Abstract*—A new neural classifier allows visualization of the training set and decision regions, providing benefits for both the designer and the user. We demonstrate the visualization capabilities of this *visual neural classifier* using synthetic data, and compare the visualization performance to Kohonen's self-organizing map. We show in applications to image segmentation and medical diagnosis that visualization enables a designer to refine the classifier to achieve low error rates and enhances a user's ability to make classifier-assisted decisions.**

***Index Terms*— Classifier, dimensionality, exploratory data analysis, multi-expert, multitask learning, neural network, reduction, visualization.**

I. Introduction

The visual neural classifier combines the information provided by several classification tasks into a visually meaningful and explanatory display. A user can interact with the display and obtain an explanation or confirmation of a classifier decision. A designer can identify difficult-to-classify input patterns that may then be applied to an additional classification stage.

Visualization is accomplished by a funnel-shaped multilayer dimensionality reduction network [2]. The dimensionality reduction network is configured to learn one or more classification tasks. If a single dimensionality reduction network does not provide sufficiently accurate classification results, a group of these dimensionality reduction networks may be arranged in a modular architecture [1]. Among these dimensionality reduction networks, we refer to those receiving the input data as *experts.* The dimensionality reduction network that combines the decisions of the experts to form the final classification decision is called a *visualization network*. Each dimensionality reduction network contains a two-neuron layer that displays the training data and the decision boundaries in a two-dimensional (2-D) space. This architecture facilitates a) interactive design of the decision function and b) explanation of the relevance of various training data to the classification decisions.

In the next three sections of this paper we describe the architecture of the visual neural classifier, the design of a visual neural classifier, and the motivation for using neural networks with two-neuron hidden layers. In Section V we describe some of the properties of the visual neural classifier. In Section VI we describe two disparate applications of the visual neural classifier: the diagnosis of mammograms and the segmentation of aerial images. The application to mammograms shows how the visual neural classifier can raise or lower a user's confidence in a classification. The application to aerial images demonstrates the ability of the visual neural classifier to enhance interactive design.

II. Architecture

Our neural classifier consists of two major parts: a set of experts and a visualization network. Each expert is a multilayer neural

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