

King Fahd University of Petroleum & Minerals
Information and Computer Sciences Department

ICS 556
Parallel Algorithms
Sections: 1 Winter 2007

Final Exam

Date: Jun. 9, 2007

Duration: 2 hours

Name:

ID#:

Please answer all of the following questions.

Make your answers exact and clearly understandable.

Write clearly any assumption you make throughout your answers.

Question 1: [16 marks]

- a) Let A be an array of n bits, and $*$ be any binary bit operation. Design an $O(\log n)$ -time algorithm that computes the result of $A[1] * A[2] * \dots * A[n]$ on an EREW PRAM computer with n processors. Explain why this algorithm is not cost-optimal.

- b) Modify your algorithm to make it cost-optimal.

Question 2: [16 marks]

a) Design an $O(\log n)$ -time algorithm that broadcasts a datum stored in a memory location called x to N processors in EREW PRAM computer.

b) Design a cost-optimal EREW PRAM algorithm that initializes an array A of size n such that $A[i] = x$, for all $i = 1, 2, \dots, n$, where x is a fixed datum saved in the memory.

Question 3: [16 marks]

a) Design a combinational circuit to compute the values $1, y, y^2, \dots, y^n$, where y is any real number and n is a positive integer. What is the cost of your algorithm?

b) Use your combinational circuit to design an EREW PRAM algorithm to evaluate the polynomial $P(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ at some point x_0 stored in the memory, i.e., to compute $p(x_0)$. Assume the values a_i are stored in an array A .

c) What is the running time of your algorithm? Is it cost optimal?

Question 4: [22 marks]

Given a rooted directed tree stored in the form of adjacency linked lists on a PRAM machine, explain how to do each of the following and state how much it costs.

- a) Computing the position (or order) of each edge in the linked list of Euler's tour of the rooted directed tree.

- b) Finding the parent of each vertex.

c) Computing the size of each subtree.

d) Numbering the leaves from left to right in the same order as they are visited by Euler's tour.

Question 5: [10 marks]

Prove that the maximal independent set problem belongs to NC^1 class by giving a CRCW PRAM randomized algorithm that solves the problem. Explain why it is in NC^1 .

Question 6: [20 marks]

- a) What is the running time and the cost of the comparison-exchange sorting algorithm on linear array network?
- b) We have studied a cost-optimal CRCW PRAM merge algorithm that merges two sorted arrays each of size at most N . What is its running time and how many processors does it use?
- c) Use the PRAM merge algorithm to design a cost-optimal CRCW PRAM mergesort algorithm to sort an array of size n . Hint: Divide (and conquer) the input into groups each of size $n/\log n$ and each to be sorted by a processor. Prove that your algorithm is cost-optimal.