ICS 353–Design and Analysis of Algorithms
Handout 1: Graphs and Trees

Definitions

1. **A graph** $G = (V, E)$ is a non-empty finite set of vertices V and a finite set of edges $E \subseteq V^2$ where each edge connects two vertices.
2. If the edges are directed (ordered pairs) then G is called **directed graph**, otherwise it is **undirected**.
3. Two vertices u and v are **adjacent to** each other if (u, v) is an edge, i.e. $(u, v) \in E$.
4. An edge e is **incident to** a vertex v if e is connected to to v, i.e. $e = (u, v)$ or (v, u) for some $u \in V$.
5. **The degree of a vertex** = the number of vertices adjacent to it = the number of edges incident to it.
6. **A path** is a sequence of vertices $v_1, v_2, \dots, v_n$ where each (v_i, v_{i+1}) is an edge, i.e. $(v_i, v_{i+1}) \in E$, for all $i = 1, \dots, n$.
7. **The length of a path** = the number of edges in it.
8. **A cycle** is a path $v_1, v_2, \dots, v_n$ where $v_1 = v_n$.
9. **A tree** is a connected graph that has no cycle.
10. **The size of a tree** is the number of vertices in it.
11. **A rooted tree** is a tree where a certain vertex is specified as a root where all other vertices are drawn away from the root (level by level). Every vertex has a parent except the root and vertices may have children except leaves.
12. **The height of a rooted tree** = the length of the longest path from the root to any leaf = The number of levels -1 .
13. **A binary tree** is a rooted tree where each vertex has at most two children called left child and right child.
14. **A complete binary tree** of height n is a binary tree where each level is full.
15. **An almost complete binary tree** of height n is a binary tree where each level is full except possibly the last one.
16. **A binary search tree (BST)** is a binary tree where the vertices have values and for any vertex with value x, the values in its left subtree is $\leq x$ and the values in its right subtree is $\geq x$.

Notes

1. Any tree of size n has $n-1$ edges.
2. Not every tree is binary.
3. Not every binary tree is complete.
4. Not every binary tree is almost complete.
5. Not every BST is complete.
6. Not every BST is almost complete.