Propositional Logic

Section 1.1

Definition

- A proposition is a declarative sentence that is either true or false but not both nor neither
- Any proposition has a truth value {T, F}

Examples

Statement	Prop	Truth Value
Today is Friday	yes	F
1+1 = 2		
I know that you hate this course		
Is that correct?		
Do not answer quickly		
I’m a liar		
X+ 2 = 0		

Examples

Statement	Prop	Truth Value
Today is Friday	Yes	F
1+1 = 2	Yes	T
I know that you hate this course	Yes	T
Is that correct?	No	
Do not answer quickly	No	
I'm a liar	No	
X+ 2 = 0	No	

Proposition Types

- A proposition could be either simple or compound.
- Simple: without logical operators
- Compound: with logical operators (connectives)

Logical Operators

- Let p & q be propositions, then the following are compound propositions:
- Negation of p: $\neg p$ = not p
- Conjunction: $p \wedge q$ = p AND q
- Disjunction: $p \vee q$ = p OR q
- Exclusive OR: $p \oplus q$ = p XOR q
- Implication: $p \rightarrow q$ = if p then q
- Biconditional: $p \leftrightarrow q$ = p iff q

Negation

- p = It is raining
- ¬ p = It is not raining

  = it is not the case that it is raining

- Truth table

P	¬ p
T	F
F	T

Conjunction

- p = it is Friday
- q = it is raining
- $p \wedge q$ = it is Friday and it is raining
- Truth Table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

- p = 1> 0
- q = monkeys can fly
- p v q = 1>0 or monkeys can fly
- Truth Table:

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

Conditional Statement: Implication

- p = I think
- q = I exist
- $p \rightarrow q$ = if I think then I exist
- Truth Table:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

If p, then q

- p = hypothesis
- q = conclusion (consequence)

- There are many ways to write a conditional statement

Other Rephrasing of Implication

- If p, then q
- If p, q
- q if p
- p is sufficient for q
    - a sufficient condition for q is p
- a necessary condition for p is q
- in order to have p true, q has to be true also
- q when p

Other Rephrasing of Implication

- $p \rightarrow q$
- p implies q
- p only if q
  - p is true only if q is also true
- q follows from p

Examples of Implication

- If you get 98% then I'll give you A+
- 98% is sufficient for A+
- If you get A+ then it doesn't mean you have 98%
- A+ is necessary for 98%
- 98% however is not necessary for A+
- 98% only if A+
- A+ follows from 98%
- 98% doesn't follow from A+

Examples of Implication

- Notice that if p is true then q must be true, however if p is not true then q may or may not be true.

Examples of Implication
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(59) قَالُوا مَن فَعَلَ هَذَا بِآلِهَتِنَا إِنَّهُ لَمِنَ الظَّالِمِينَ

(60) قَالُوا سَمِعْنَا فَتًى يَذْكُرُهُمْ يُقَالُ لَهُ إِبْرَاهِيمُ

(61) قَالُوا فَأْتُوا بِهِ عَلَى أَعْيُنِ النَّاسِ لَعَلَّهُمْ يَشْهَدُونَ

(62) قَالُوا أَأَنتَ فَعَلْتَ هَذَا بِآلِهَتِنَا يَا إِبْرَاهِيمُ

(63) قَالَ بَلْ فَعَلَهُ كَبِيرُهُمْ هَذَا فَاسْأَلُوهُمْ إِن كَانُوا يَنطِقُونَ

(64) فَرَجَعُوا إِلَى أَنفُسِهِمْ فَقَالُوا إِنَّكُمْ أَنتُمُ الظَّالِمُونَ

(65) ثُمَّ نُكِسُوا عَلَى رُؤُوسِهِمْ لَقَدْ عَلِمْتَ مَا هَؤُلَاء يَنطِقُونَ

(66) قَالَ أَفَتَعْبُدُونَ مِن دُونِ اللَّهِ مَا لَا يَنفَعُكُمْ شَيْئاً وَلَا يَضُرُّكُمْ

(67) أُفٍّ لَّكُمْ وَلِمَا تَعْبُدُونَ مِن دُونِ اللَّهِ أَفَلَا تَعْقِلُونَ

(68) قَالُوا حَرِّقُوهُ وَانصُرُوا آلِهَتَكُمْ إِن كُنتُمْ فَاعِلِينَ

Examples of Implication
سورة الأنبياء 59 - 68

- It is very clear that the prophet Ebrahim didn’t lie because he said that

If they speak, then the biggest one did it

BiConditional Statement

- $p = 1 < 0$
- q = monkeys can fly
- $p \leftrightarrow q$ = $1 < 0$ if and only if monkeys can fly
- Truth Table:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

BiConditional Statement

- $p \leftrightarrow q$ = p if and only if q

  = p if q and p only if q

  = "$q \rightarrow p$" and "$p \rightarrow q$"

  = "$p \rightarrow q$" $\wedge$ "$q \rightarrow p$"

- p is necessary & sufficient for q
- p and q always have the same truth value

Theorem

- p and q are logically equivalent, i.e.,
  $p \equiv q$ if and only if $p \leftrightarrow q$ is always true

p	q	$p \equiv q$	$p \leftrightarrow q$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

Example

Logically equivalent

$\neg p \vee q \equiv p \rightarrow q$

p	q	$\neg$p	$\neg$p v q	p $\rightarrow$ q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Precedence of Logical Operators

- The following operators are sorted according to their precedence

$$\neg \;,\; \wedge \;,\; \vee \;,\; \rightarrow \;,\; \leftrightarrow$$

- This means for example

$$\Big(\big(\,(p \wedge (\neg q))\; \vee\; p\; \vee\; q\;\big) \rightarrow q\Big) \leftrightarrow \big(\,(p \wedge (\neg q))\; \vee\; p\,\big)$$