Rules of Inferences

Section 1.5

Definitions

- Argument: is a sequence of propositions (premises) that end with a proposition called conclusion.
- Valid Argument: The conclusion must follow from the truth of the previous premises, i.e., all premises → conclusion
- Fallacy: is an invalid argument or incorrect reasoning.
- Rules of inference: rules we follow to construct valid arguments.

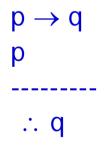
Valid Arguments in Propositional Logic

- If we rewrite all premises (propositions) in any argument using only variables and logical connectors then we get an argument form.
- Thus, an argument is valid when its form is valid.
- Valid argument doesn't mean the conclusion is true.

Example

• Argument:

- If you have a password, then you can login to the network.
- You have a password
- Therefore you can login to the network.
- Argument Form:



 So it is a valid argument with correct conclusion

Example

• Argument:

- If $x \neq 0$, then $x^2 > 1$
- But ½ ≠0
- Thus $\frac{1}{4} > 1$
- Argument Form:

 $p \rightarrow d$

∴ q

 So it is a valid argument with wrong conclusion

Rule	Name	Tautology
$p \rightarrow q$	Modus Ponens	$p \land (p \rightarrow q) \rightarrow q$
∴ q		
$\neg q$ $p \rightarrow q$ \dots $\therefore \neg p$	Modus Tollens	$\neg q \land (p \rightarrow q) \rightarrow \neg p$

Rule	Name	Tautology
$p \rightarrow q$		
$q \rightarrow r$	Hypothetical	$(p \rightarrow q) \land (q \rightarrow r)$
	Syllogism	\rightarrow (p \rightarrow r)
$\therefore p \rightarrow r$		
$p \lor q$		
_ p	Disjunction	$(p \lor q) \land \neg p \to q$
	Syllogism	
.∵ q		

Rule	Name	Tautology
p p ∨ q	Addition	$p \rightarrow p \lor q$
p∧q p	Simplification	$p \lor d \to b$

Rule	Name	Tautology
р		
q	Conjunction	$p \land q \rightarrow p \land q$
$\therefore p \land q$		
$p \wedge q$		
$\neg p \lor r$	Resolution	$(p \lor q) \land (\neg p \lor r) \rightarrow q \lor r$
$\therefore q \lor r$		

Examples

- "It's below freezing and raining now. Therefore it's below freezing"
- Argument form:

 $p \wedge q$

∴р

Simplification Rule

Examples

- "If x>1, then $1/x \in (0,1)$. If $x \in (0,1)$, then $x^2 < x$. Therefore, if x> 1, then $1/x^2 < 1/x$."
- Argument Form:
 - $b \rightarrow d$
 - $\mathsf{q}\to\mathsf{r}$

 $\therefore p \rightarrow r$

Rule: Hypothetical Syllogism

Using Rules of Inference to Build Arguments

Show that the hypotheses

- It's not sunny this afternoon and it's colder than yesterday.
- We will go swimming only if it's sunny
- If we don't go swimming, then we will take a canoe trip.
- If we take a canoe trip, then we will be home by sunset.

lead to the conclusion "we will be home by sunset"

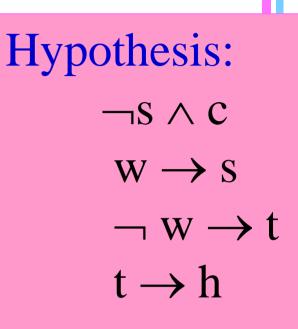
Using Rules of Inference to Build Arguments

the hypotheses

- It's not sunny this afternoon and it's colder than yesterday.
- We will go swimming only if it's sunny
- If we don't go swimming, then we will take a canoe trip.
- If we take a canoe trip, then we will be home by sunset.

the conclusion

"we will be home by sunset"



Conclusion: h

Using Rules of Inference

 $\neg S \land C$ $\neg S$ $W \rightarrow S$ $\neg W$ $\neg w \rightarrow t$ t $t \rightarrow h$

: h

hypo simplification hypo Modus Tollens hypo **Modus Ponens** hypo

Modus Ponens

Using Rules of Inference to Build Arguments

Show that the hypotheses

- If you send me an email message, then I'll finish writing the program.
- If you don't send me an email, then I'll go to sleep early.
- If I go to sleep early, then I'll wake up feeling refreshed.

lead to the conclusion "if I don't finish writing the program then I'll wake up feeling refreshed"

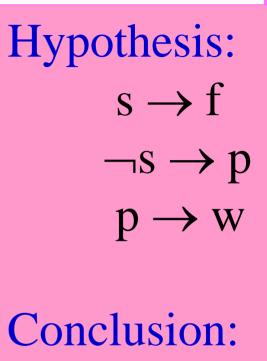
Using Rules of Inference to Build Arguments

the hypotheses

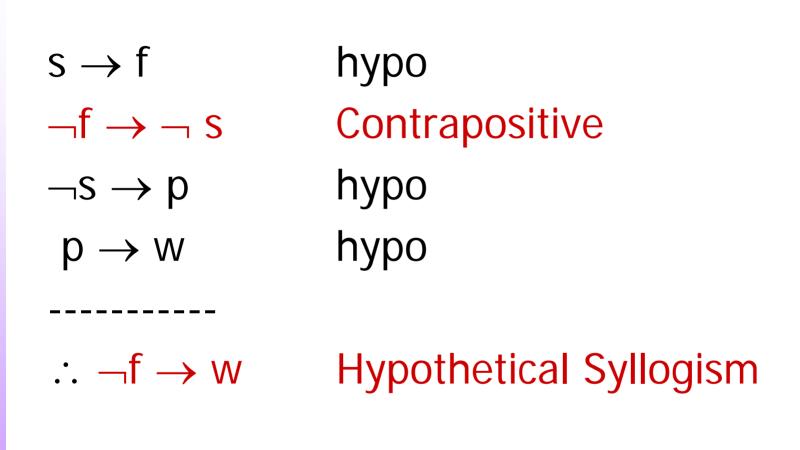
- If you send me an email message, then I'll finish writing the program.
- If you don't send me an email, then I'll go to sleep early.
- If I go to sleep early, then I'll wake up feeling refreshed.

the conclusion

"if I don't finish writing the program then I'll wake up feeling refreshed"



 $\neg f \rightarrow w$



Fallacies

- Incorrect reasoning based on contingencies and not tautologies.
- 1. Fallacy of affirming the conclusion: $(p \rightarrow q) \land q \rightarrow p$
- Example: If you solve every problem in this book, then you'll pass the course. You did passed the course. Therefore, you did solved every problem in this book.

Fallacies

- 2. Fallacy of denying the hypothesis: $(p \rightarrow q) \land \neg p \rightarrow \neg q$
- Example:
- Since you didn't pass the course, then you didn't solve every problem.
- Since you didn't solve every problem, then you didn't pass the course.

Rules of Inference for Quantified Statements

Universal Instantiation:

 $\forall x p(x)$

... p(c)
 Universal Generalization:
 p(c) for arbitrary c

 $\therefore \forall x p(x)$

Rules of Inference for Quantified Statements

Existential Instantiation:

 $\exists x p(x)$

... p(c) for some c
Existential Generalization: p(c) for some c

 $\therefore \exists x p(x)$

Combining Rules of Inference

- Universal Modus Ponens:
 Universal Instantiation + Modus Ponens
 ∀ x (P(x) →Q(x))
 P(a)
 - ∴ Q(a)

Combining Rules of Inference

Universal Modus Tollens:
 Universal Instantiation + Modus Tollens
 ∀ x (P(x) →Q(x))
 ¬Q(a)

∴ ¬ P(a)