Warm Up Exercises

Section 1.1

5.h

- p = "Swimming at the New Jersey shore is allowed"
- q = "Sharks have been spotted near the shore"
- $\neg p \wedge (p \vee \neg q)$ = " ? "

= Swimming at the New Jersey shore is not allowed and either Swimming at the New Jersey shore is allowed or Sharks have not been spotted near the shore

7.f

- p = “It is below freezing”
- q = “It is snowing”
- It is either below freezing or it is snowing, but it is not snowing if it is below freezing = ?

$= (p \vee q) \wedge (p \rightarrow \neg q)$

7.g

- p = “It is below freezing”
- q = “It is snowing”
- Below freezing is necessary and sufficient for it to be snowing = ?

= $p \leftrightarrow q$

- p is necessary for q means $q \rightarrow p$
- p is sufficient for q means $p \rightarrow q$

True or false:

- 13.a: If 1+1=2 then 2+2 = 5

  False

- 13.b: If 1+1=3 then 2+2 = 4

  True

- 13.d: If monkeys can fly, then 1 +1 =3

  True

Truth Tables

- Have fun with this Java Applet

Propositional Equivalences

Section 1.2

Definitions

- Tautology is a compound proposition that is always true (independent of the truth values of the single propositions)
- Contradiction is a compound proposition that is always false.
- Contingency is a compound proposition that is neither a tautology nor a contradiction.
- p and q are logically equivalent ($p \equiv q$) iff $p \leftrightarrow q$ is a tautology.

Examples

- $p \vee \neg p$ is a tautology
- $p \wedge \neg p$ is a contradiction

Proof

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Use T for Tautology & F for Contradiction

- $p \vee \neg p \equiv T$
- $p \wedge \neg p \equiv F$
- $T \wedge \neg T \equiv F$
- $T \vee p \equiv T$
- $T \wedge p \equiv p$
- $F \vee p \equiv p$
- $F \wedge p \equiv F$
- $(p \vee \neg p) \wedge q \equiv q$
- $(p \wedge \neg p) \vee p \equiv p$
- $p \rightarrow T \equiv T$
- $F \rightarrow p \equiv T$
- $p \wedge q \rightarrow p \equiv T$

Examples of Equivalences

- $p \rightarrow q \equiv q \vee \neg p$

p	q	$\neg p$	$p \rightarrow q$	$q \vee \neg p$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Examples of Equivalences

- $\neg (p \wedge q) \equiv \neg q \vee \neg p$ De Morgan's Law

p	q	$\neg p$	$\neg q$	$\neg (p \wedge q)$	$\neg q \vee \neg p$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

Known Equivalences

- Double Negation Law:

  $\neg(\neg p) \equiv p$

- Identity Laws:

  $p \wedge T \equiv p$

  $p \vee F \equiv p$

- Domination Laws:

  $p \wedge F \equiv F$

  $p \vee T \equiv T$

- Idempotent Laws:

  $p \wedge p \equiv p$

  $p \vee p \equiv p$

Known Equivalences

- Commutative Laws:

  $p \wedge q \equiv q \wedge p$

  $p \vee q \equiv q \vee p$

- Negation Laws:

  $p \wedge \neg p \equiv F$

  $p \vee \neg p \equiv T$

- Associative Laws:

  $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

  $p \vee (q \vee r) \equiv (p \vee q) \vee r$

Known Equivalences

- Distributive Laws:

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

- De Morgan's Laws:

$$\neg (p \wedge q) \equiv \neg p \vee \neg q$$ NAND

$$\neg (p \vee q) \equiv \neg p \wedge \neg q$$ NOR

- Absorption Laws:

$$p \vee (p \wedge q) \equiv p$$

$$p \wedge (p \vee q) \equiv p$$

Known Equivalences

All of these laws can be proved easily by using truth tables.

More Equivalences

- $p \rightarrow q \equiv \neg p \vee q$
- $p \rightarrow q \equiv \neg q \rightarrow \neg p$ contrapositive
- $\neg (p \rightarrow q) \equiv p \wedge \neg q$
- $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
- $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
- $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
- $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$
- $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
- $\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Example

Prove that $\neg (p \to q) \equiv p \land \neg q$

Proof

$\neg (p \to q) \equiv \neg (\neg p \lor q)$

$\equiv \neg (\neg p) \land \neg q$ De Morgan's

$\equiv p \land \neg q$ double negation

Example

Prove that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

Proof

$(p \wedge q) \rightarrow (p \vee q)$

$\equiv \neg (p \wedge q) \vee (p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$ De Morgan's

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$ Associative

$\equiv T \vee T \equiv T$

Example

Simplify

$\neg (p \vee (\neg p \wedge q))$

$\equiv \neg p \wedge \neg (\neg p \wedge q)$	De Morgan's
$\equiv \neg p \wedge (p \vee \neg q)$	De Morgan's
$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	Distributive
$\equiv F \vee (\neg p \wedge \neg q)$	Negation
$\equiv (\neg p \wedge \neg q)$	Identity
$\equiv \neg (p \vee q)$	De Morgan's

Wake-up Exercises

11.d

Prove that $(p \wedge q) \rightarrow (p \rightarrow q) \equiv T$

Proof

$\equiv (p \wedge q) \rightarrow (p \rightarrow q)$

$\equiv \neg (p \wedge q) \vee (p \rightarrow q)$

$\equiv (\neg p \vee \neg q) \vee (\neg p \vee q)$

$\equiv \neg p \vee \neg p \vee \neg q \vee q$

$\equiv \neg p \vee T \equiv T$

11.e

Prove that $\neg (p \to q) \to p \equiv T$

Proof

$\equiv \neg (\neg (p \to q)) \vee p$

$\equiv (p \to q) \vee p$

$\equiv (\neg p \vee q) \vee p$

$\equiv (\neg p \vee p) \vee q$

$\equiv T \vee q \equiv T$

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Determine if $(\neg q \land (p \rightarrow q)) \rightarrow \neg p$

is T, F, or contingency

Tautology !

Prove it.

25

Prove $(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

Proof

$(p \to r) \lor (q \to r)$

$\equiv (\neg p \lor r) \lor (\neg q \lor r)$

$\equiv \neg p \lor \neg q \lor r$

$\equiv \neg (p \land q) \lor r$

$\equiv (p \land q) \to r$

Definition

- Dual of a compound Proposition is the same proposition but
- each $\wedge$ is replaced with $\vee$
- each $\vee$ is replaced with $\wedge$
- each T is replaced with F
- each F is replaced with T

- Dual of $(\neg p \wedge T) \vee (q \vee F)$ is
  $(\neg p \vee F) \wedge (q \wedge T)$