ICS 252–Discrete Structures, Fall 2005

Final Exam

Date: Jan. 26th, 2006 **Duration: 120 minutes**

Name:

ID#:

Section #:

This is a closed-book and closed-notes exam. No calculators are allowed.

Notation: The following symbols are used throughout this exam.

$\mathbb{N}$	$= \{1, 2, 3, \ldots\}$	$\chi(G)$	the chromatic number of a graph G
λ	the empty (null) string	w^R	the reversal of the string w
ϕ	the empty set	$L(M)$	the language recognized by a finite state automata M
Σ	an alphabet	$L(r)$	the language recognized by a regular expression r

Q.#	Marks	Scores	Remarks
1	17		
2	8		
3	8		
4	9		
5	14		
6	14		
7	12		
8	10		
9	8		
Total	100		

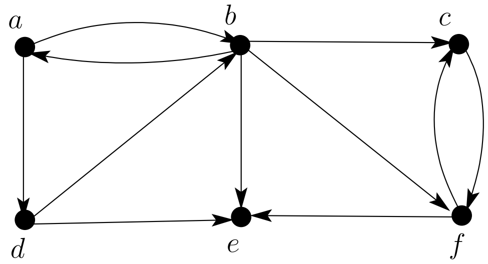
Question 1: [$6 + 5 + 6$ **marks**]

1. For which values of n are these graphs bipartite?

   a) The complete graph K_n:

   b) The cycle C_n:

2. Find the strongly connected components of the following graph?



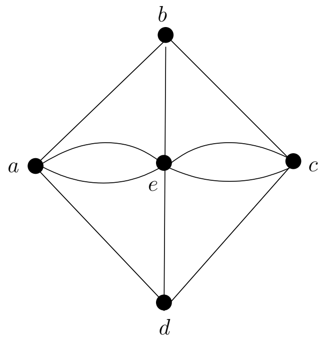
3. Find the following chromatic numbers.

   (a) $\chi(W_n)$, where $n \in \mathbb{N}$ and $n \geq 3$.

   (b) $\chi(K_{n,m})$, where $n, m \in \mathbb{N}$.

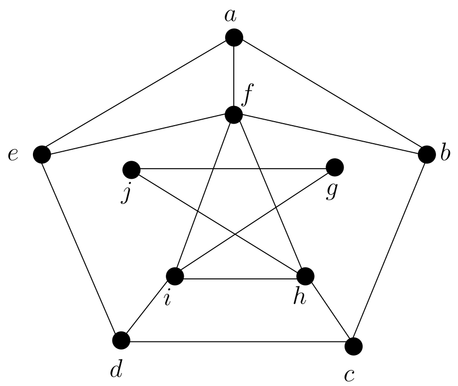
Question 2: [4 × 2 **marks**]

1. Determine if the following graph has an Euler circuit, an Euler path but no Euler circuit, or none. Circle the correct answer and write down the vertices of the circuit or the path (if any exist).



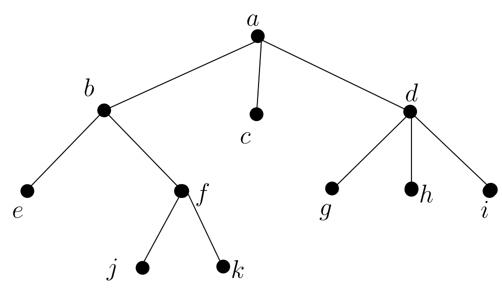
Euler Circuit　　　　Euler path but no Euler circuit　　　　None

2. Determine if the following graph has a Hamilton circuit, a Hamilton path but no Hamilton circuit, or none. Circle the correct answer and write down the vertices of the circuit or the path (if any exist).



Hamilton Circuit　　　　Hamilton path but no Hamilton circuit　　　　None

Question 3: [4 × 2 **marks**]



Given the above tree, write the order of the vertices into which they are visited by using:

1. Preorder traversal:

2. Postorder traversal:

Question 4: [3 × 3 **marks**]

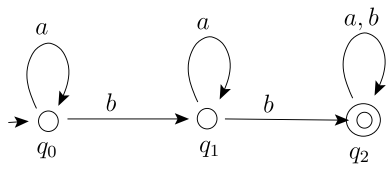
1. How many edges must be removed from a connected graph with n vertices and m edges to produce a spanning tree?

2. Apply your previous answer to the K_5 and find a spanning tree for it. *Draw the spanning tree and state how many edges are removed.*

3. How many different spanning trees does K_5 have?

Question 5: **[2 + 4 × 3 marks]**

Consider the following finite state machine M over the alphabet $\Sigma = \{a, b\}$.



1. Is M a DFA, an NFA or both?

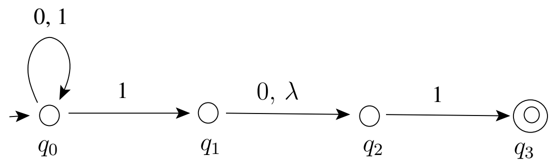
2. Find two strings x and y each of length exactly 4 such that x is accepted by M and y is not.

3. Find a regular expression r such that $L(r) = L(M)$.

4. Draw a DFA that recognizes the complement of $L(M)$, i.e. $\Sigma^* - L(M)$.

Question 6: [$4 + 6 + 4$ **marks**]

Consider the following State diagram of the machine N over the alphabet $\Sigma = \{0, 1\}$.



1. Write the formal definition of the machine N.

2. Determine whether N accepts or rejects each of the following strings.

   (a) 10001

   (b) $(110000)^R$

   (c) 0010010

3. Find a regular expression r such that $L(r) = L(N)$.

Question 7: [3×4 **marks**]

State whether the following languages defined over the alphabet $\Sigma = \{a, b\}$ are regular or not. For regular languages prove your answer by stating one of the related theorems that you have studied, or by giving a regular expression, a DFA, or an NFA.

1. $A = \{x \in \Sigma^* \mid \text{ the length of } x \geq 3\}$

2. $B = \{x \in \Sigma^* \mid x \text{ starts with "}a\text{" and ends with "}b\text{"}\}$

3. $A^* \cap B$, where A and B are as above.

4. $\left\{x \in \Sigma^* \mid x = a^n b^n, \text{ for some } n \leq 2^{10}\right\}$

Question 8: [$2 \times 2 + 3 \times 2$ **marks**]

1. Let $\Sigma = \{0, 1\}$. For each of the following regular expression r find a string $x \notin L(r)$ or write **none** if no such string exists.

   (a) $(01 + 1 + 00)^*$

   (b) $(0^*1)^*(10^*)^*$

2. Write a regular expression for each of the following languages over $\Sigma = \{a, b\}$.

   (a) $\{x \in \Sigma^* \mid x$ starts with "ab" and ends with "bba"$\}$. *Note that abba belongs to the language.*

   (b) $\{x \in \Sigma^* \mid x$ contains even number of a's $\}$

Question 9: [4×2 **marks**]

1. Using no more than **4 states** draw the state diagram of a DFA that recognizes the language of all strings over $\{0, 1\}$ that have even number of 0s **and** end with the substring 01. *DFA with more than 4 states will NOT be graded.*

2. Using no more than **4 states** draw the state diagram of an NFA that recognizes the language $L(a^*(abb + aa)b^*)$. *NFA with more than 4 states will NOT be graded.*