ICS 251 Foundations of Computer Science
Final (25/12/1997)
Time allowed: 2 hours

- In all the questions, you have to clearly justify your answers. **out of 24**
- Simple yes or no answers carries no value.
- Questions are worth three points each.

Name: ______________________________ **ID#:** __________

Q1: Let $A = B = C = Z$, and let $f : A \rightarrow B$, $g : B \rightarrow C$ be defined as: $f(a) = a - 1$ and $g(b) = b^2 - 2b + 1$; then

a) Find $(g \circ f)(11)$.

b) Is g one-to-one?

c) Is g onto?

Q2: a) For the set $A = \{1,2,3\}$, find all the permutations on A.

b) Compute the product $(1,2)\circ(2,3)\circ(1,2,3)$.

Q3: Let $A = \mathbf{Z}$ (all the integers) and let $a, b \in A$. Define the relation R on A as

aRb iff $a^2 + b^2$ is even.

Is R an equivalence relation?

Q4: Let $A = \{1,2,3,4\}$ and let the relation R on A be defined as
$R = \{(1,1),(1,3), (2,4), (3,1), (3,4),(4,2)\}$
Use Warshall's algorithm to compute the transitive closure of the relation R.

Q5: Let $(G,*)$ be a group and let d be some element of G.
Let the function $f : G \rightarrow G$ be defined as $f(x) = d*x*d^{-1}$, for all $x \in G$.
Show that f is an isomorphism.

Q6: Let G be the set of all real numbers and let $a*b = a+b+99$.
Is $(G,*)$ a group?

Q7: Let $A = \{1,2,4,5,7,8\}$ and let the operation $*$ be defined as
$a*b = ab \bmod 9$.

a) Draw the multiplication table of the group $(A,*)$.

b) What is the inverse of 4?

c) Find a subgroup S of $(A,*)$ such that $|S|=2$.

d) Can you find a subgroup S of $(A,*)$ with $|S|=4$? Why or why not.

Q8: a) Given the prime number $p = 97$, find the inverse of 49 mod 97; i.e. find a number y such that $49y \equiv 1 \pmod{97}$ [equivalently, $49y \bmod 97 = 1$] (Hint: use the GCD algorithm.)

b) Fill in the blank: $8^{97} \equiv$ _____ (mod 97)? Justify your answer.