ECON (206)
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Review of Growth Models

1. FEATURES OF ECONOMIC GROWTH

a. Definition

Economic growth refers to an increase in a country's ability to produce goods and services. The advantage of economic growth is that an increase in real national income allows more goods for consumption.

b. Developing Countries

A developing country or less developed country (LDC) is one which is not yet fully industrialized and tends to have the following features:
i) Agriculture is more important than manufacturing.
ii) There is limited specialization and exchange.
iii) There are not enough savings to finance investment.
iv) Population is expanding too rapidly for available resources.
v) A low standard of living.

As opposed to a developing country, a developed country is more fully industrialized and has a high standard of living.

c. Barriers to Economic Growth

A country can increase production if it increases the amount of resources used or makes better use of existing factors. Economic growth is more difficult if:
i) A country lacks the infrastructure (underlying capital) to produce goods more efficiently. There are three types of infrastructure:
  x) Basic including electricity, road and telephone networks;
  y) Social including schools, hospitals and housing;
  z) Industrial including factories and offices.
ii) A country lacks the machines or skilled labor needed to manufacture modern goods or services.
iii) A country lacks the technical knowledge.
iv) Workers are not prepared to accept specialization and the division of labor.
v) Population growth is too rapid.
vi) A country has too large a foreign debt.

d. Disadvantages of Economic Growth

i) Increased noise, congestion and pollution.
ii) Towns and cities may become overcrowded.
iii) Extra machines can be produced only by using resources currently involved in making consumer goods.
iv) A traditional way of life may be lost.
v) People may experience increased anxiety and stress.

2. THEORIES OF ECONOMIC DEVELOPMENT

We will first look at two models of economic development, the Lewis model and the Harrod-Domar Model. These models focus on economic development of the LDC's rather than explain growth itself. Later, we will look at several growth models that look at growth from any economies perspective.

a. Lewis's Dual Sector Model of Development: The Theory of Trickle Down

i. Assumption of the Lewis Model:

Lewis proposed his dual sector development model in 1954. It was based on the assumption that many LDCs had dual economies with both a traditional agricultural sector and a modern industrial sector. The traditional agricultural sector was assumed to be of a subsistence nature characterized by low productivity, low incomes, low savings and considerable underemployment. The industrial sector was assumed to be technologically advanced with high levels of investment operating in an urban environment.

ii. Growth Trickle Down in Lewis Model:

Lewis suggested that the modern industrial sector would attract workers from the rural areas. Industrial firms, whether private or publicly owned could offer wages that would guarantee a higher quality of life than remaining in the rural areas could provide. Furthermore, as the level of labor productivity was so low in traditional agricultural areas people leaving the rural areas would have virtually no impact on output. Indeed, the amount of food available to the remaining villagers would increase as the same amount of food could be shared amongst fewer people. This might generate a surplus which could them be sold generating income. Those people that moved away from the villages to the towns would earn increased incomes and this crucially according to Lewis generates more savings. The lack of development was due to a lack of savings and investment. The key to development was to increase savings and investment. Lewis saw the existence of the modern industrial sector as essential if this was to happen. Urban migration from the poor rural areas to the relatively richer industrial urban areas gave workers the opportunities to earn higher incomes and crucially save more providing funds for entrepreneurs to investment.

A growing industrial sector requiring labor provided the incomes that could be spent and saved. This would in itself generate demand and also provide funds for investment. Income generated by the industrial sector was trickling down throughout the economy.

iii. Problems of the Lewis Model:

The idea that the productivity of labor in rural areas is almost zero may be true for certain times of the year however during planting and harvesting the need for labor is critical to the needs of the village.
The assumption of a constant demand for labor from the industrial sector is questionable. Increasing technology may be labor saving reducing the need for labor. In addition if the industry concerned declines again the demand for labor will fall.

The idea of trickle down has been criticized. Will higher incomes earned in the industrial sector be saved? If the entrepreneurs and labor spend their new found gains rather than save it, funds for investment and growth will not be made available.
The rural urban migration has for many LDCs been far larger that the industrial sector can provide jobs for. Urban poverty has replaced rural poverty.

b. Harrod-Domar Growth Model

i. Basic Proposition of the H-D Model:

H-D suggests savings provide the funds which are borrowed for investment purposes.
The model suggests that the economy's rate of growth depends on:
x) The level of saving
y) The productivity of investment i.e. the capital output ratio
For example, if $10 worth of capital equipment produces each $1 of annual output, a capital-output ratio of 10 to 1 exists. A 3 to 1 capital-output ratio indicates that only $3 of capital is required to produce each $1 of output annually.

ii. Implication of the H-D model:

1) Economic growth depends on the amount of labor and capital. As LDCs often have an abundant supply of labor it is a lack of physical capital that holds back economic growth and development.
2) More physical capital generates economic growth. Net investment leads to more capital accumulation, which generates higher output and income.
3) Higher income allows higher levels of saving.

iii. The H-D Model:

Assume:
S= Savings Rate; s= Savings Rate
K= Capital Sock;
I= Investment
k= (K/Y) Ratio
Assume also that $I = \Delta K$.
If S=sY and if k= (K/Y) and we assume that k is a constant, then we can get:

$$\frac{\Delta K}{\Delta Y} = k$$

In equilibrium, S=I. Substituting the values, we get:

$$sY = \Delta K \Rightarrow sY = k\Delta Y \Rightarrow \frac{\Delta Y}{Y} = \frac{s}{k}$$

Growth rate of GDP = savings rate x inverse of the capital output ratio
Intuition: The growth rate of output rises as savings levels rise but is limited by the rate at which capital can be converted into output.

iv. Further Implications of the H-D Model:

The key to economic growth is to expand the level of investment both in terms of fixed capital and human capital. To do this policies are needed that encourage saving and/or generate technological advances which enable firms to produce more output with less capital i.e. lower their capital output ratio.

v. Problems of the H-D Model:

1) Economic growth and economic development are not the same. Economic growth is a necessary but not sufficient condition for development
2) Practically it is difficult to stimulate the level of domestic savings particularly in the case of LDCs where incomes are low.
3) Borrowing from overseas to fill the gap caused by insufficient savings causes debt repayment problems later.
4) The law of diminishing returns would suggest that as investment increases the productivity of the capital will diminish and the capital to output ratio rise.
5) Assumes that complementary factors like managerial skills, skilled labor, and administration abilities are available.
6) There is an efficient system that will convert savings into capital (e.g. role of political structure).
7) External forces may nullify even the best developmental strategies.
8) Development is more than throwing together some key components and expecting the economy to grow.

3. GROWTH MODELS

We will look at several growth models. The first set of models is called exogenous growth models while the second group of models is called endogenous growth models.

A. Model 1: SOLOW (Neoclassical) Growth Model: Exogenous Growth Model

a. Idea behind the Solow Model:

Working in the middle 1950s, a little less than ten years after Harrod and Domar, Robert Solow (MIT) developed a model of economic growth based not on the assumption of a fixed coefficients production but rather on a neoclassical aggregate production function – one that, realistically, allows for substitution between one factor of production and another.

b. The Basic Solow Model:

The neoclassical function can be expressed as a relationship between output per worker (Y/L) and capital (i.e., productive assets) per worker (K/L).

$$Y/L = F(K/L, 1) \qquad (1)$$

Or more simply

$$y = f(k) \qquad (2)$$

The inward-bowed production isoquants of the neoclassical production function imply that there are diminishing marginal returns to capital (or labor),

In the Solow model, growth in average living standards (Y/L or y) occurs because of an improvement in the capital-labor ratio (K/L or k). As in the Harrod-Domar model, this latter improvement depends on savings and depreciation rates. But it also depends on growth in the working population – a factor neglected in the Harrod-Domar model that of course influences k's denominator.

$$\Delta K/K = s\ Y/K - d\ K/K = s\ Y/K - d$$

Next comes a definition of $\Delta k/k$.

$$\Delta k/k = \Delta K/K - \Delta L/L$$

This can be rearranged, with n (the rate of growth in the working population) substituted for $\Delta L/L$.

$$\Delta K/K = \Delta k/k + n$$

We now have two equations with $\Delta K/K$ on the right-hand side. These can be combined.

$$\Delta k/k = s\ Y/K - n - d$$

Finally, multiplying both sides by k yields a definition of change in the capital-labor ratio.

$$\Delta k = s\ y - (n + d)\ k \qquad (3)$$

Equation (3) shows that capital accumulation depends on the difference between total savings(s y) and minimum investment requirement ((n + d) k) which is needed to keep the existing capital stock active. Any time $s\ y > (n + d)\ k$, $\Delta k > 0$.

Solow's model had two characterization of the growth process:

a) Solow defined something called "**steady state**". **Steady state** refers to the long run equilibrium level of capital per capita and output per capita, at which k and y does not grow any further. In mathematical terms, steady state refers to the level at which $\Delta k = \Delta y = 0$. Substituting this into equation (3), we get:

$$s\,y = (n + d)\,k \qquad (4)$$

Equation (4) suggests that long run steady state equilibrium will be found where total savings in the economy is exactly equal to the total minimum investment requirement.

Figure 1 shows Solow Growth Model. There are several things to notice:

1) The production function and the savings function are both concave, which indicates diminishing marginal returns to physical capital.

2) When savings is more than minimum investment requirement (left of k^*), both per capita capital and output grows. When savings is above minimum investment requirement (right of k^*), both per capita capital and output decreases.

3) Steady state long run equilibrium is found where savings is exactly equal to the investment requirement, where the savings function crosses the straight line, at k^* level. At k^* level, $\Delta k = \Delta y = 0$.

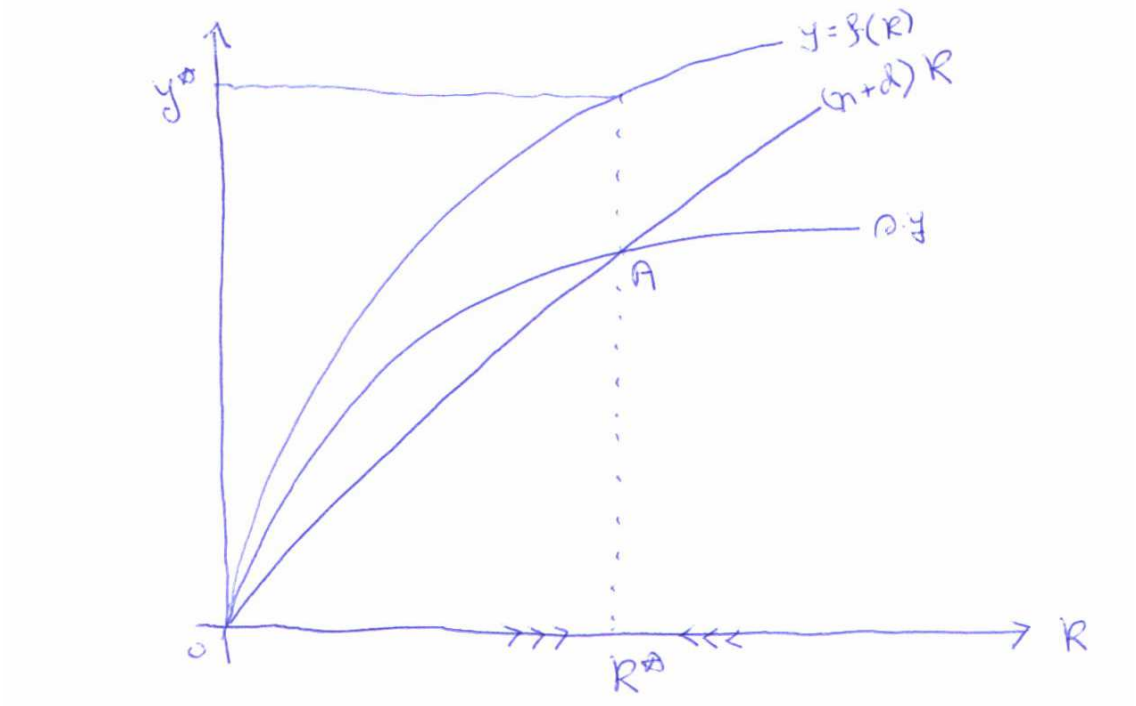

FIGURE 1: Solow GROWTH MODEL

b) Solow also defined something called the '**Golden Rule'**. This corresponds to the maximum possible consumption per capita at the steady state. The capital stock and output level at which this consumption level is found are also called the **Golden Rule level of capital and output.** This is found where the marginal product of capital is equal to the sum of population growth rate and the depreciate rate:

MPK (k^*) = (n+d)

Empirical analysis suggested that for most of the economies, the steady state per capita capital stock is below the golden rule level of capital stock.

c. Implication of the Solow Growth Model

1) Growth of any economy depends on three factors, savings rate(s), population growth (n) and the depreciation rate (d).
2) Any factor that raises savings rate, decreases population growth or reduces depreciation rate for the economy, increases economic growth.
3) Economic growth is constant or stagnant. Once the economy reaches k^*, growth stops. Point A is **a stable** equilibrium.

d. Limitations of Slow Growth model

1) Growth is exogenous, depends on factors that the economy or the policy makers cannot control.
2) Economic growth is limited.

e. Clue for the examination

1) What happens when a policy raises savings rate of the economy?
2) What happens when there is a natural disaster?
3) What happens if population growth is endogenous?

B. Model 2: Extension of Solow Growth Model: Endogenous Population Growth

In this extension, we only relax the assumption that population growth is exogenous. Rather, we assume that population growth is function of income:

$$n=f(y) =n(y)$$

Graphically, in figure 2, we see that the endogenous population growth leaves everything unchanged, except it changes the minimum investment requirement line. The line is no longer a straight line but a curve. The graph has several properties:

1) Since income growth is higher at the beginning of growth experience, the population growth is rapid. As a result, the investment requirement line is below the savings function in the short run
2) In the medium run, the investment requirement line goes above the savings function.
3) In the long run, the investment requirement line first falls below the savings function, then rises again and never comes below the savings function.

The investment requirement line crosses the savings function at several points. We will distinguish between three kinds of intersection:

1) Points like A are called the **Poverty Trap** (Remember Malthus). If capital stock is below k_1, the economy moves back to k_1 capital stock. If the capital stock is above k_1, the economy also comes back to k_1. This equilibrium is also stable.

2) Points like B are called **Unstable Equilibrium.** At any capital stock level left of k_2, the economy moves back to k_1. At any capital stock level above k_2, it never come back to k_2.
3) Points like C are called long run equilibrium. It has two important properties:
    a. If the capital stock is below k_3 but above k_2, the economy moves back to k_3. Hence it is **stable**.
    b. If the capital stock is above k_3, the economy comes back to k_3. Hence, it is **stable** long run equilibrium.

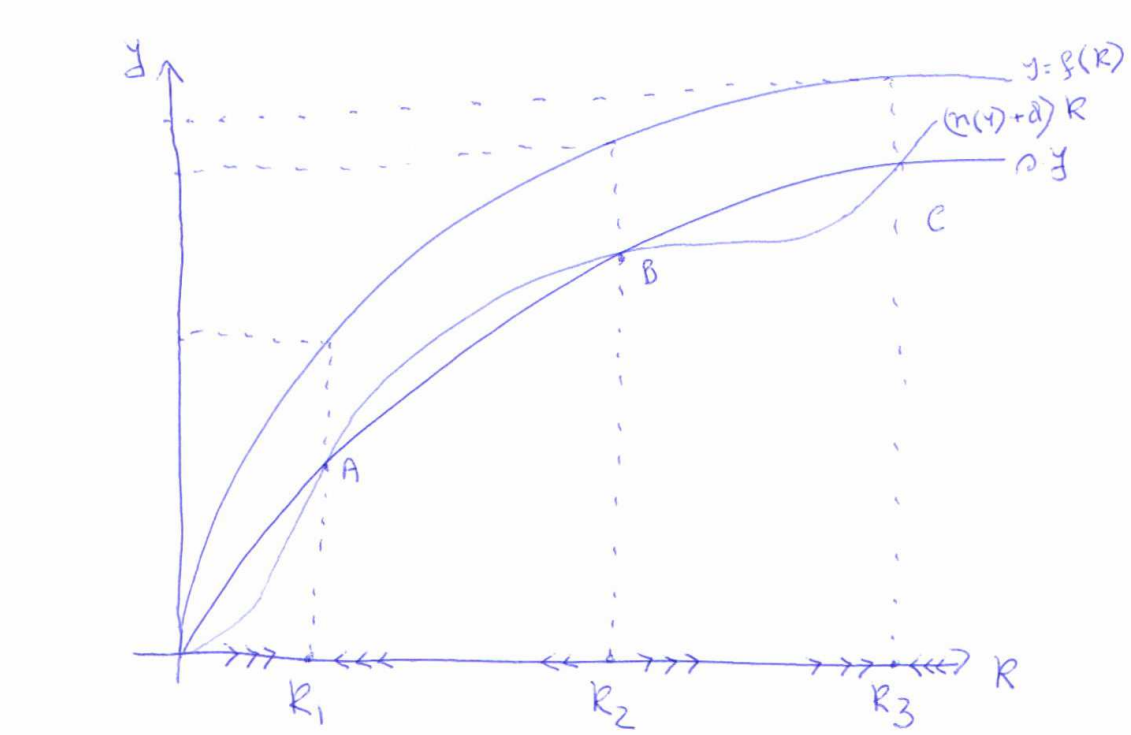

FIGURE 2: SOLOW GROWTH MODEL WITH ENDOGENOUS POPULATION GROWTH

C. Model 3: Endogenous Growth Models (Romer 1992)

a. Motivation

1. The persistent growth in GDP per capita across countries over a long period of time undermines the exogenous growth models.
2. It has been observed that poor countries in the last 60 years have been able to catch up with the richer countries (catch up effect) through investment in human capital R&D, and education. This raises the importance of other factors of production than labor and physical capital which were the subject matter of most of the old growth and development models.
3. The idea of diminishing returns to scale on physical capital seems like a economically plausible idea, which has been pursued by all the previous models. But physical

capital might have positive externality which might lead to constant or even increasing returns to scale of even physical capital

b. Main Assumption of the Romer Model

Romer(1992) assumed that physical capital might have external effects, which means that invention of one physical capital by one firm might lead to benefit to other firms. Then physical capital might have constant returns to scale.

c. Main Model

If physical capital has constant returns to scale and production depends only on physical capital, then:

$$Y = aK$$

$$\Rightarrow \Delta K = sY = saK$$

$$\Rightarrow \frac{\Delta K}{K} = \frac{\Delta Y}{Y} = sa$$

The model has two simple and powerful implications:

1) GDP and capital growth rate is constant. This means that economy continues to grow in the long run. Therefore growth is **unbounded.**
2) If savings rate increases or marginal product of physical capital (a) increases, then the economy grows faster and unbounded.

In figure 3, we see that because of CRT assumption on the physical capital, the savings function never crosses the investment requirement line. As a result, both output per capita and capital stock per capita grows unboundedly. In this model, there is **no steady state**.

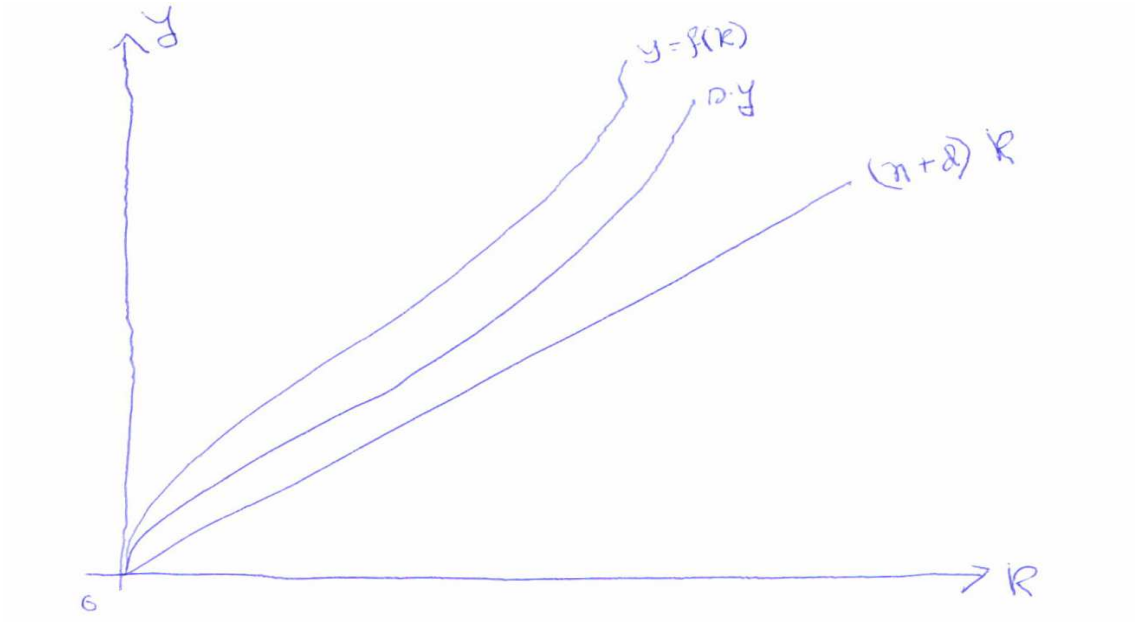

FIGURE 3: ENDOGENOUS GROWTH MODEL WITH CONSTANT RETURNS TO SCALE

D. Model 4: Lucas Two Sector Growth Model (1988)

a. Motivation

The Nobel Laureate economist brought the idea of human capital into a neoclassical production for the first time and showed that human capital can be the source of unbounded growth for an economy.

b. Assumption

1. There are two kinds of capital in the economy, human capital and physical capital.
2. The economy allocates more resources in physical capital because the return from physical capital is faster (short run) and the return on human capital takes time (long run).

c. The Model

We will analyze the implication of the model by using a graph. Figure 4 shows the implication of the two sector model in a graph similar to the one used for other models. The graph only shows relationship between physical capital and output growth. We see several very important aspects of the growth process:

1) In the short run, the return to physical capital dominates the return to human capital. As physical capital is subject to diminishing returns, savings rises above the investment requirement but quickly falls below it.
2) In the long run, returns from human capital begin to dominate the returns to physical capital. Savings shoots up, goes beyond the investment requirement and grows exponentially there after.

Based on the fluctuations of the physical capita stock, we get two kind of equilibrium:

1) In the short run, the economy reaches equilibrium such as point A. This is a stable equilibrium because if the economy goes below or above k_1, the economy always comes back. This equilibrium is **steady state** equilibrium. It is also **poverty trap** equilibrium.
2) In the long run, returns from human capital forces the economy to move to a point like B. This is an **unstable** equilibrium because if the economy is left of k_2, the economy moves back to k_1. Furthermore, if the economy moves above k_2, the economy never come back to k_1. Growth of per capita GDP and capital stock is **unbounded** after k_2.

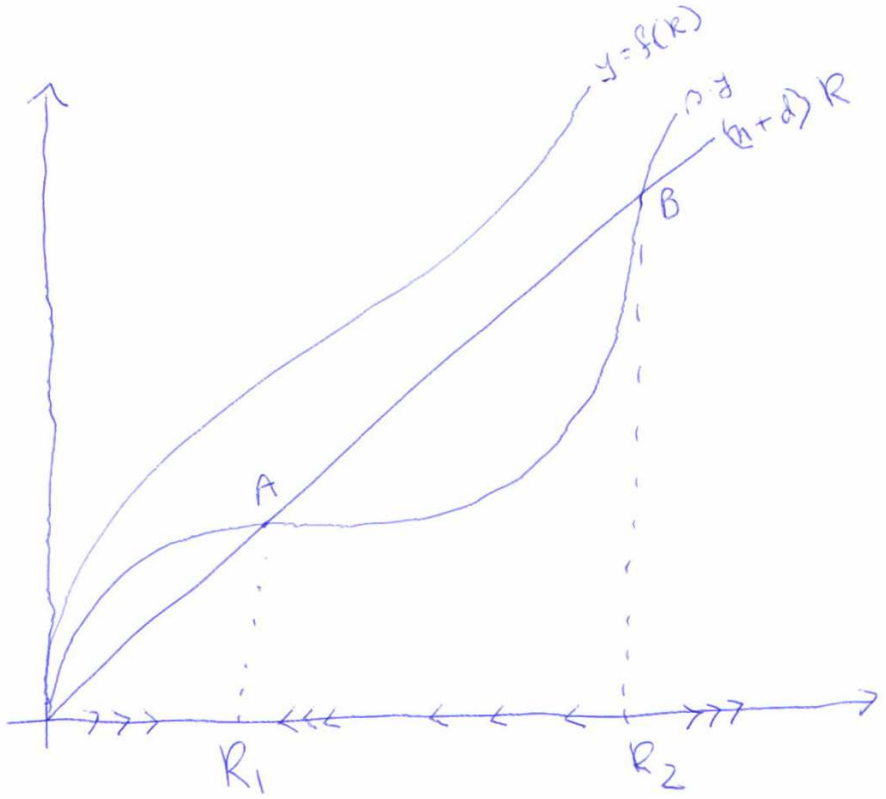

FIGURE 4: LUCAS TWO SECTOR ENDOGENOUS GROWTH MODEL