CHAPTER-6

A. OBJECTIVE OF THIS CHAPTER

In this chapter we will do the following:

- Look at some stylized facts about economic growth in the World.
- Look at two Macroeconomic models of exogenous economic growth
    a) Malthusian model
    b) Solow model.
- Do policy experiment under different models.

B. STYLIZED FACTS OF ECONOMIC GROWTH IN THE WORLD

1. Before industrial revolution in about 1800, standards of living differed little over time and across countries.

2. Since the industrial revolution, per capita income growth has been sustained in the richest countries. In the USA, average growth of per capita income has been about 2% since 1869

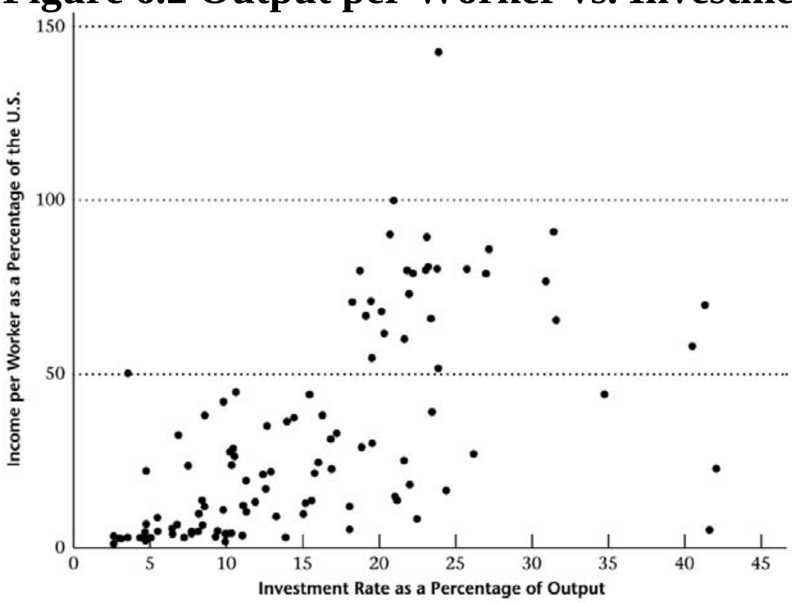
- The slope of the log per capita income(which measures income growth) is roughly constant
- The great depression and WWII was the only exceptions

Figure 6.1 Natural Log of Real per Capita Income in the United States, 1869–2002

3. There is a positive correlation between the rate of investment and output per worker

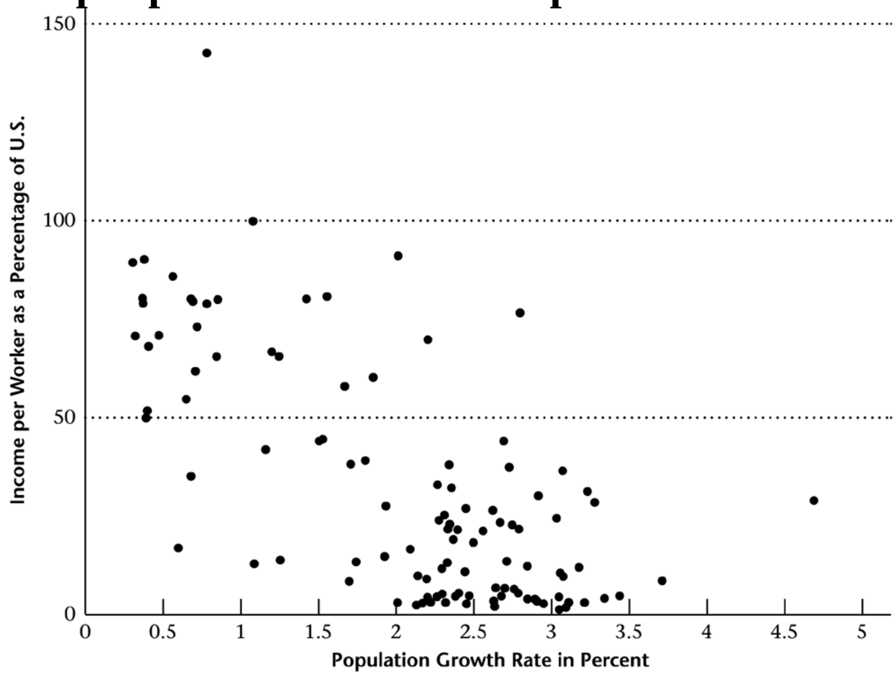
- It seems more productive countries were investing more. This makes sense.

Figure 6.2 Output per Worker vs. Investment Rate



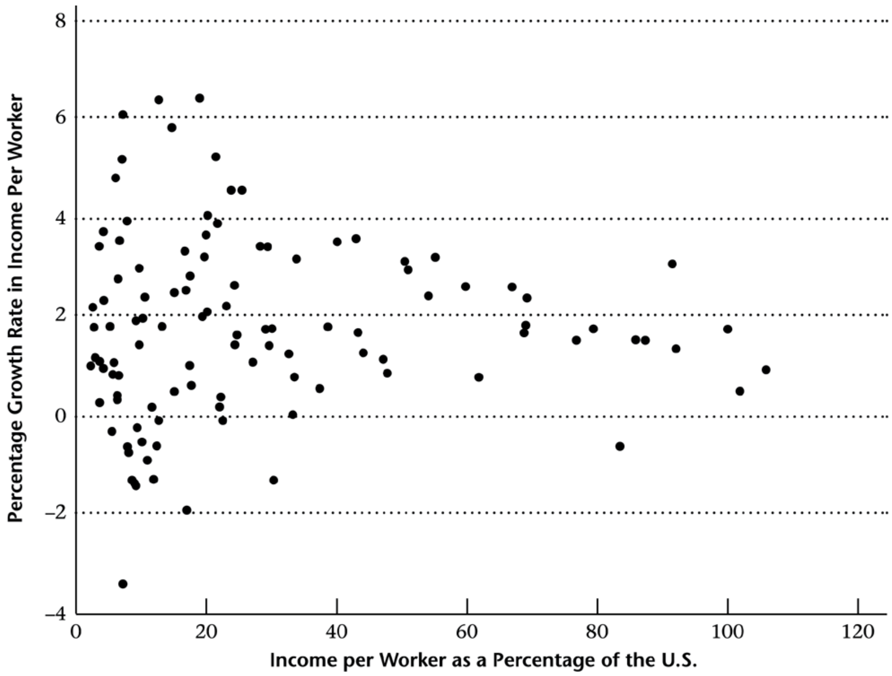
4. **There is negative correlation between the population growth and the output per worker across countries.**
    - Seems like more populated countries were less productive. This is highly unintuitive. Probably Law of Diminishing returns played a role.

Figure 6.3 Output per Worker vs. the Population Growth Rate



5. **Differences in per capita incomes increased dramatically among countries of the world between 1800 and 1950 with gap widening between the countries of Western Europe, United States, Canada, Australia and New Zealand, as a group and the rest of the world.**
    - End of colonization?
    - What else? Industrial Revolution?

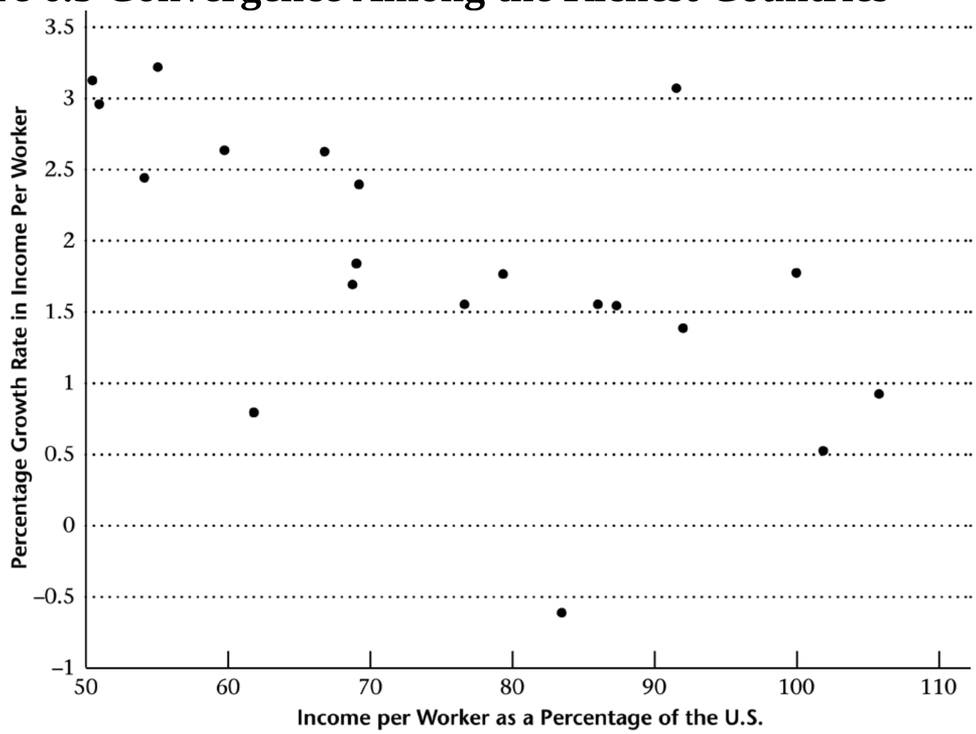
Figure 6.4 No Convergence among All Countries



6. **There is essentially <u>no</u> correlation across countries between the <u>level</u> of output per worker in 1960 and the <u>average</u> rate of <u>growth</u> in output per worker for years 1960-1995.**
   - Standard of living would converge if poor countries grow at a faster rate than the rich countries. If this happens, then there would be a negative relationship between the growth rate of income (bigger for poorer countries) and the level of income (which is smaller for poorer countries). <u>This means that the standard of living has not converged across countries.</u>

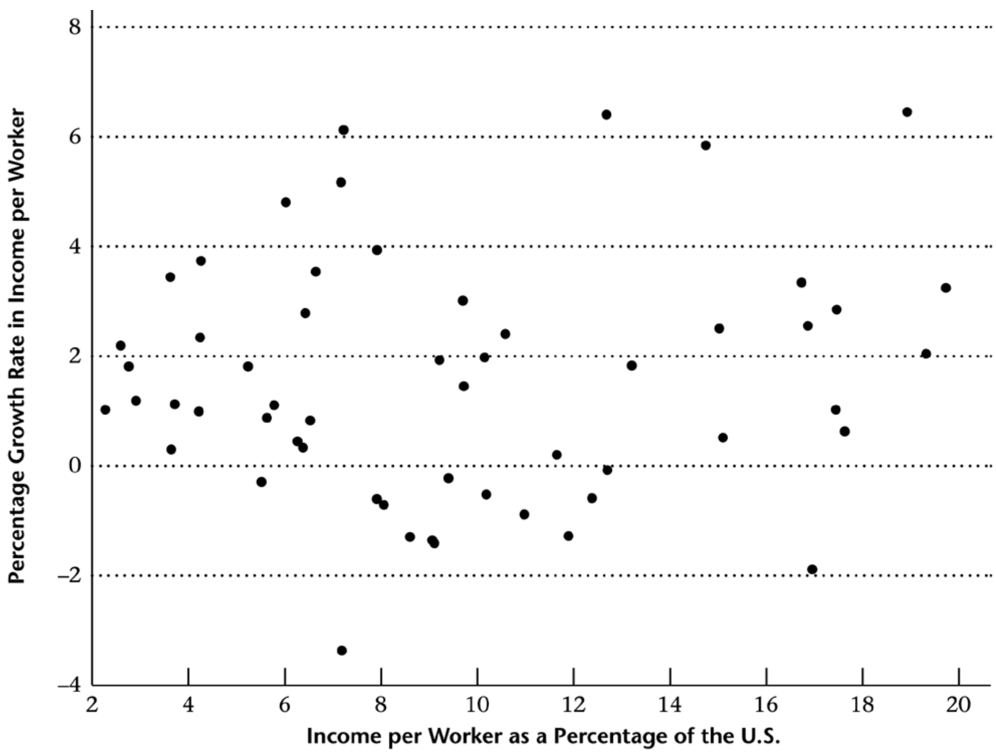
7. **Among the richest countries there is a negative correlation between the level of output per worker in 1960 and the average rate of growth in output per worker for years 1960-1995**
   - Seems like standard of living were converging among richer countries.

Figure 6.5 Convergence Among the Richest Countries



8. Among the poorest countries, there is essentially no correlation between the level of output per worker in 1960 and the average rate of growth in output per worker for the years 1960-1995

Figure 6.6 No Convergence among the Poorest Countries



Critical Thinking

Q: Why are the stylized facts of economic growth are important for us?
Answer: These facts will motivate the construction of different models of economic growth. We see that growth is probably effected by three factors:

1) Population growth.
2) Technology(Industrial Revolution)
3) Human Capital (Japan, Korea, India vs African countries)

C. MALTHUSIAN MODEL OF ECONOMIC GROWTH

1) Basic Argument of the Model

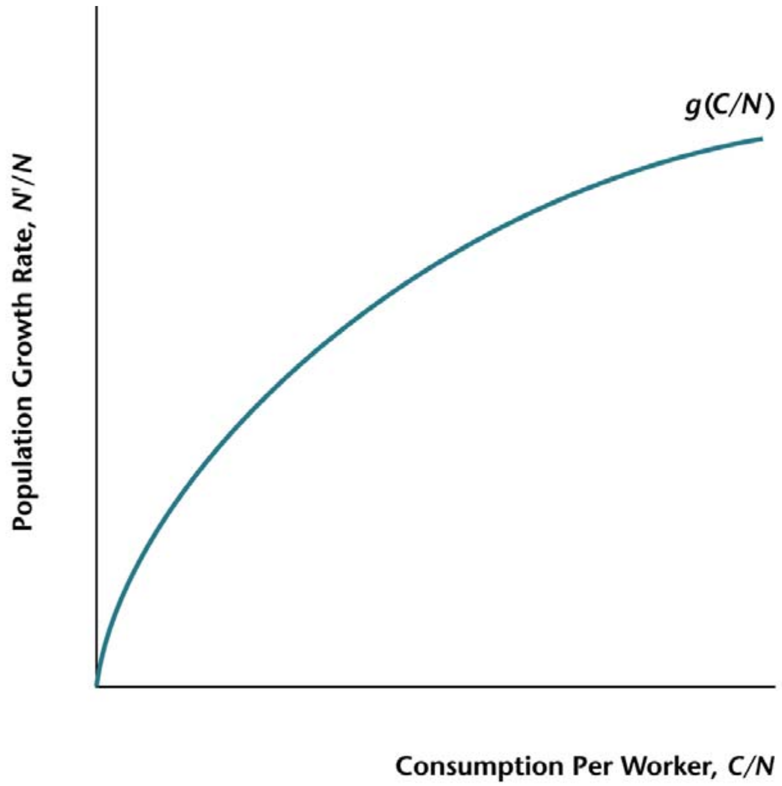
- Advance in food technology would increase population growth.
- With higher population growth the average consumption/capita falls.
- Population and consumption will grow over time.
- No increase in standards of living unless limits to growth.
- Malthus was pessimistic about increase in standard of living without intervention.

2) Formal Malthusian Model

- This is **dynamic model** but just consider 2 periods (current/future)
- The aggregate production function has two inputs, Land (L) and labor (N). The production function looks like:
  Y= zF(L,N) -------(1)
  Here Z is the **TFP** which has the usual previous properties. Y is thought as food (or perishable good) which cannot be stored (no savings in this model).
- We assume Production satisfies **CRS.**
- Land is fixed. There is **no** government.
- Workers work at a wage arte **W. Normalize N=h=1.** This means:
  a) N is both the labor force and population (representative worker).
  b) There is no leisure decision.

- Population growth depends on consumption per worker:
  $\frac{N'}{N} = g\left(\frac{C}{N}\right)$-------(2)
  Where the LHS is the population growth rate (N' is the future population) and the RHS is function of consumption per worker$\left(\frac{C}{N}\right)$. We assume that g is a concave function. Hence $\frac{N'}{N}$ is a concave function of $\frac{C}{N}$.

Figure 6.7 Population Growth Depends on Consumption per Worker in the Malthusian Model



- In equilibrium, all goods are consumed:
  C = Y (because G=NX=I=0). This gives us, by substituting C for Y in (1):
  C = zF(L, N)-----(3)
- Subbing (3) into (2):
  $\frac{N'}{N} = g\left(\frac{zF(L,N)}{N}\right)$----(4)
- CRS implies:
  xzF(L, N) = zF(xL, XN) ---(5)
  Suppose without loss of generality, we

Assume x = $\frac{1}{N}$. Then equation (5) looks like:

$\frac{zF(L,N)}{N} = zF\left(\frac{L}{N},1\right)$-----(6)

Subbing (6) into (4):

$N' = g\left(zF\left(\frac{L}{N},1\right)\right)N$ ----(7)

- Equation (7) gives future population as a function of current population.
- Assume $F(L,N) = L^{\alpha} N^{1-\alpha}$ and $g\left(\frac{C}{N}\right) = \left(\frac{C}{N}\right)^{\gamma}$.

  Then, from (2):

$$\frac{N'}{N} = g\left(\frac{C}{N}\right) = \left(\frac{C}{N}\right)^{\gamma} = \left(\frac{zF(L,N)}{N}\right)^{\gamma} = \left[zF\left(\frac{L}{N},1\right)\right]^{\gamma} = \left[z\left(\frac{L}{N}\right)^{\alpha} 1^{1-\alpha}\right]^{\gamma}$$

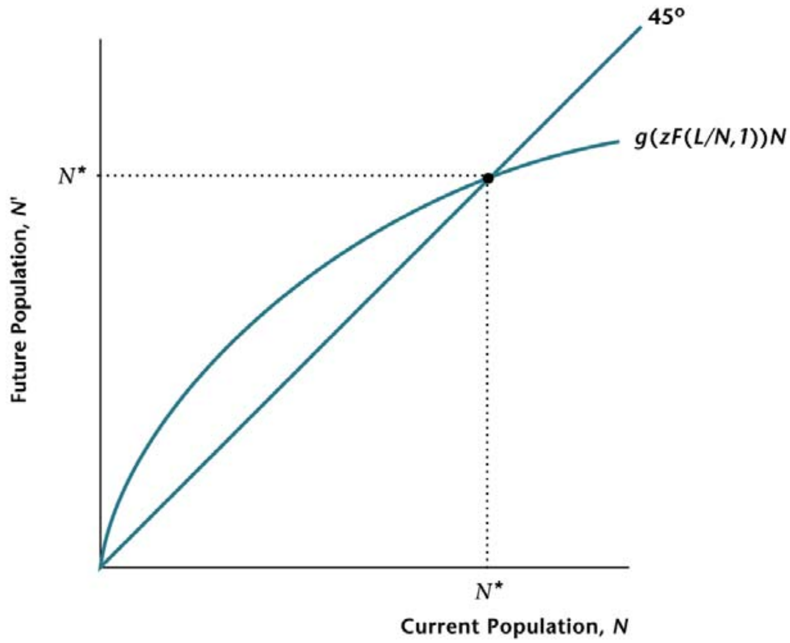
$$\Rightarrow \frac{N'}{N} = \left[z\left(\frac{L}{N}\right)^{\alpha} 1^{1-\alpha}\right]^{\gamma} = z^{\gamma}\left(\frac{L}{N}\right)^{\alpha\gamma} \text{------(8)}$$

$\Rightarrow N' = z^{\gamma}(L)^{\alpha\gamma} N^{1+\alpha\gamma}$

$0 \prec \alpha \prec 1$

With $0 \prec \alpha \prec 1, 0 \prec \gamma \prec 1$, the RHS is a concave function of N.

Figure 6.8 Determination of the Population in the Steady State



- In the graph, the equality in equation (8) is denoted by the intersection between the 45 degree line and the concave $g\left(\frac{C}{N}\right)N$ line. This is called a **steady state** point. This means that if the value of today's population is N*, then there would be a constant growth of population (in this case no population growth at all) and in future, the population would again be N* and it will **forever**.
- It also points out three important things:
  1) If N < N*(points below the intersection), then $N' \succ N$ and population increases in future.
  2) If $N \succ N^*$(points above the intersection), then $N' \prec N$ and population decreases in future.
  3) Only at N = N* population comes to a rest. Hence N = N* is the **long run** equilibrium for population.
- Since L is fixed, the long run equilibrium of consumption would be given by equation (1), which now reads: $C^* = zF(L, N^*)$

3) Analysis of Steady State in Malthusian Model

- Because of CRS, equation(1) can be written as:

  $\frac{Y}{N} = zF\left(\frac{L}{N},1\right)$

  Using $y = \frac{Y}{N}$ and $l = \frac{L}{N}$ and $c = \frac{C}{N}$ to denote output, land and consumption per worker, we have:

  $y = zf(l)$

  Where $zf(l)$ is the per-worker production function?

- Since in equilibrium, c = y, then in equilibrium,

  $c = zf(l)$ -----(9)

  Finally, equation (4) can be written as:

  $\frac{N'}{N} = g(c)$ ---(10)

  In the steady state, $N = N'$. Hence (10) can be written as:

  $g(c) = 1$

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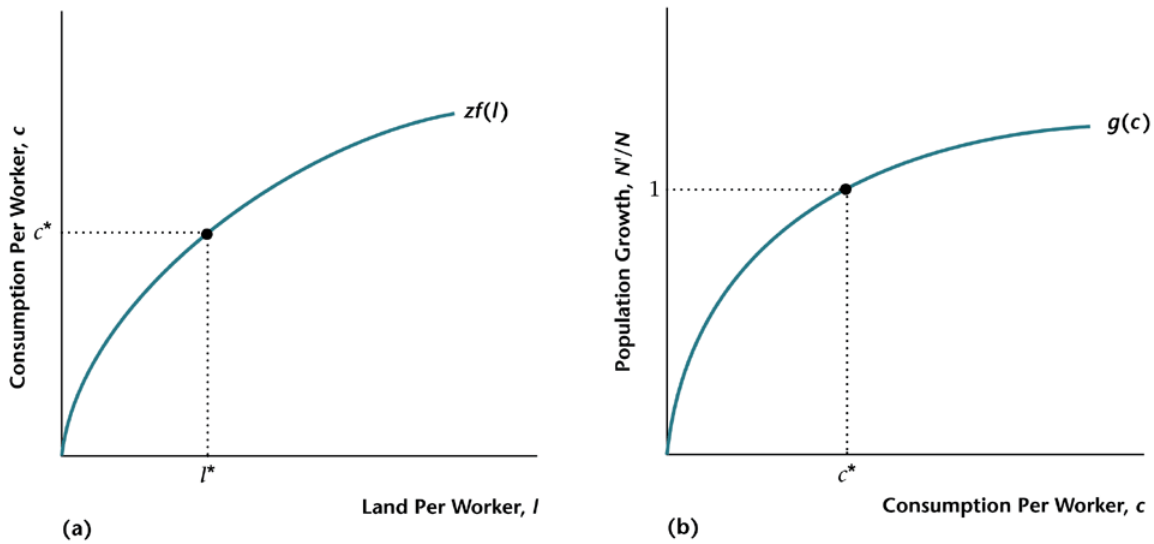
Figure 6.10 Determination of the Steady State in the Malthusian Model



- In the graph 6.10(a), we have expressed equation (9) and in graph 6.10(b), we have expressed equation (10). So, in steady state, consumption c* is determined from equation (10) which implies no population growth.
- Given, the c*, the quantity of land per worker l* is determined from equation (9). Since land is fixed, we can determine the steady state population is determined by,

$N^* = \frac{L}{l^*}$ ----(11)

Very Very Important Critical Thinking

- Nothing effects the consumption per worker in the steady state or in the long run (we can see that from figure 6.10.b).
- Given the consumption per worker in the long run, we can determine the land per worker, which determines the population in the long run (from equation 10 and 11). **Thus fixed population determines fixed standard of living with fixed land.**

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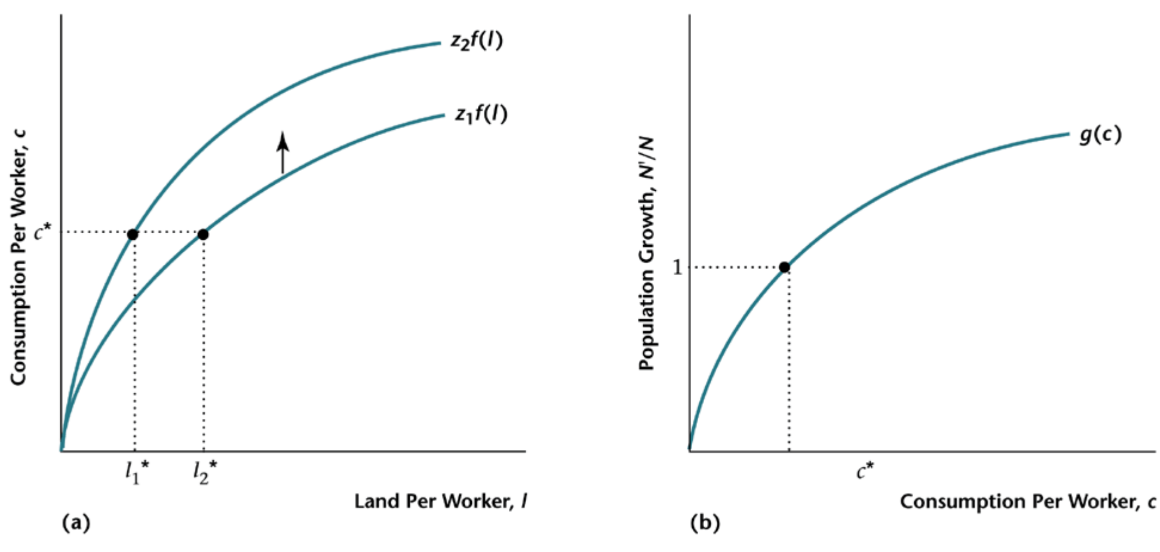
4) Experiment: Effect of z on the Steady State in Malthusian Model

- We consider an experiment where z increases. This is interpreted as an improvement of technology.
- We will consider the effect of z as follows:

a. Long Run Effect

- Production function pivots upward. This increases the productivity of the worker. Hence land per worker **decreases**. Since there was no population growth up to that point, consumption per worker would be fixed.
- There is a decline of l* from $l_1$to l_2 given the same consumption per worker c*. This is evident in figure(b) and (a)

Figure 6.11 the Effect of an Increase in z in the Malthusian Model



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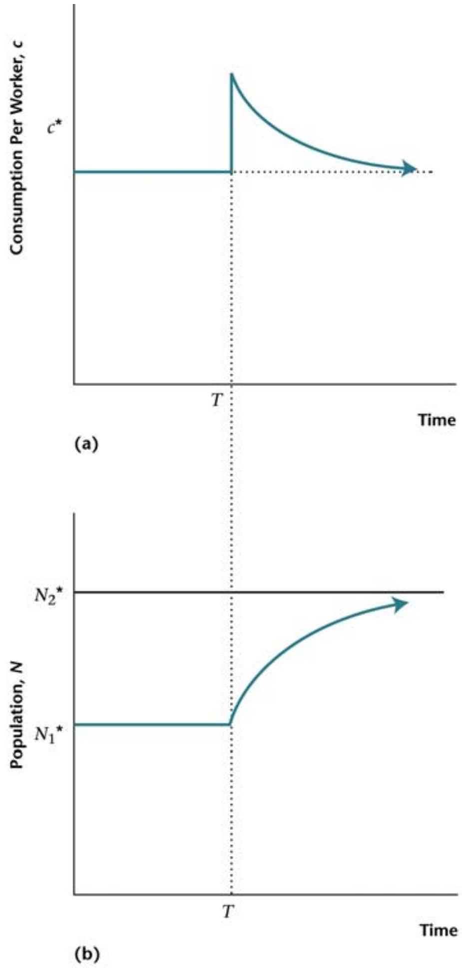
b. Adjustment Process: Going from SR to LR

- A decline of l* from l_1 to l_2 increases population from $N_1^* = \frac{L}{l_1^*}$ to $N_2^* = \frac{L}{l_2^*}$. Eventually, in the LR population will converge to their steady state value
- As an immediate effect of z at time T, consumption increases but converges to its steady state value because population will remain constant in the LR

Figure 6.12 Adjustment to the Steady State in the Malthusian Model When z Increases



Important Summary

- In the SR, an increase in the z decreases the per worker land. This productivity gain will increase the population.
- But due to the law of diminishing returns, this gain disappears in the LR. In the LR population thus converges to its steady state value.
- In the SR, in increase in z also causes consumption per worker to go up. But since in the LR population converges to its steady state value (no population growth at all), consumption per worker converges to its steady state value. Thus in the LR , we see no change in C*, N* and a decline in l*

Punch Line: Increase in z causes c* to **go up**. But this also causes N* to up. Therefore, c* eventually **goes down**. Thus there is no improvement in standard of living

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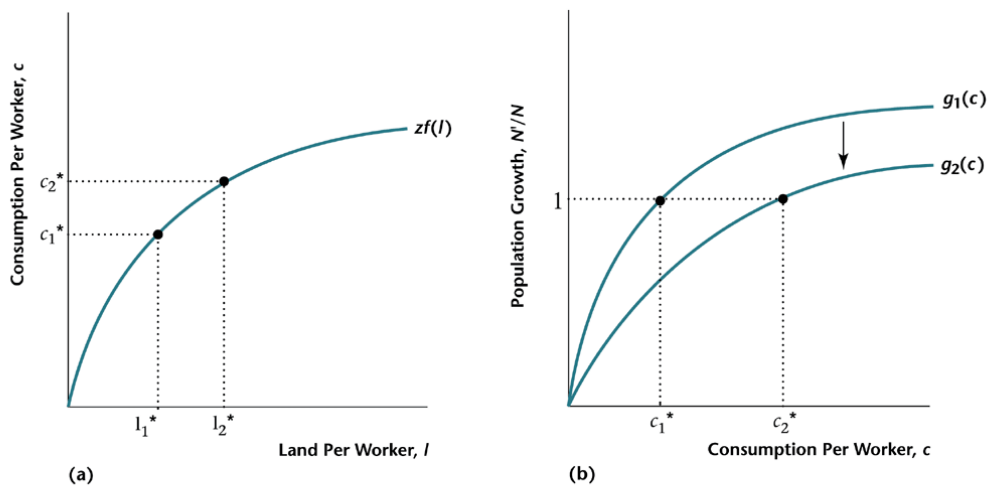
5) Implication of the Malthusian Model: Population control

a. Malthus proposed a state-mandated population control (example: one child per family in china). This would reduce rate of population growth and increase consumption per worker. This will thus increases in standard of living

Figure 6.13 Population Control in the Malthusian Model



b. In the graph (b), population control shifts the $g(c)$ function downward. This causes an increase in c* from c_1^* to c_2^*.

c. Population control also increases land per worker which increases output per worker. So, in the LR everyone is better-off.

6) Criticism of Malthus Model

- In the richest countries, we see a decline in the birth rate despite increase in productivity. This contrasts with Malthus's idea.
- There have been increases in standard of living even in the highly populated countries (India, china).
- Malthus did not allow for effect of increase in capital on the production process. The only other input in his production was land which is by nature fixed.
- Malthus did not account for the effect of other economic forces on the population growth, like reduction of birth rate because people can now raise fewer children making them more productive.

D. SOLOW MODEL: EXOGENOUS GROWTH

1) Basic Argument of the Model

- It is a simple model.
- It makes sharp predictions about the following:
    a) What are the sources of economic growth?
    b) What causes living standard to increase over time?
    c) What happens to the level and growth rate of aggregate incomer when savings rate or the population growth rate rises?
    d) What should happen to relative standard of living across countries when either population or income level increases?
- Solow model is more optimistic about prospects of long run improvements in standard of living than the Malthusian model.
- In this model sustained increase in standard of living can occur if there is technological improvement. This contrasts directly with the Malthusian model and is much more realistic.

2) Formal Solow Model

Solow model is a dynamic competitive equilibrium macroeconomic model. Hence we formulate the macro model as follows:

- Specify consumer's behavior.
- Specify Producers behavior.
- Define Competitive equilibrium.
- Look at only SS equilibrium

a) Consumer

- This is dynamic equilibrium where there are many periods. We will only focus on consumer's decision for today and future (this will also be a two period model).
- There is population growth in the economy:

  $N' = (1+n)N$ ----(12)

  Where n is the growth rate of population from today to future. This growth rate is **exogenous.**

  $n \succ -1$, So that we can have a situation where $n \prec 0$ which means that population can decrease over time as well.
- There is no leisure decision in this model. The consumer has one unit of time which he enelastically supplies as labor to the firm. Thus population is also identical to the labor force in this model.
- Consumers receive an aggregate income Y (which can be thought as consisting of labor income and dividend income). But now he faces two decisions:
  How much to consume this period?
  How much to consume next period?
  Therefore, how much to save this period?

We assume consumers consume a constant fraction of their income. The remaining is saved:

$C = (1-s)Y$ ---(13)

Therefore, we assume the consumer also saves a constant fraction of their income:

$S = sY$ --------(14)

Here s is the savings rate of the economy.

- In this model, there is no government (so no tax) and no trade. Thus the income expenditure identity looks like:

  $C + S = Y$ --------(15)

b)The Representative Firm

- The firm's production technology is same as previous:

  $Y = zF(K, N)$------(16)

  So, we now have two inputs to the production process, Labor (N) and Capital (K).
- Production technology exhibits CRS. Therefore, we can write:

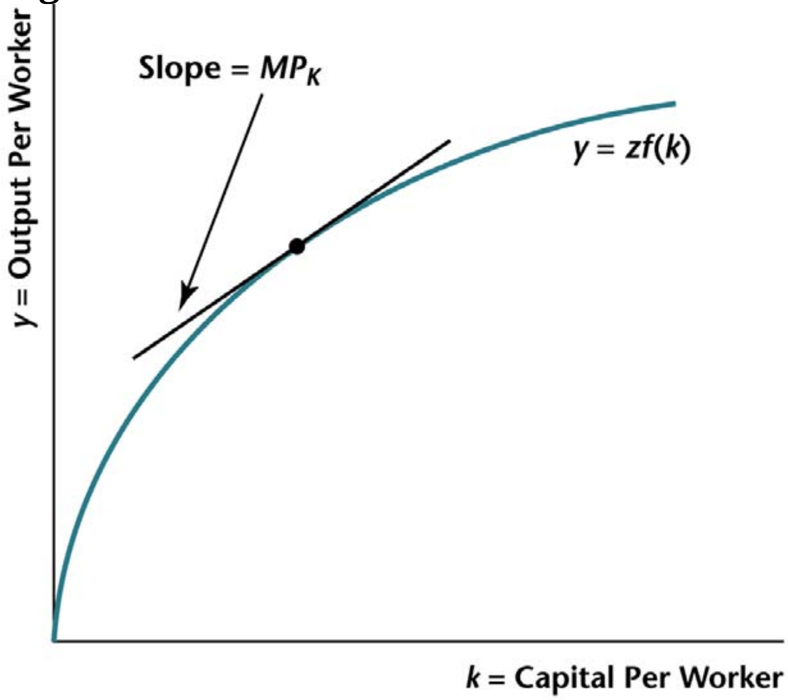
  $\frac{Y}{N} = zF\left[\frac{K}{N}, 1\right]$-------(17)

  Define $\frac{Y}{N} = y, \frac{K}{N} = k$, then (17) can be written as:

  $y = zf(k)$------------(18)

Equation (18) is the per-worker production function

Figure 6.14 The Per-Worker Production Function



- We assume that some of the capital wears out each period. Assume a constant depreciation rate of d. Also there is investment in building new capital every period. But this new capital can only comes into operation next period. Thus future period's capital stock looks like: $K' = (1-d)K + I$ -----(19)

c) Competitive equilibrium in the Solow Model

- The competitive equilibrium in this model set of prices (price of labor-real wage and price of capital-rental rate) such that:
  1. Consumers maximize their utility.
  2. Producers' maximize their profit.
  3. Market clears.

     There are three markets:
     i) **Labor Market**: Since labor is supplied enelastically, labor market always clears at any real wage.
     ii) **Capital Market**: Capital market clears if :

        $S = I$ ----------(20)
     iii) **Goods market**: Since consumer saves in this model by accumulating capital (we assume part of income which is saved can be translated into capital without any difficulty. This is an assumption), the goods market, clearing condition. considering (20), goods market clearing can be written as:

        $C + I = Y$ --------(21)
- Using equation (13) and (19), equation(21) can be written as:

  $Y = (1-s)Y + K' - (1-d)K$ ---(22)

Rearranging, we get:

$K^{'} = sY + (1-d)K$ -----------(23)

- Subbing (16) into (23):

$K^{'} = szF(K,N) + (1-d)K$ ----(24)

- Dividing both side of (24) by N:

$\frac{K^{'}}{N} = \frac{szF(K,N)}{N} + \frac{(1-d)K}{N}$ ----(25)

- Multiply both side of (25)by $1 = \frac{N^{'}}{N}$:

$\frac{K^{'}}{N}\frac{N^{'}}{N} = \frac{szF(K,N)}{N}\frac{N^{'}}{N} + \frac{(1-d)K}{N}\frac{N^{'}}{N}$

Using (12), the above equation can be written as:

$k^{'}(1+n) = szf(k) + (1-d)k$ -------(26)

Which can be further simplified to:

$k^{'} = \frac{szf(k)}{(1+n)} + \frac{(1-d)k}{(1+n)}$ ------------(27)

Equation (27) expresses future capital as a concave function of the present capital

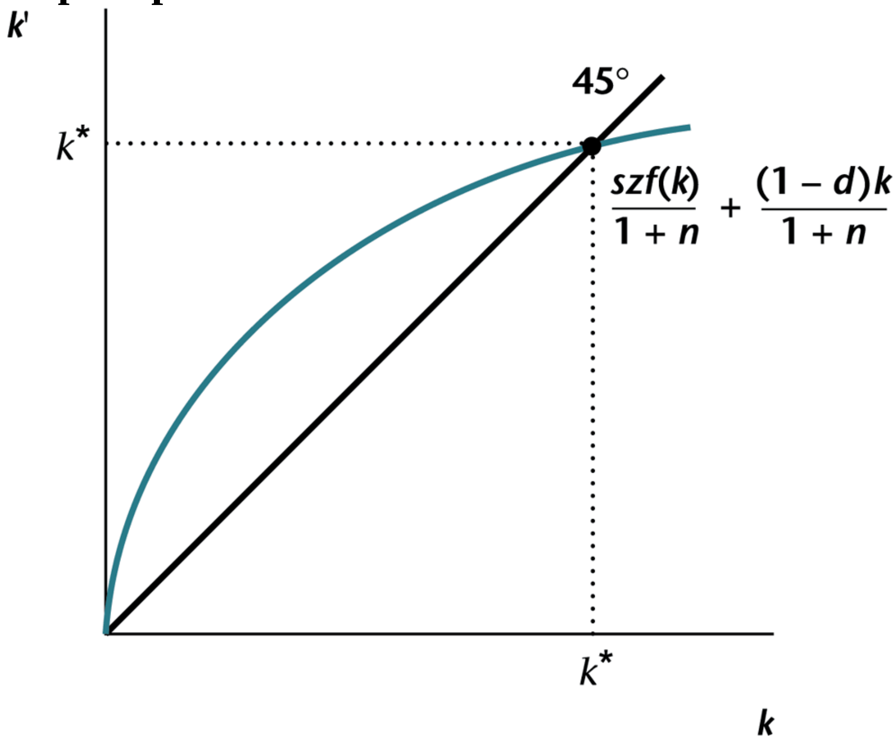
d) Steady state Competitive equilibrium in the Solow Model

The steady state CE in the solow model has to satisfy equation (27). In addition, the steady state would be a situation where

$k^{'} = k = k^{*}$

Where k* is the steady state value of k

Figure 6.15 Determination of the Steady State Quantity of Capital per Worker



- We see the following:

1. At any $k \prec k^*$, then $k' \succ k$. This means that future capital stock increases over time. So, in this case, current investment is so large that despite depreciation and population growth, per-capita capital stock increases over time.
2. At any $k \succ k^*$, then $k' \prec k$. This means that future capital stock declines. In this case current investment is not strong enough compared to depreciation and population growth.
3. Only at $k = k^*$, we see $k' = k$. Thus future capital is the same as present capital. This is the long run **steady state** equilibrium

3) Implication of Steady state analysis of Solow Model

The implication of the steady state is very important and interesting:

- In the steady state k converges to a constant value k*. This implies that in the SS, output per worker is also constant. From (18):

  $y^* = zf\left(k^*\right)$

Thus if the savings rate is constant at s, labor grows at a constant rate of n and the TFP or solow residual z is also constant, the solow model implies that output per worker is also constant. So, if output per worker is a measurement of living standard, then under the above conditions, there would not be any improvement in the living standard. This is similar to a prediction made by the Malthusian model.

- The return on investment is equal to the MP_k. Since MP_k also satisfies the law of diminishing returns, the increase in capital stock reduces the return on investment. Since in this case, we need more and more capital to produce the same output, the new investment just only keeps up with the depreciation and the growth of labor force but cannot contribute to any improvement to per capita output.
- Although capital per worker is constant in SS, the level of capital stock increases over time. From the definition of k, we see:

$$K = k^*.N$$

Since k* is constant and N grows at a constant rate of n, K must also increase at the rate of n.

- Similarly aggregate output also grows at a constant rate of n,

$$Y = y^* N = zf\left(k^*\right)N$$

- Also since: $I = sY = szf(k^*)N$
  Thus Investment also grows at the constant rate of n.
- Finally, $C = (1-s)zf(k^*)N$
  Hence consumption also grows at a constant rate of n

Critical Summary

- In steady state all the aggregate variable grows at a constant rate which is equal to the growth rate of population. Thus population growth determines the aggregate economy.
- Per capita variables such as k and y are all constant in SS. Thus there is no improvement in standard of living.
- The only way we can some changes in the per capita variable or growth in this model if any of the exogenous variables, such as n, s or z changes. Hence Solow model is also an exogenous growth model, just like the Malthusian model

4) Policy experiment on the steady state equilibrium of the Solow Model

- **Basic objective**

We will analyze how changes in the various exogenous variables affect the steady state outcome in the solow growth model. In order to do that, we will modify our equilibrium condition in equation (27): with $k' = k = k^*$, equation (27) looks like:

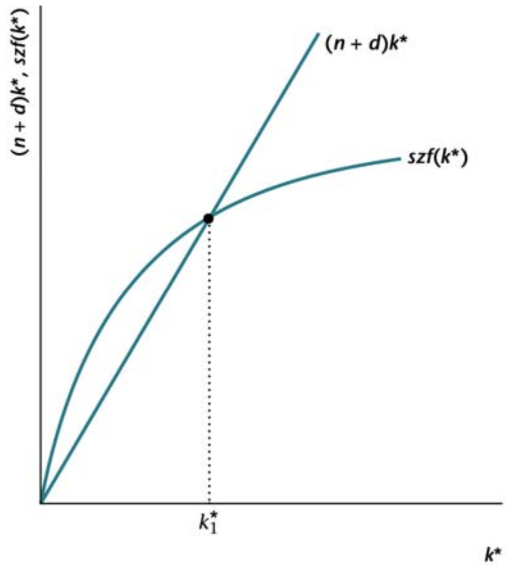
$$k^* = \frac{szf(k^*)}{(1+n)} + \frac{(1-d)k^*}{(1+n)}$$

Which can be further simplifies to:

$$szf(k^*) = (n+d)k^* \text{------}(28)$$

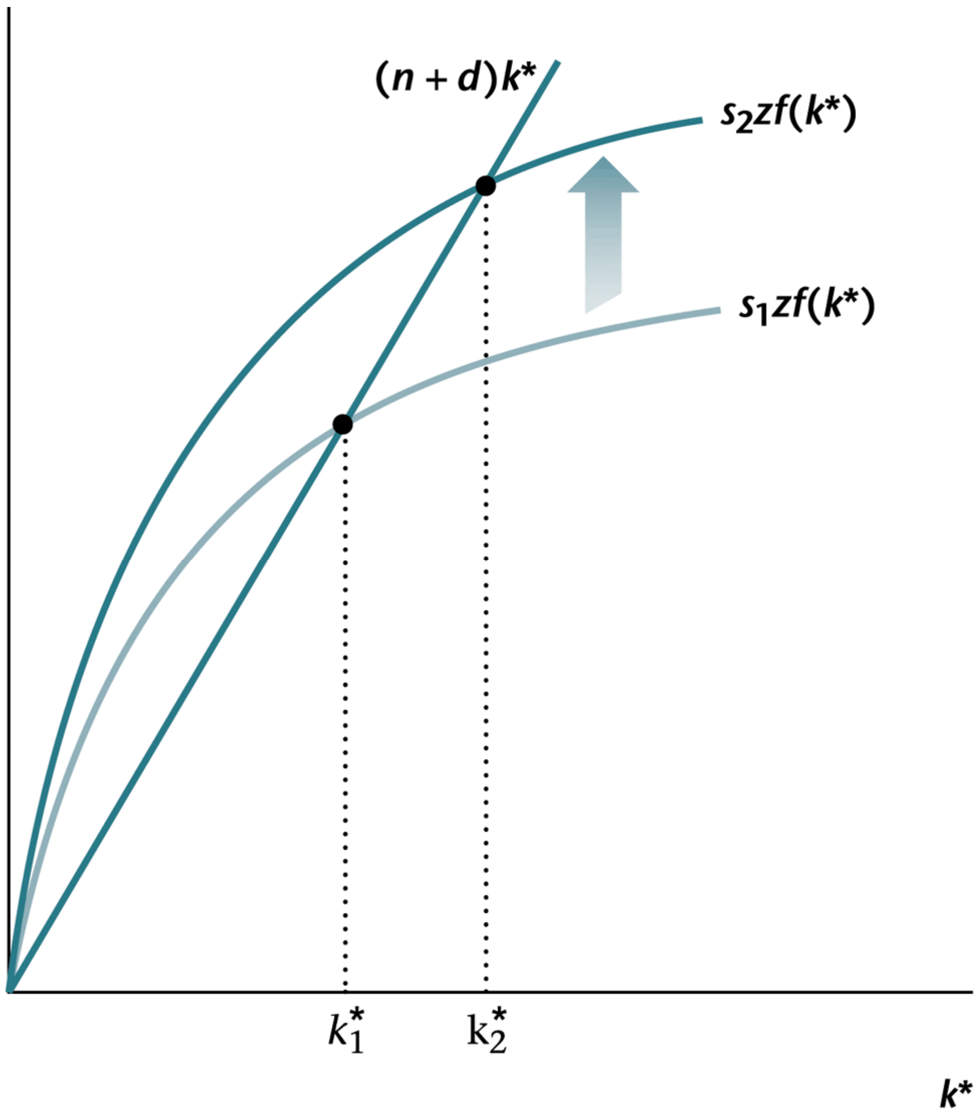
Equation (28) now determines only the steady state value of capital per worker, k^*. This equation will be very useful because if there is a change in the exogenous variables such n, s, or z, this equation will tell us how the k^* changes as well.

Figure 6.16 Determination of the Steady State Quantity of Capital per Worker



- **Effect of an increase in savings rate**
  1. An increase in the savings rate will increase the rate of investment. This will cause an increase in the capital per worker which will not only increase aggregate C, I, Y, but also increase per capita output y. thus improvement in standard of living can be achieved.
  2. In the graph below, an increase in the savings rate pivots the $szf(k^*)$ curve up. This causes steady state capital to go up from k_1^* to k_2^*. This will result in:
     a. An increase in per capita output from: $y^* = zf(k^*)$. this will also increase the aggregate income from: $Y = y^* N = zf(k^*)N$
     b. An increase in aggregate investment from: $I = sY = szf(k^*)N$
     c. An increase in aggregate consumption from: $C = (1-s)zf(k^*)N$

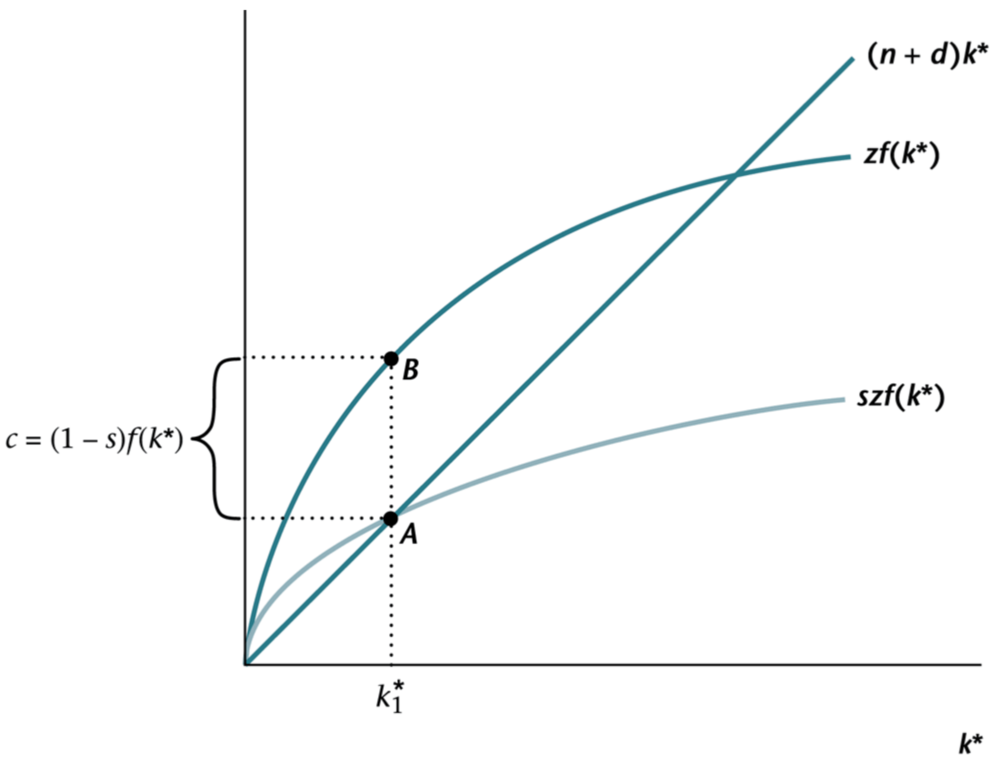
Figure 6.17 Effect of an Increase in the Savings Rate on the Steady State Quantity of Capital per Worker



- **Effect of an increase in savings rate on per capita Consumption**
    1. The effect of an increase in savings rate on per capita consumption seems **<u>obvious</u>**. **But it is not**.
    2. In steady state, consumption per worker is denoted as:
    $c = (1-s)zf(k^*) = zf(k^*) - szf(k^*) = Output(inSS) - Savings(inSS)$

    Graphically, the steady state consumption would be the vertical difference between the per worker production function $f(k^*)$ and the savings function $szf(k^*)$. This is shown in the graph below.

Figure 6.19 Steady State Consumption per Worker

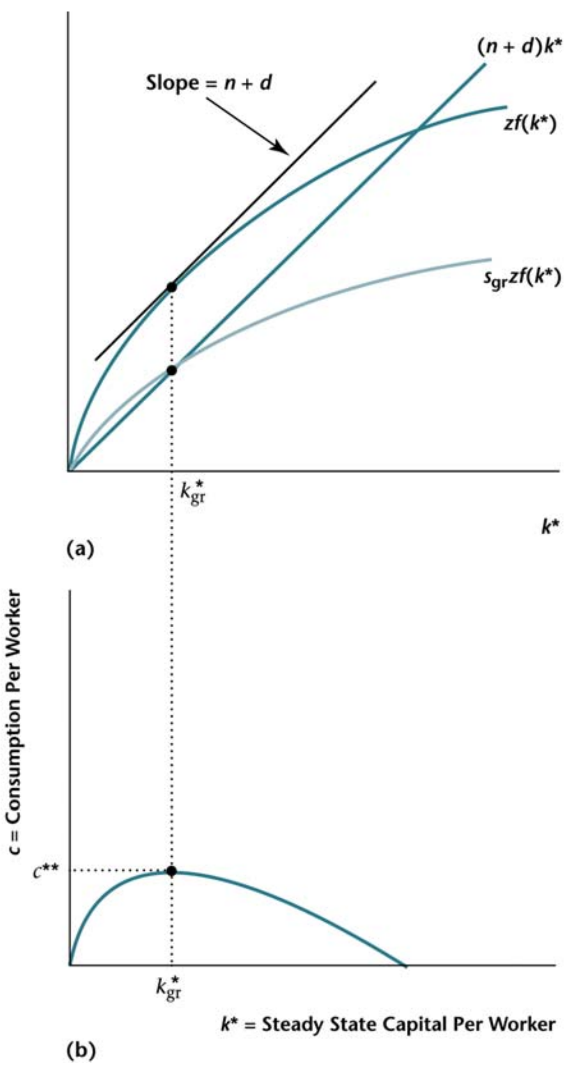


3. In the graph, steady state consumption is also the difference between per capita production and $(n+d)k^*$. Thus: $c = zf(k^*) - (n+d)k^*$-------(29)

4. Per capita consumption is maximized where the gap between $zf(k^*)$ and $(n+d)k$ is maximized. This is found at k^*_1.

<u>The steady state capital that maximizes per capita consumption is known as the golden rule quantity of capital per worker</u>. This is denoted as k_{gr}. Hence k_1 is our k_{gr} for this model.

Figure 6.20 the Golden Rule Quantity of Capital per Worker



5. At the golden rule level, from the graph, we see that slope of the per capita production function (figure 6.20.a) which is equal to the MP_K is equal to the slope of the straight line which is $(n+d)k^*$. Thus the condition for golden rule level of capital per worker is that at that capital per worker level:

$$MP_k = (n+d)k^* \text{------}(30)$$

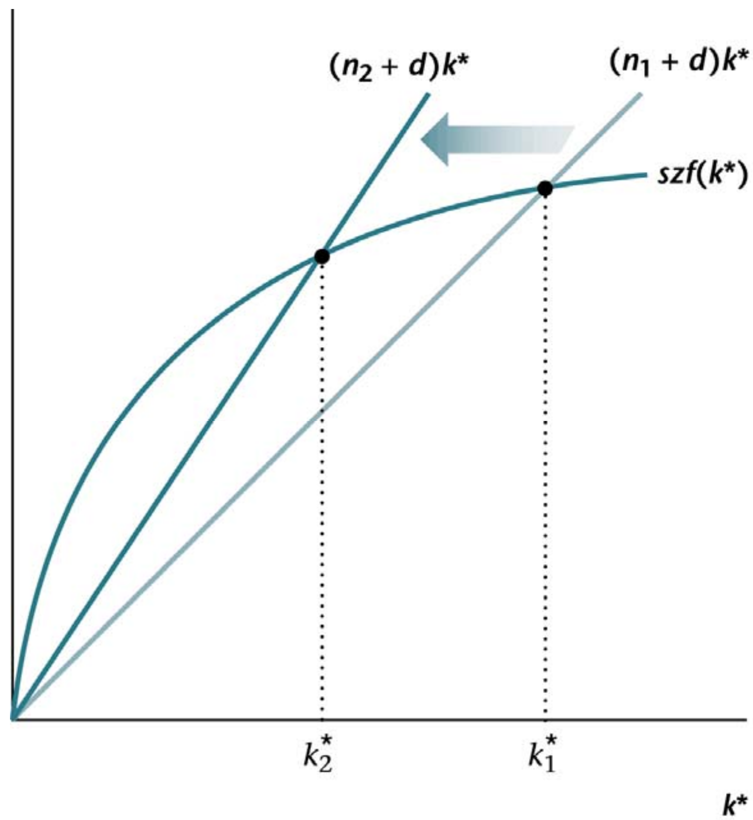
6. The savings rate at which the golden rule of capital per worker is achieved is known as the **golden rule savings rate**. This is denoted as s_{gr}.
7. The relationship between consumption per worker and capital per worker is derived in figure 6.20(b). we see:
   a. It seems that c^* increases with k^* first, reaches a maximum point, and then declines.
   b. Consumption per worker is maximized when the capital per worker reaches its Golden rule level.

Very Very Critical Summary

- When $k \prec k_{gr}$, an increase in savings rate **increases** consumption per worker, c^*
- $k \succ k_{gr}$ an increase in savings rate **decreases** consumption per worker, c^*

- **Steady state effect of an increase in the labor growth**
    1. An increase in labor growth rate forces to have an increase in the aggregate output. But the per capita output deceases. Also capital per worker decreases
    2. Graphically, we see an increase in labor growth shifts the $(n+d)k^*$ curve to the left. This causes k^* to go down from k_1^* to k_2^*. This decline in steady state capital per worker will lead to a decline in output per worker production. Thus the standard of living goes down.

Figure 6.21 Steady State Effects of an Increase in the Labor Force Growth Rate



- **Steady state effect of an increase in the total factor productivity(TFP)**
  1. An increase in TFP, Solow residual or z causes an increase in the capital per worker which is caused by a direct increase in output per worker.
  2. Graphically, we see an increase in TFP from $z_1$to z_2 shifts the per worker production function up. This causes k^* to go up from k_1^* to k_2^*. Thus the standard of living goes up too.

Figure 6.22 Increases in Total Factor Productivity in the Solow Growth Model