Chapter-4

A. OBJECTIVE OF THE CHAPTER

In this chapter, we will learn the following:

1) The difference between static and dynamic macro model.
2) Formulate a static model of representative agent and analyze his optimization decision making process. We will focus on some specific issues:
    i) Significance of substitution and income effect in the decision making(micro foundation of utility maximization)
    ii) Analyze the production function of the representative firm and relate that with the micro foundation of production maximization.
    iii) Analyze various comparative static exercise such as increasing income, change in the total productivity.
    iv) Analyze the profit maximization behavior of the representative firm.

B. DEFINITION OF A MACRO MODEL

Macro models analyze how economic decisions are made in a macro economy (as opposed to a micro model where we focus to decisions made in a micro economy) by all the agents in the economy. As mentioned in chapter-1, a macro model has the following components:

1) A specification on how the consumers and firms interact in the economy.
2) A set of goods that consumers want to consume.
3) A specification of the consumer's preference over the set of goods.
4) A specification of technology available to the firms for producing the set of goods.
5) A specification of the resources available to the economy.

C. TYPES OF MACRO MODELS BASED ON TIME HORIZONS

Based on time horizons, macro models can be divided into categories:

1) **Static:** In these models agents make decisions for one period only. This simplification is done to illustrate basic macroeconomic principles. Example: one period representative agent model.
2) **Dynamic:** In these models agents make decisions for more than one period of time, probably for their entire life. Example: two period representative agent models, Life cycle model of income hypothesis.

Critical Thinking

In the representative agent model we do not need to specify how the consumers and the producers interact in the economy because the consume is also the producer

D. SPECIFICATION OF MARKET INTERACTION

We will always assume that consumers and producers in this economy interact with each other **competitively.** This means that consumer and the producer are both price taker. Consumers will want optimize their utility subject to their budget constraint while taking the price of the goods as given. The producers would want to maximize their profit by satisfying their production objective while taking prices as given.

- Agents in the model might act competitively, but there might not have any monetary transaction. There would be competitive price in the economy but the price would not be paid in terms of money. An economy like this is known as **Barter Economy.**

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E. THE REPRESENTATIVE CONSUMER

Representative Agent: Robinson Crusoe

-Ron Embleton

1) **Specification of Goods**

We will analyze a very simple economy where the consumer has only two goods to consume, a consumption good(**C**) and leisure (**L**).

2) **Specification of preference**

Consumer's utility over the goods is defined by a Utility function:

U=U(C, L)

- Consumer's preference and hence his utility function satisfies some general properties of preference:
  - **a) Strict preference relation**: if consumer has two bundle of goods (C1,L1) and (C2,L2) to choose from and he wants to choose the first bundle, he can that if and only if bundle 1 strictly gives him more utility. So, strict prefers imply**,**

    $$U(C_1, L_1) > U(C_2, L_2)$$

ii) **Indifferent preference relation**: If both the bundle gives the consumer the same utility, he is indifferent between them. It is denoted as,

$$U(C_1, L_1) = (C_2, L_2)$$

iii) Weak preference relation: it is defined as,

$$U(C_1, L_1) \geq (C_2, L_2)$$

- Consumer's preference relation would also be complete, continuous and transitive. It means the following:
  a) **More is always preferred to less**. More of one or both goods in the bundle give the consumer more utility.
  b) **The consumer like sto diversifies his or her consumption bundle.** Consumer likes to consume more combination of goods rather than only one good.
  c) **Consumption and Leisure are both Normal Goods**. A good is a normal good for the individual if his consumption for those goods increases when his income increases. If it does not, then the good is an **Inferior good**.

3) Graphical representation of consumer preference

Graphically, the consumer preference is represented by an Indifference curve(IC). The IC connects a set of points, with these points representing consumption bundles among which the consumer is indifferent.

- The indifference curve has two properties**:**
  a) **It is downward sloping**. This follows from property 1 consumer preference.
  b) **It is convex to the origin, or bowed in towards the origin**. This follows from property 2 consumer preference.
  c) **Higher IC implies Higher Utility**.

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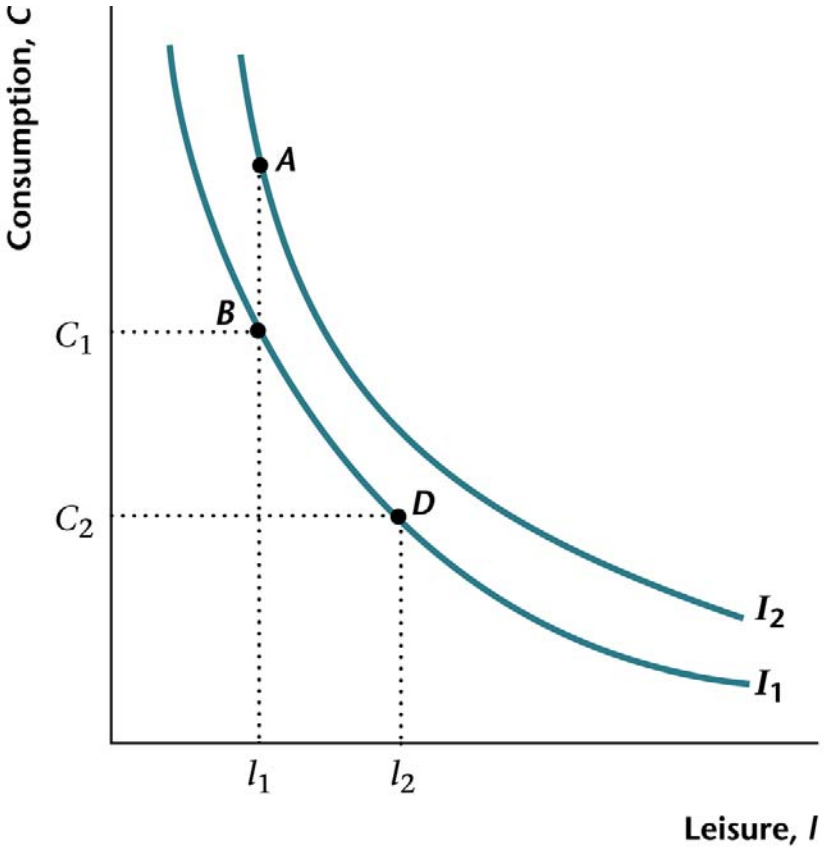
Figure 4.1 indifference curves



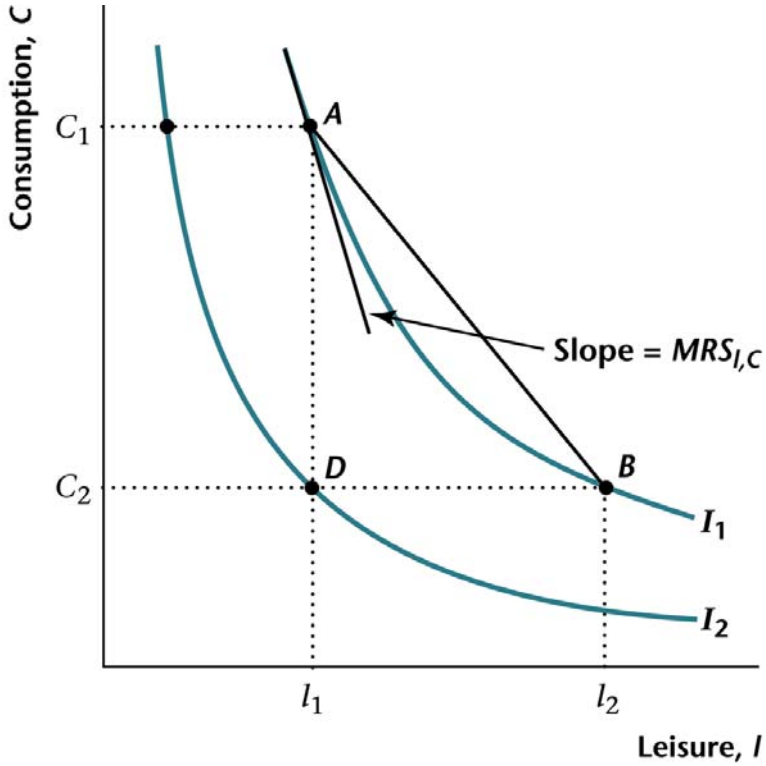
- **The Marginal Rate of Substitution,** denoted as $MRS_{l,c}$, is the rate at which the consumer is willing to substitute leisure for consumption goods. Graphically, the MRS is represented by the slope of the IC. **MRS is always negative**. Mathematically,

  $U = U(C, l)$

  $dU = MU_C dC + MU_l dl = 0$

  $MRS = dC/dl = -MU_l/MU_C$

Figure 4.2 Properties of Indifference Curves



4) Specification of Consumer's constraint

a) **Basic Intuition:** You cannot always get everything you like. You always have some income restriction or other resource constraint. This is summarized in the budget constraint and the time constraint.

b) Consumer's working **time constraint** or **labor supply** constraint:

$L + N^S = h$

Where, L = leisure

N^S = Working time

h= total time available (24 hours)

c) Consumers budget constraint would be a summary of his income and his consumption possibilities:

- We assume the consumer has three sources of income:
  a) The consumer earns wage by working. By spending one unit of labor time, he gets ***w*** unit of consumption good. Thus his wage is actually his **real wage** because the price of the consumption good is assumed to be 1(Consumption good is the numeraire good). His total **labor income = wN^S**
  b) The representative consumer is also the owner of the firm. So, he gets part of the firm's profit. This is **dividend income** and is denoted as π.
  c) The consumer also pays a **lump sump tax,** denoted a **T.** This is his negative income.
- The **budget constraint** of the consumer can be summarized as follows:
  $C = wN^S + \pi - T$
  Substituting the time constraint:
  $C = w(h - L) + \pi - T$
  Here real wage is the price of "leisure"
  With further simplification:
  $C + wL = wh + \pi - T$
  Here, RHS = real disposable income of the consumer
  LHS = implicit expenditure on two goods
- Mathematically, the equation for the budget line looks like: $C = -wL + wh + \pi - T = -wL + (wh + \pi - T)$ **----(1)**
  Where the first term on the RHS includes slope (w) and the second term includes the vertical intercept.

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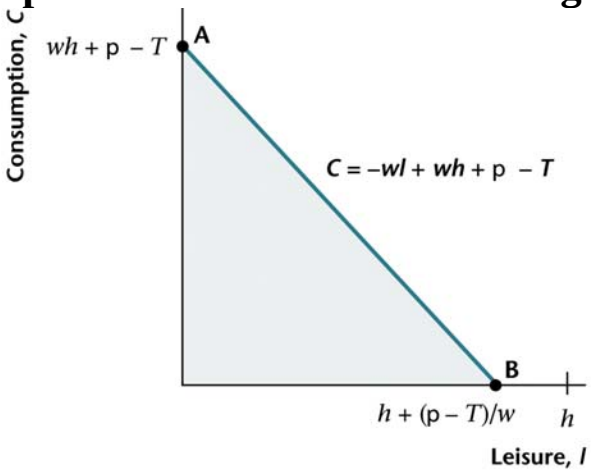
5) Graphical Representation of Different Kind of budget constraint

We will focus on two different kinds of budget constraint:

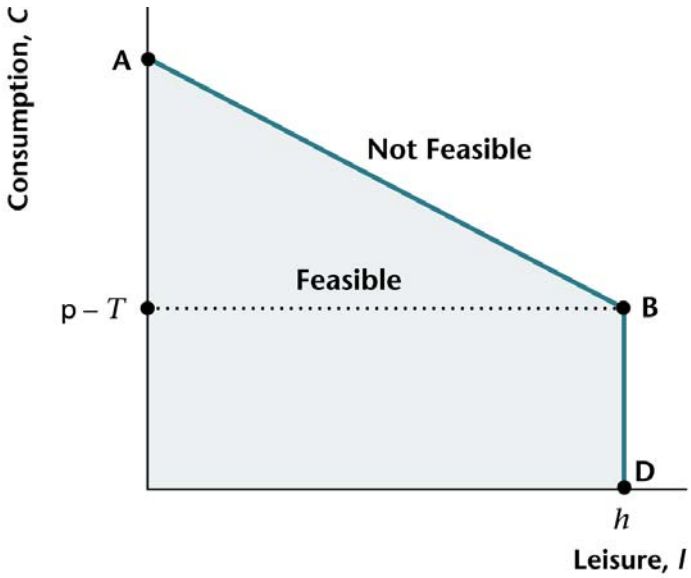
a) **When T > π: T**his is the normal case

Figure 4.3 Representative Consumer's Budget Constraint (T > π)



b) **When T < π:** We have a kink in our budget constraint.

Figure 4.4 Representative Consumer's Budget Constraint (T < π)



Critical Question: why do we have a kinked Budget line when T < π?
Answer: The consumer cannot consume leisure more than ***h*** hours. At point B, leisure is h =L. But even at that level the consumer can consume some positive level of consumption goods,
With h=L, equation (1) gives, C = π-T > 0. By throwing away some of his profit income, the consumer can consume some consumption goods and still enjoy full level of leisure

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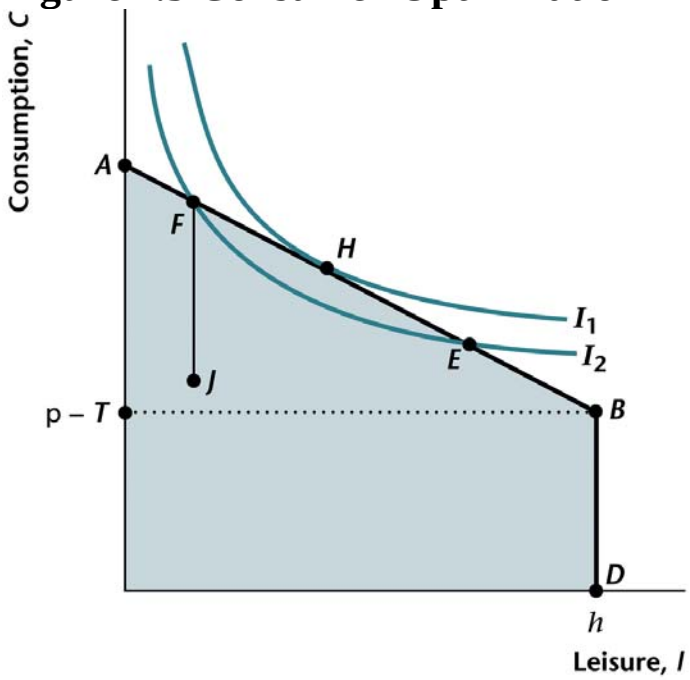
6) Consumer Optimization

- Preferences + constraints = optimized choices (Rational)
- Optimal consumption bundle (c,l) pair that is highest possible U that is in feasible set.
- Optimal consumption bundle is the (c,l) pair that is on the highest IC and is on or inside the consumer's budget constraint.

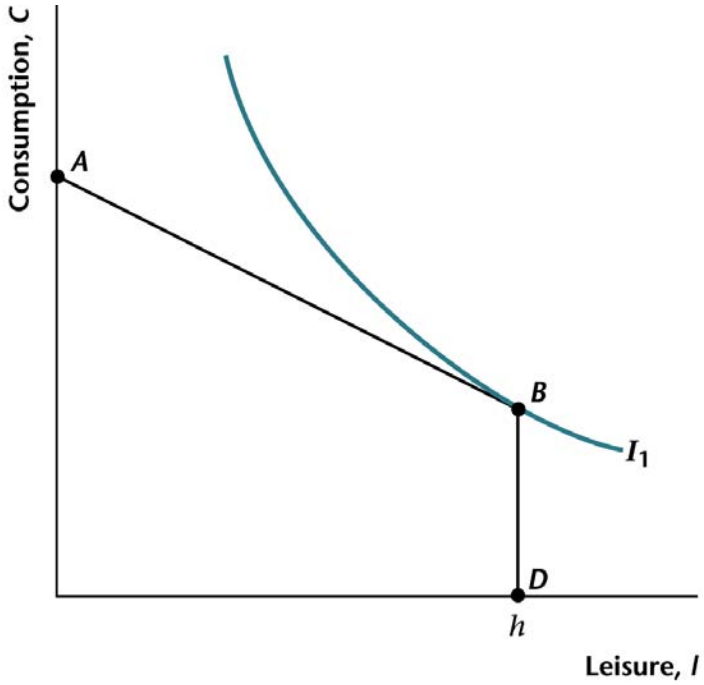
Figure 4.5 Consumer Optimization



- In the graph, Point H is the optimal consumption bundle.

Critical Question: Can we have optimal consumption bundle chosen at the kink?
Answer: Optimal consumption bundle at the kink is a **theoretical possibility**. In figure 4.6, consumer can choose point B. but at point B, consumer does not work at all. This is **inconsistent** with actions taken by firms and consumer (if consumer does not work, firms cannot produce anything). Therefore, ***this situation cannot happen***

Figure 4.6 the Representative Consumer Chooses Not to Work



- At the optimal point, the IC is tangent to the budget line. Therefore, at the optimal consumption bundle,
  Slope of IC = MRS_{lc} = Slope of the budget line = w
  The fore, at the optimum,
  MRS = relative price of leisure in terms of consumption goods (w)

Critical Question

Why do we want to equate the MRS with the relative price at the optimum?

7) Policy Experiments

- We will analyze how does the consumer's optimal decision changes when ever there are changes in his:
  a) Profit income or tax.
  b) His wage.

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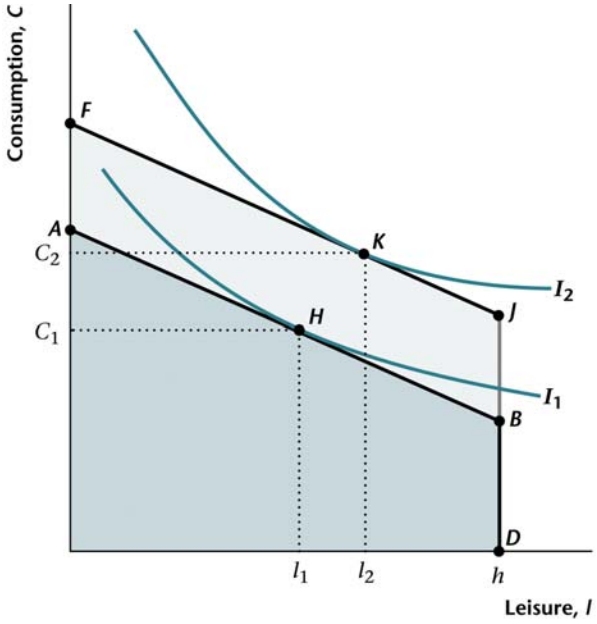
7. A) Effect of a Decrease in Lump Sum Taxes or increase in π

- The effect of increase in π or decrease in T is the same, they increase π -T.
- We will only consider case when $\pi > T$
- An increase in the π -T is a **Pure Income effect,** because this only increases the disposable income of the consumer.
- There is **no Substitution Effect because** the relative price of the goods (w) is unchanged.
- An increase in the π -T leads to an **upward shift** of the budget line. This leads to a new equilibrium between a higher IC and the new upward shifted Budget line. This results in a new optimal consumption bundle that involves **more** of both goods.

Figure 4.7: An Increase π – T for the Consumer



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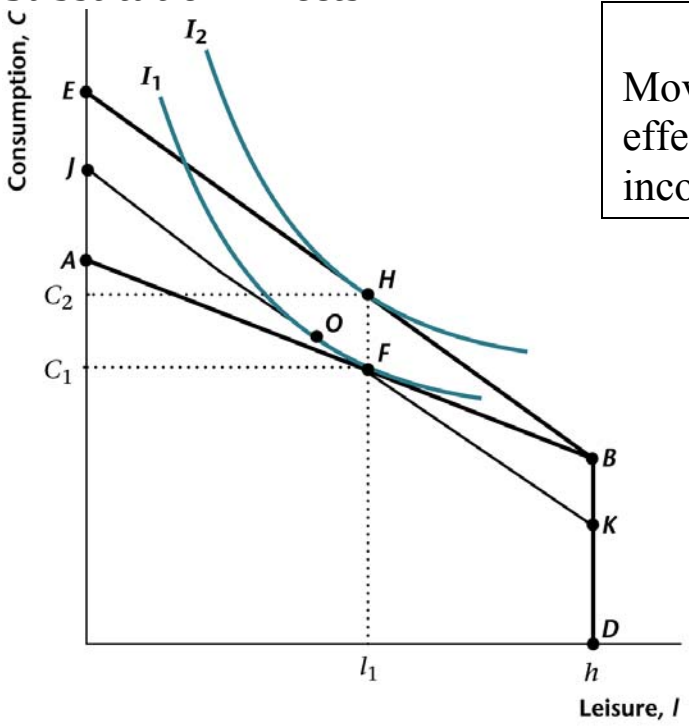
Critical Question

Why do we have an increase in the consumption of both goods when there is **only** an increase in income?

7. B) Effect of an increase in wage

- Increase in the wage (relative price of leisure) causes the budget line to pivot around the kink.
- **Substitution effect** causes consumption (cheaper) to increase and leisure (more expensive) to decrease.
- **Income effect** causes for both consumption and leisure to increase.
- **The total effect (TE) or PE** will depend on the relative strength of SE and IE.
  a) If SE>IE, leisure goes down
     (PE negative)
  b) If SE<IE, leisure goes up
     (PE positive).

Figure 4.8 Increase in the Real Wage Rate—Income and Substitution Effects



Note

Movement from F to O is substitution effect and movement from O to H is income effect

Critical Question

Q: What happen to consumption of leisure in figure 4.8? Why did this happen?
Q: Can we write the decomposition of PE for leisure and for C?

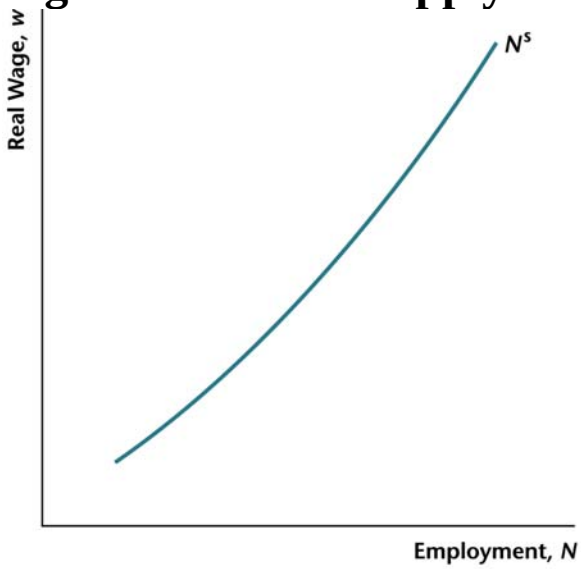
8) Labor Supply decision of the consumer

- **The labor supply curve relates** labor supply to the price of labor
- Ns (w) = h - l (w)
- The effect of an increase in wage on the supply of labor is summarized as follows:
- $\frac{\partial N^S}{\partial w} = -l'(w)$ =?
- The sign of $l'(w)$ is ambiguous. Hence the effect of an increase in wage on the labor supply is also ambiguous. So, whether the labor supply curve is upward sloping or downward sloping will depend on the sign of $l'(w)$
- If $l'(w)$>0, labor supply goes down
- If $l'(w)$<0, labor supply goes up
- If SE < IE, $l'(w)$ > 0(Consumption of leisure goes up)

  Then $\frac{\partial N^S}{\partial w}$<0

  If SE > IE, $l'(w)$< 0, then $\frac{\partial N^S}{\partial w}$>0.

Figure 4.9 Labor Supply Curve



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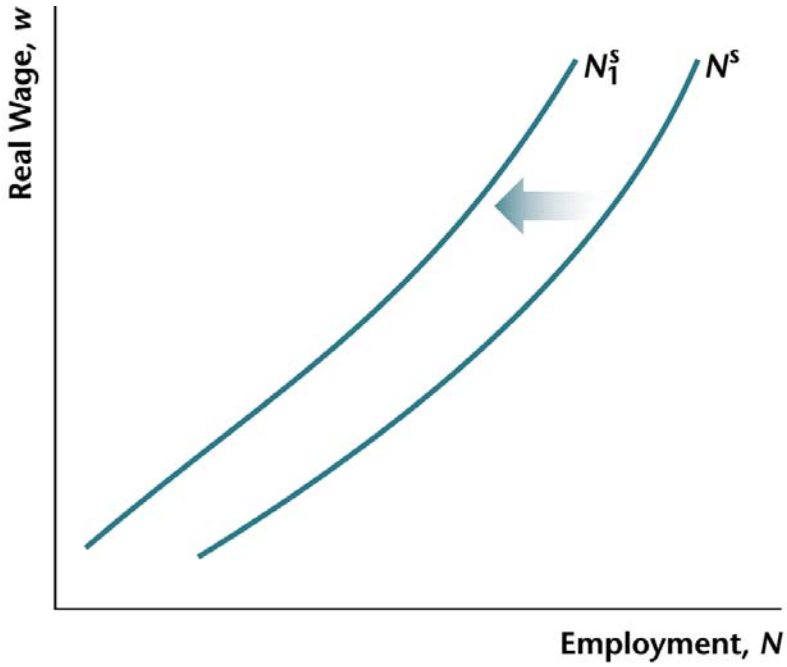
- The effect of an increase in π -T is always positive because it is an exogenous increase in income. It also is a pure IE. Therefore, it causes a shift in the labor supply curve.

Figure 4.10 Effect of an Increase in Dividend Income or a Decrease in Taxes



Critical Question

Q: Is Leisure a normal good? Is labor Supply curve actually upward sloping?
Answer: There is a lot of controversy over this. Some people claim that at a lower level of real wage, leisure is a normal good. But at a higher real wage, leisure is an inferior good. So, at the lower level, when there is an increase in real wage, we see an upward sloping labor supply curve. But at a higher real wage, if wage increases, labor supply actually goes down. Hence people claim that Labor supply curve is actually **backward bending.** This implies that Leisure probably has **Inferior Good Syndrome**

9) Three Examples of Consumer Preference

- We will talk about one general example and two extreme examples of consumer preference and analyze how these preference behaviors affect our analysis of optimal consumption bundle.
- **Cobb –Douglas Preference**: The consumers Utility function looks like:

  $U(C,L) = C^{\alpha}L^{1-\alpha}, \alpha \in [0.1]$

  The budget constraint is the same as before,

  C + wL = wh + π -T

  The Lagrange,

  £ = $C^{\alpha}L^{1-\alpha} + \lambda\{\text{wh} + \pi - \text{T} - \text{C} - \text{wL}\}$

  First order conditions,

  $\frac{\partial £}{\partial \text{C}} = \alpha C^{1-\alpha}L^{1-\alpha} + \lambda(-) = 0$

  $\frac{\partial £}{\partial \text{L}} = (1-\alpha)C^{\alpha}L^{-\alpha} + \lambda(-w) = 0$

  Solving, we get,

  $L = \frac{(1-\alpha)C}{\alpha w}$

  Substituting that into the BC:

  $\overset{*}{C} = \alpha(wh + \pi - T)$

  $\overset{*}{L} = \frac{(1-\alpha)(wh + \pi - T)}{w}$

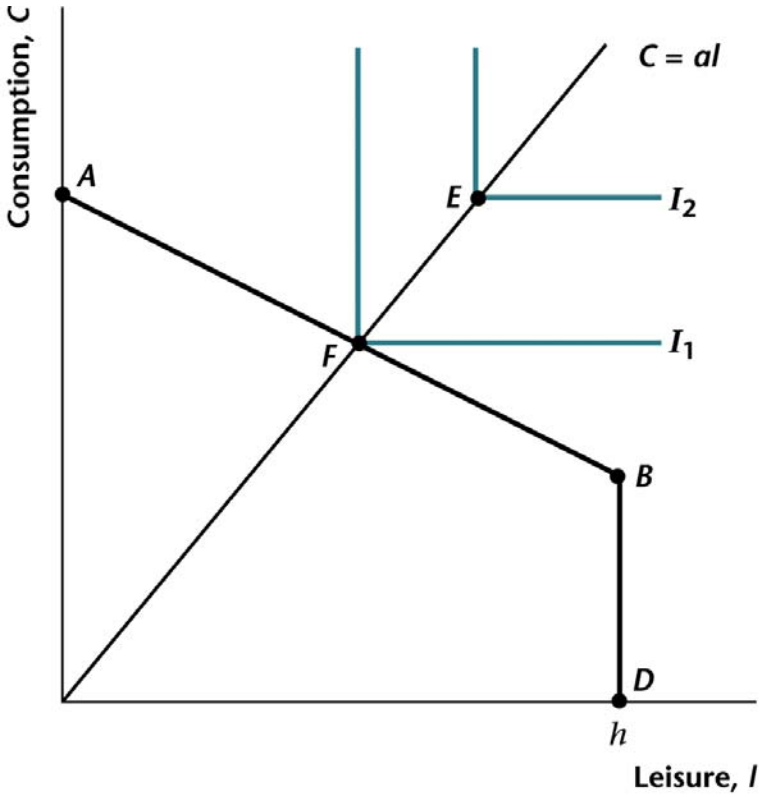
  Graphically, the equilibrium looks like figure 4.5
- **Perfect Complements Preference**: The consumers Utility function looks like:

  $U(C,L) = Min(\frac{C}{a}, L)$

  This implies: $\frac{C}{a} = L \Rightarrow C = aL$

  Subbing in the BC: $L = \frac{wh + \pi - T}{a + w}$ and $C = \frac{a(wh + \pi - T)}{a + w}$

Figure 4.11 Perfect Complements



- **Perfect Substitute Preference**: The consumers Utility function looks:

$$U(C,L) = \frac{C}{a} + L$$

Critical Thinking

Do the rest as Homework. Show equilibrium graphically

10) IE and SE and Labor Supply in the USA

- Real wage has an upward trend in USA. But in some part real wage is declining and in some part real wage is increasing
- Average weekly hours of work (labor supply) has a downward trend in USA. In some part labor increases and in some part labor supply decreases.

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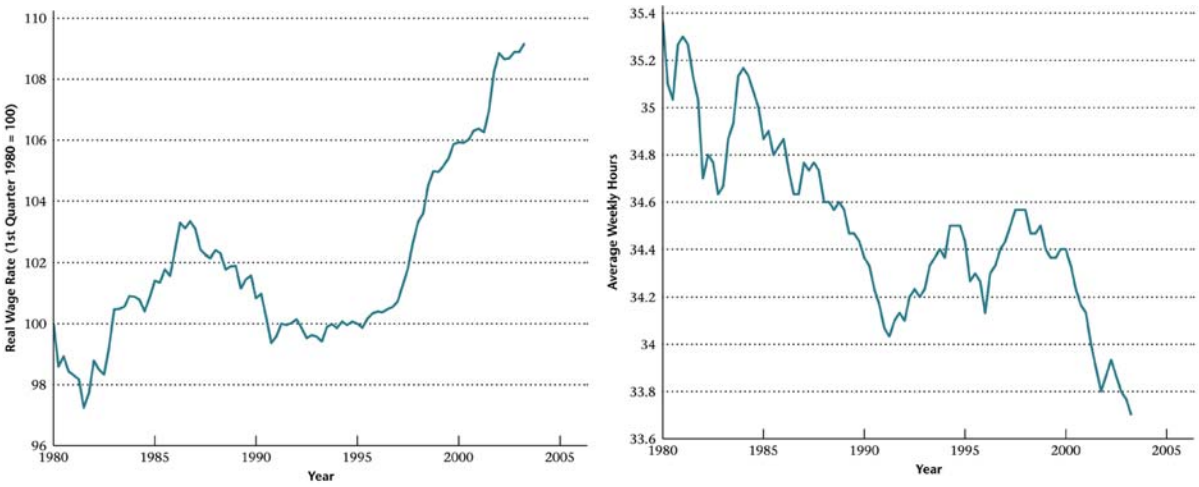
- In 1980-1990, labor supply and real wage had a **Negative relationship.**
- 1990-95, they had **positive relationship.**
- 1997-2005 they again had **negative relationship.**
- **So the labor supply trend in USA is very puzzling.**



Very Important Critical Thinking

Q: How can we have a downward sloping labor Supply curve?

Answer: two ways:

a) If leisure is normal good but SE < IE, or

Critical Thinking

Q: How can we explain the trend in labor supply for USA?

Answer: Two clues:

1. Decomposition of IE and SE and their relative strength in different times.
2. Inferior good syndrome

F. THE REPRESENTATIVE FIRM

1) Specification of Production Technology

The Production technology of the firm is represented by the Production Function (PF) which describes the technological possibilities for converting factor inputs into outputs(Y).

- We will assume the firm has only two inputs, Labor (N^d) and Capital (K).
- The algebraic expression for PF is as follows:
  $Y = zF(K, N^d)$
  Where: **Y**=output of the consumption good
  z=Total Factor Productivity
  K=quantity of capital inputs in the production process.
  N^d= Quantity of labor inputs measured as the total number of hours worked by employees of the firm
  F=Function describing the production relationships between the inputs.
- We are looking at a one period static model of production. Hence this is a short run production technology. Hence K assumed to be **fixed** and N^d is assumed to be **variable**.
- The term Z captures the technological sophistication of the production process.
- **An increase in z will lead to an increase in the productivity of both inputs** (Handmade Candy VS Hershley, Homemade VS bakery Bread).

2) Graphical Representation of the Production Function

- Graphically, the production function is represented by a concave upward sloping graph**.**
- **The marginal Product (MP)** of a factor of production is the additional output that can be produced with one additional unit of that factor input, holding all other quantities of inputs. Graphically, MP of different

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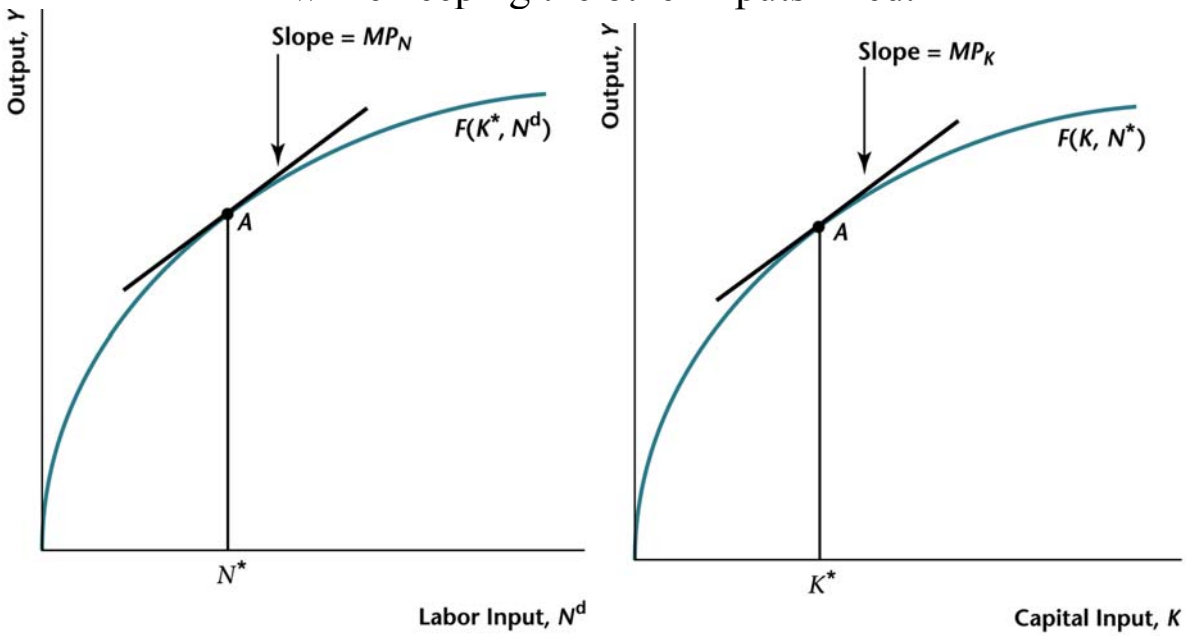
inputs would be the slope of the production function while keeping the other inputs fixed.



3) **Properties of the Production Function**

The Production function has five properties:

*a) The production exhibits **constant returns to scale** (CRS). This means for any constant x:*

$$zF(xK, xN^d) = xzF(K, N^d)$$

- The idea of CRS is that if all the inputs are doubles, then output doubles as well.
- **Increasing returns to scale** (IRS) means that doubling the inputs increases the output more than double.
- **Decreasing returns to scale** (DRS) means that doubling the inputs increases the output less than double.

Critical Thinking

1. IRS is a feature of large firms. So IRS means that large firms are more efficient than small firms.
2. DRS is a feature of small firms. So, DRS means that small firms are more efficient than large firms.
3. CRS means that the economy behaves the same as if they were many small firms or if there are few large firms. That is why our representative firms have CRS so that the difference between many small firms vs few large firms are **unimportant**.

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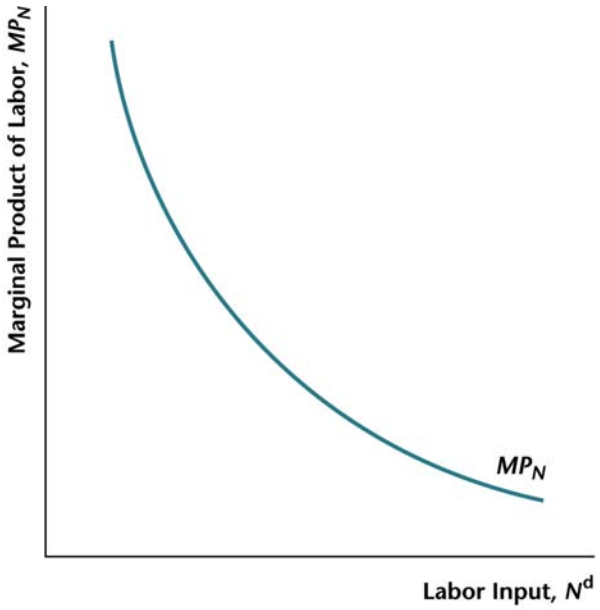
b) The production function has the property that output increases when either the capital input or the labor input increases.

- This means that $MP_N>0$, $MP_K>0$
- Graphically it means that the PF has a positive or upward slope.

c) The marginal product of labor decreases as the quantity of labor increases.

- Graphically, the slope of the PF decreases as labor (left figure above).
- This means that labor follows the **Law of Diminishing returns.**
- This means that the MP_N is a downward sloping curve

Figure 4.16 Marginal Product of Labor Schedule for the Representative Firm



d) The marginal product of capital decreases as the quantity of capital increases.

- Graphically, the slope of the PF decreases as capital (right figure above).
- This means that capital follows the **Law of Diminishing returns.**

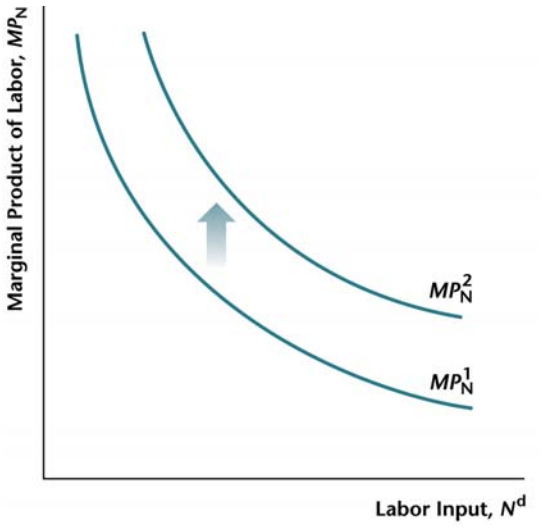
e) The marginal product of labor increases as the quantity of capital input increases.

- Labor and capital are thought as complements in the production process.
- An increase in the capital shifts the MP_N curve to the right.
- Suppose two people are working in one machine. If there is an increase in the machine, the marginal product of both workers can increase because they can **specialize**.

Critical Thinking

Increasing the capital can lead to **specialization** because each worker can work only on one type of job. Thus Specialization creates **division of Labor** and leads to increase in **productivity** (marginal)

Figure 4.17 Adding Capital Increases the Marginal Product of Labor



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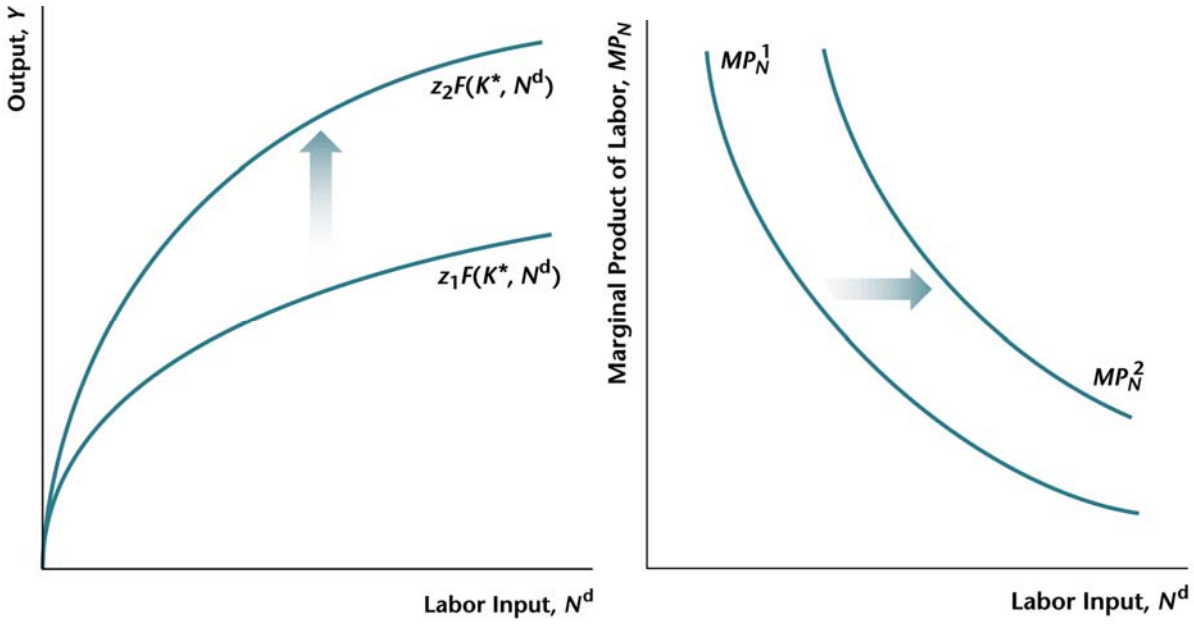
4) Properties of the Total Factor Productivity(TFP)

a) **Effect of changing TFP**: An increase in the total factor productivity has two effects:

- An increase in z makes both labor and capital more productive because more output can be produced with the existing inputs. This shifts the production function to the right.
- The marginal product of labor increases. This means that the MP_N curve shifts to the right.

Total Factor Productivity Increases **Effect on the MP_N Curve**



Critical Thinking

Increase in z has similar effect on MP_N as an increase in capital

b) Factors Affecting the (TFP)

There would be four kinds of factors that affect the total factor productivity**:**

- **Technological Innovation.** This is viewed as increase in technological sophistication and is the most **obvious** way of increasing TFP. Examples are new assembly lines (Ford), introducing new technology (Computers), and new management techniques.
- **Government Regulation.** This is the most **unobvious** means of changing TFP. Examples are government requiring installing pollution abatement equipment (this increases capital to production but adds nothing to production). This reduces TFP.
- **Weather.** For agricultural production, weather plays a very important role(less rain, more rain, flood, **Katrina**)
- **Change in the relative input price.** This is another not so obvious means of changing TFP. An increase in the relative price of energy (gasoline, oil) causes both the productivity of labor and capital go down, causing z to go down.

Critical Thinking

1) There are similarity between factors that effect TFP and factors that affect the supply curve of goods.
2) Gasoline can either be an input and also intermediate goods. Increase in the relative price increases the production of gasoline, but does not increase the TFP in the gasoline sector. But increase in the gasoline price leads to a decline in the TFP in sectors that use gasoline as input.

5) Measuring TFP for USA: Solow Residual

- People have tried to measure the z in the following way:
  With Cobb-Douglas Production function:
  $Y = zK^{\alpha}(N^{d})^{1-\alpha}, \alpha \in [0,1]$
  $$z = \frac{Y}{K^{\alpha}(N^{d})^{1-\alpha}}$$
  Where z is known as the **Solow residual** after the famous economist.
- Y, K, N^d can all measured. Y is a measure of GDP from NIPA calculation, K is the measure of total capital expenditure from NIPA and N^d is a measure of total employment, from the population survey by BLS.
- **Prescott** calculated the value of α to be 0.36.

6) Profit Maximization problem of the Representative firm

- Profit is defined as:
  $\pi = zF(K, N^{d}) - wN^{d}$
  Where firm seeks to maximize the difference between the output and the variable input. This is because the fixed capital cost is thought as a **sunk cost,** which the firm can never recover.
- So, the only choice that the firm has is to choose its labor input to maximize its profit.
- The firm maximizes its output by hiring labor up to $N^d=N^*$, where the cost of hiring the additional worker (the wage **w**) is just equal to the MP of that worker. Hence profit maximization satisfies:
  $MP_N = w$

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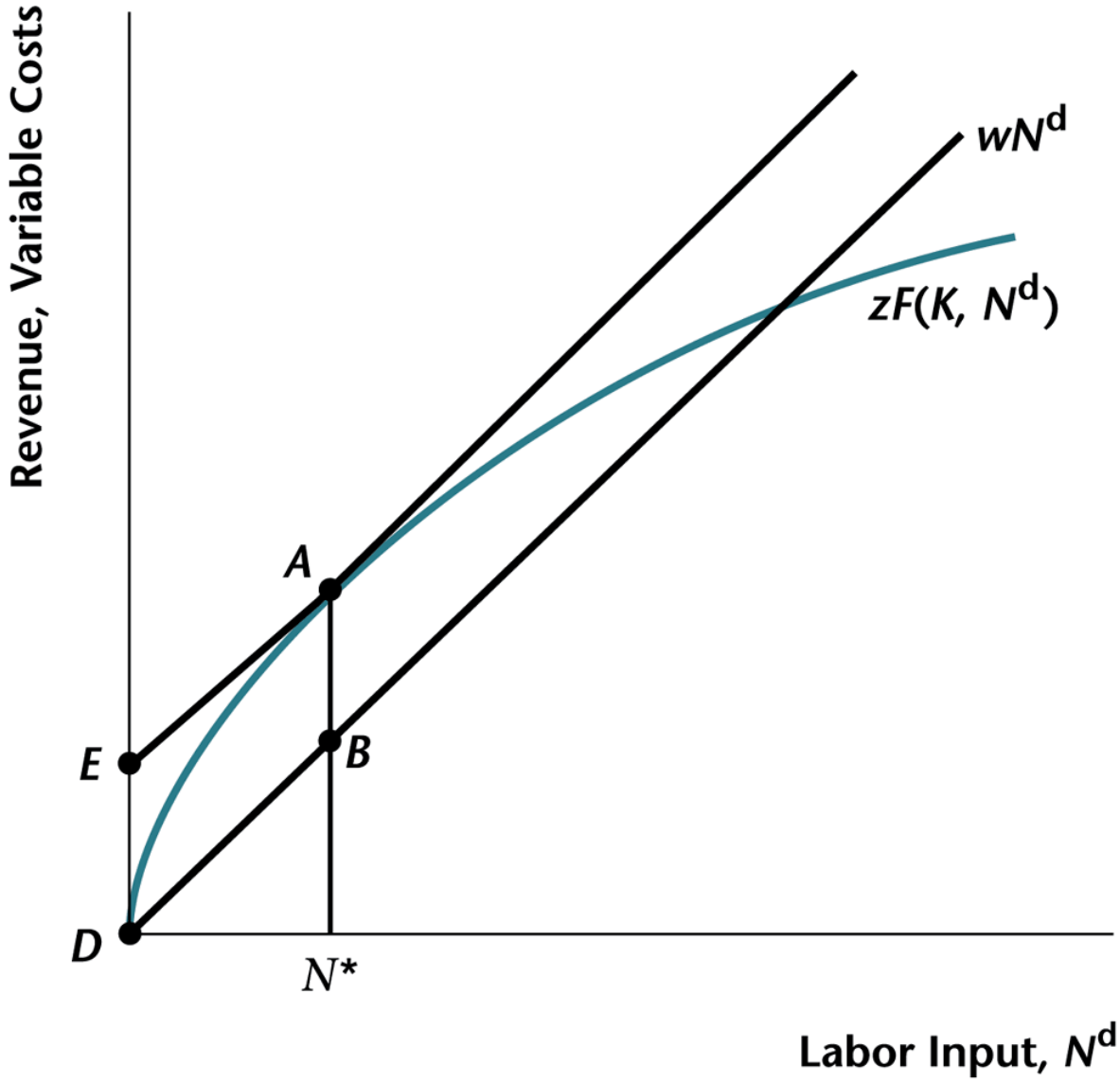
- Graphically, the profit maximizing labor input is determined at the point where PF (Total Product) graph is tangent to the Total cost (TC) line.

Figure 4.21 Revenue, Variable Costs, and Profit Maximization



Critical Thinking

1) For a firm, maximizing the difference between output and total cost or only total variable cost gives the same amount of equilibrium labor input N*.
2) Notice to maximize profit, the firm is not only maximizing output but also minimizing cost. This is done by determining the appropriate level of variable input.
3) The firm will always behave as a profit maximizer. That means it will always equal $w = MP_N$. Since this is true for each unit of labor the firm hires, the MP_N curve is also the labor demand curve for the firm

Figure 4.22 The Marginal Product of Labor Curve Is the Labor Demand Curve of the Profit-Maximizing Firm