E322: Intermediate Macroeconomics

Homework 5: Solutions

1. The amount of land increases, and, at first, the size of the population is unchanged. Therefore, consumption per worker increases. However, the increase in consumption per worker increases the population growth rate. In the steady-state, neither c^* nor l^* are affected by the initial increase in land. This fact can be discerned by noting that there will be no changes in either of the panels of Figure 6.10 in the textbook, which figure is also reproduced as Figure 6.2, below.

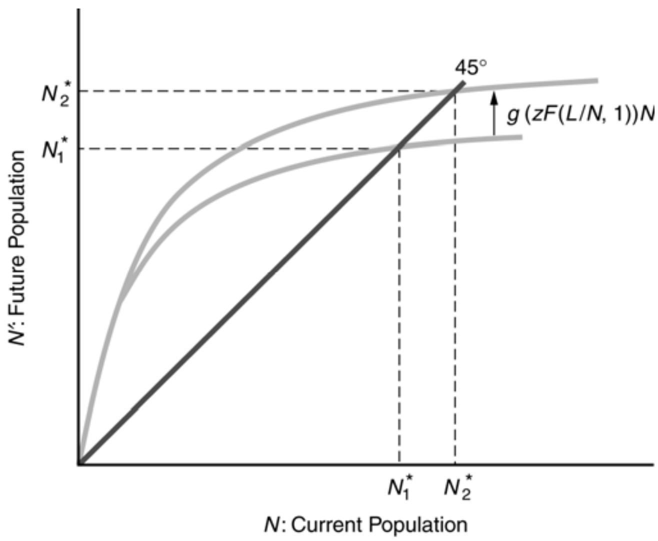


Figure 6.1

2. A reduction in the death rate increases the number of survivors from the current period who will still be living in the future. Therefore, such a technological change in public health shifts the function, $g(c)$ upward. In problem #1 there were no effects on the levels of land per worker and consumption per worker. In this case, the $g(c)$ function in the bottom panel of Figure 6.2 shifts upward. Equilibrium consumption per worker decreases. From the top panel of Figure 6.2, we also see that the decrease in consumption per worker requires a reduction in the equilibrium level of land per worker. The size of the population has increased, but the amount of available land is unchanged.

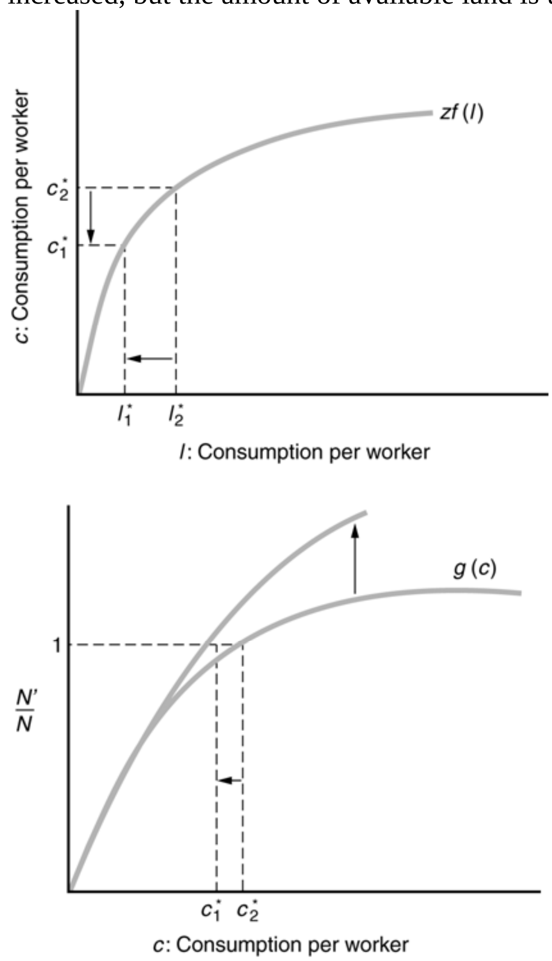


Figure 6.2

3. For the marginal product of capital to increase at **every** level of capital, the shift in the production function is equivalent to an increase in total factor productivity.

(a) The original and new production functions are depicted in Figure 6.3.

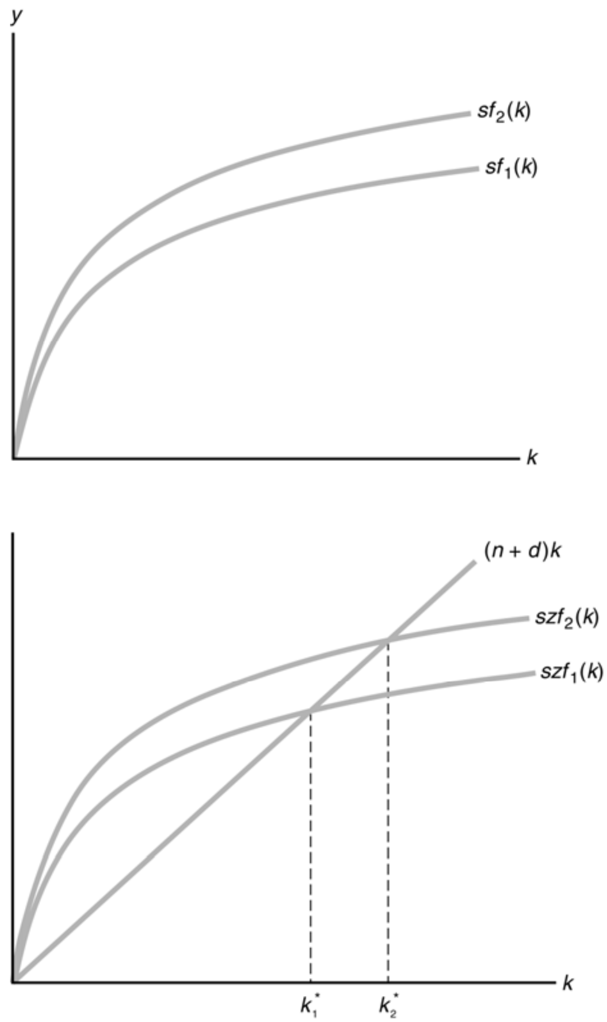


Figure 6.3

(b) Equilibrium in the Solow model is at the intersection of $szf(k)$ with the line segment $(n+d)k$. The old and new equilibria are depicted in the bottom panel of Figure 6.3. The new equilibrium is at a higher level of capital per worker and a higher level of output per worker.

(c) For a given savings rate, more effective capital implies more savings, and in the steady state there is more capital and more output. However, if the increase in the marginal product of capital were local, in the neighborhood of the original equilibrium, there would be no equilibrium effects. A twisting of the production function around its initial point does not alter the intersection point.

5. A destruction of capital.

(a) The long-run equilibrium is not changed by an alteration of the initial conditions. If the economy started in a steady state, the economy will return to the same steady state. If the economy were initially below the steady state, the approach to the steady state will be delayed by the loss of capital.

(b) Initially, the growth rate of the capital stock will exceed the growth rate of the labor force. The faster growth rate in capital continues until the steady state is reached.

(c) The rapid growth rates are consistent with the Solow model's predictions about the likely adjustment to a loss of capital.

7. Government spending in the Solow model.

(a) By assumption, we know that T + G, and so we may write:

$$K' = s(Y - G) + (1 - d)K = sY - gN + (1 - d)K$$

Now divide by N and rearrange as:

$$k'(1 + n) = szf(k) - sg + (1 - d)k$$

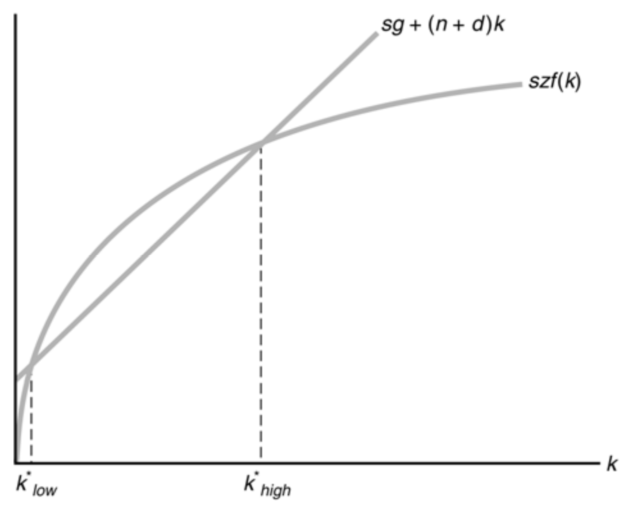
Divide by $(1 + n)$ to obtain:

$$k' = \frac{szf(k)}{(1+n)} - \frac{sg}{(1+n)} + \frac{(1-d)k}{(1+n)}$$

Setting k = k', we find that:

$$szf(k^*) = sg + (n + d)k^*.$$

This equilibrium condition is depicted in Figure 6.5.



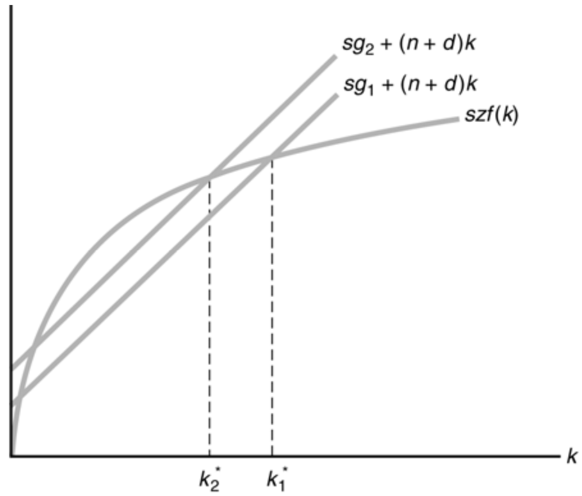


Figure 6.5

(b) The two steady states are also depicted in Figure 6.5.

(c) The effects of an increase in g are depicted in the bottom panel of Figure 6.5. Capital per worker declines in the steady state. Steady-state growth rates of aggregate output, aggregate consumption, and investment are all unchanged. The reduction in capital per worker is accomplished through a temporary reduction in the growth rate of capital.

8. The Golden Rule quantity of capital per worker, k^*, is such that $MP_K = zf'(k^*) = n + d$. A decrease in the population growth rate, *n*, requires a decrease in the marginal product of capital. Therefore, the Golden Rule quantity of capital per worker must increase. The golden rule savings rate may either increase or decrease.

9. Production linear in capital: $\frac{Y}{N} = z\frac{K}{N} = zf(k) \Rightarrow f(k) = k$

(a) Recall equation (20) from the text, and replace $f(k)$ with k to obtain:

$$k' = \frac{(sz + (1 - d)}{(1 + n)}k$$

Also recall that $\frac{Y}{N} = zk \Rightarrow k = \frac{1}{z}\frac{Y}{N}$ and $k' = \frac{1}{z}\frac{Y'}{N'}$. Therefore:

$$\frac{Y'}{N'} = \frac{(sz + (1 - d))}{(1 + n)}\frac{Y}{N}$$

As long as $\frac{(sz + (1 - d))}{(1 + n)} > 1$, per capita income grows indefinitely.

(b) The growth rate of income per capita is therefore:

$$g = \frac{\frac{Y'}{N'} - \frac{Y}{N}}{\frac{Y}{N}} = \frac{(sz + (1-d))}{(1+n)} - 1$$

$$= \frac{sz - (n+d)}{(1+n)}$$

Obviously, g is increasing in s.

(c) This model allows for the possibility of an ever increasing amount of capital per worker. In the Solow model, the fact that the marginal product of capital is declining in capital is the key impediment to continual increases in the amount of capital per worker.