

E322: Intermediate Macroeconomics
Homework 4: Solutions

2. $u = al + bC$

(a) To specify an indifference curve, we hold utility constant at \bar{u} . Next rearrange in the

form:
$$C = \frac{\bar{u}}{b} - \frac{a}{b}l$$

Indifference curves are therefore linear with slope, $-a/b$, which represents the marginal rate of substitution. There are two main cases, according to whether $\frac{a}{b} > w$ or $\frac{a}{b} < w$.

The top panel of Figure 4.2 shows the case of $\frac{a}{b} < w$. In this case the indifference curves are flatter than the budget line and the consumer picks point A, at which $l = 0$ and

$C = wh + \pi - T$. The bottom panel of Figure 4.2 shows the case of $\frac{a}{b} > w$. In this case the indifference curves are steeper than the budget line, and the consumer picks point B, at which $l = h$ and $C = \pi - T$. In the coincidental case in which $\frac{a}{b} = w$, the highest attainable indifference curve coincides with the budget line, and the consumer is indifferent among all possible amounts of leisure and hours worked.

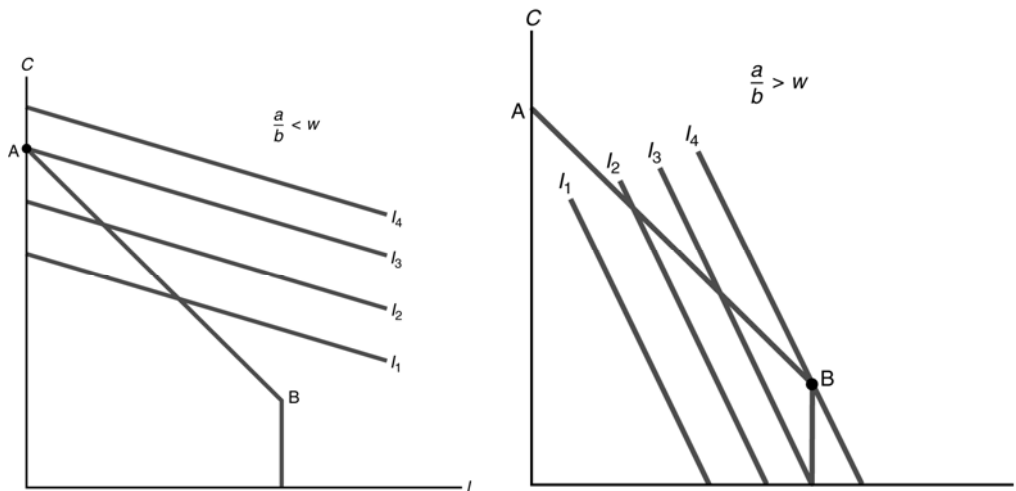


Figure 4.2

- (b) The utility function in this problem does not obey the property that the consumer prefers diversity, and is therefore not a likely possibility.
- (c) This utility function does have the property that more is preferred to less. However, the marginal rate of substitution is constant, and therefore this utility function does not satisfy the property of diminishing marginal rate of substitution.

3. When the government imposes a proportional tax on wage income, the consumer's budget constraint is now given by:

$$C = w(1-t)(h-l) + \pi - T,$$

where t is the tax rate on wage income. In Figure 4.3, the budget constraint for $t = 0$, is FGH. When $t > 0$, the budget constraint is EGH. The slope of the original budget line is $-w$, while the slope of the new budget line is $-(1-t)w$. Initially the consumer picks the point A on the original budget line. After the tax has been imposed, the consumer picks point B. The substitution effect of the imposition of the tax is to move the consumer from point A to point D on the original indifference curve. The point D is at the tangent point of indifference curve, I_1 , with a line segment that is parallel to EG. The pure substitution effect induces the consumer to reduce consumption and increase leisure (work less).

The tax also makes the consumer worse off, in that he or she can no longer be on indifference curve, I_1 , but must move to the less preferred indifference curve, I_2 . This pure income effect moves the consumer to point B, which has less consumption and less leisure than point D, because both consumption and leisure are normal goods. The net effect of the tax is to reduce consumption, but the direction of the net effect on leisure is ambiguous. Figure 4.3 shows the case in which the substitution effect on leisure dominates the income effect. In this case, leisure increases and hours worked fall. Although consumption must fall, hours worked may rise, fall, or remain the same.

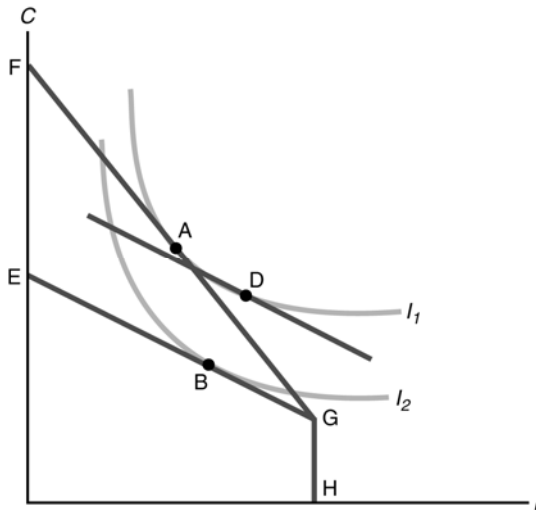


Figure 4.3

10. The level of output produced by one worker who works $h - l$ hours is given by

$$Y = zF(K, h - l).$$

This equation is plotted in Figure 4.10. The slope of this production possibilities frontier is simply $-MP_N$.

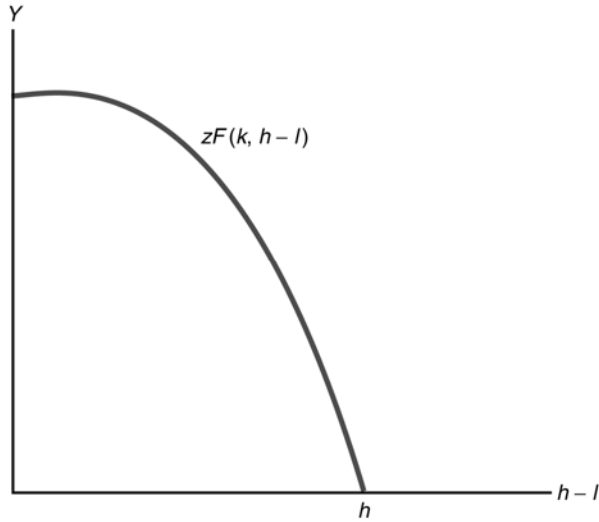


Figure 4.10

11. $Y = zK^{0.3}n^{0.7}$
- $Y = n^{0.7}$. See the top panel in Figure 4.11. The marginal product of labor is positive and diminishing.
 - $Y = 2n^{0.7}$. See Figure 4.11.
 - $Y = 2^{0.3}n^{0.7} \approx 1.23n^{0.7}$. See Figure 4.11.
 - See the bottom panel of Figure 4.11.

$$z = 1, K = 1 \Rightarrow MP_N = 0.7n^{-0.3}$$

$$z = 2, K = 1 \Rightarrow MP_N = 1.4n^{-0.3}$$

$$z = 1, K = 2 \Rightarrow MP_N = 2^{0.3} \times 0.7n^{-0.3} \approx 0.86n^{-0.3}$$

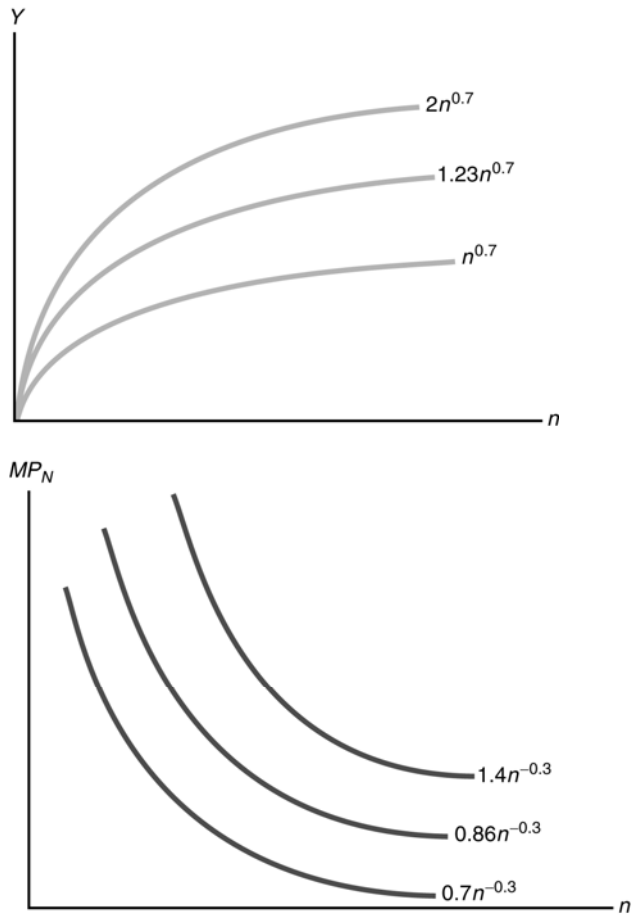


Figure 4.11