What Determines Specific Schooling Decisions in the USA? A Dynamic General Equilibrium Analysis

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Abstract

Aggregate public school enrollment in primary education in the USA in the last 100 years has been roughly constant at 0.88 or 88% of the total enrollment. This contradicts with the conventional wisdom and the "popular press" which argues that there have been significant changes in the quality of education and the cost of education itself over this long period, although the latter claim has been challenged by a recent paper by Fernandez and Rogerson (2001). Also there appears to be a divergence between the qualities of education in private vs. public schools, indicated by various sources. This paper tries to investigate the reason why the fraction of public school enrollment has been constant over such a long period of time. I use a canonical model of schooling decisions which is widely used in literature and try to analyze the effect of income inequality, mean income and changes in the quality of education on the public enrollment. My approach sharply contrasts with the existing literature which mainly focuses on the role of schooling decisions on income inequality. Using a parametric model, I identify the threshold income level below which parents send their kids to public school and above which they send their kids to private school. Analytical results show how this threshold income level changes with the income inequality of the economy and how the changes in the threshold income effect the enrollment decisions. Under the assumption of no quality change in education and an unchanged real cost of education, I show that the model calibrated to 1989 USA data can match the aggregate enrollment figures for the USA almost perfectly. I then show that the model, applied to each individual state, can also match their enrollment decisions, although not uniquely. Therefore, the paper draws support to the empirical work of Fernandez and Rogerson (2001).

Key Words: Overlapping Generations, Public School, Private School, Enrollment, Threshold Income, Income Inequality

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1 Introduction

Public school enrollment in primary education in the USA in the last 100 years has been roughly constant at 0.88 or 88% of the total enrollment. This is despite significant changes in average income, income inequality and even according to some, changes in the quality of education. This paper tries to investigate the reason why the fraction of public school enrollment has been constant over such a long period of time. A very simple model will be developed to analyze the effect of income inequality, average income and changes in the quality of education on the

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public school enrollment. The model will define a threshold income level below which parents send their kids to public school and above which they send their kids to private school. Then, some calibration and empirical exercise will be conducted to match both national and state-level USA data on public school enrollment.

2 Literature Review

The literature on the relationship between income and schooling is extensive and falls into two broad categories. First, there are a long list papers that try to understand the relationship in a theoretical environment. For example, Glomm and Ravikumar (1992) makes a seminal contribution by analyzing the endogenous relationship between income inequality and how it is effected by parental choice of public versus private school. In another seminal paper, Epple and Romano (1996) setup a theoretical model where schooling decision and schooling financing(tax) is determined simultaneously. The authors define a threshold income level below which parents will send their children to public school. This threshold level will be a function of the tax that parents pay to finance public school education. On the empirical side, there is a long list of papers that try to identify the factors that effect schooling decisions. For Example, Goldhaber (1998) tries to investigate the relationship public school expenditure and private school enrollment and finds no strong relationship. Cohen-Zada and Justman (2005) finds a strong latent demand for religious education. Gemello and Osman (1984) analyze which economic, social, religious, and ethnic characteristics are significantly related to the private school choice. Fernandez and Rogerson (2001) finds that two most important determinants of spending per student are personal income and number of students. Also they find that the quality of public school education has remained almost unchanged.

The present paper differs from the previous literature in three aspects. First, it extends the literature developed by Glomm and Ravikumar (1992) but goes beyond their scope by looking at the effect of inequality on the threshold income. Second, the paper attempts to match USA national and state-level data with the model by using calibration as well as Generalized Method of Moments (GMM). Third, the paper analyzes the effect of inequality on the schooling decisions rather than vice versa which has been the norm in the literature. While the existing literature identifies the causal relationship between schooling and income by assuming that the former effect the latter, I will analyze how income inequality effects schooling decisions. So far Catalina (2006) is the only paper that has taken this approach.

The paper is organized in the following; Chapter 3 explains the theoretical model and develops several propositions regarding the determination of the threshold income and the effect of income inequality on the threshold income and schooling. Chapter 4 provides some numerical results that exploits the relationship between the threshold income, income inequality and schooling decisions. Chapter 5 reports three kinds of empirical results. First, it reports the results from the calibration exercise where the model is calibrated to both USA national and state-level income data to see whether the model can predict enrollment figures that can match USA data. Second, GMM method will be applied to estimate the parameters of the model to see whether the model can generate private and public school enrollment similar to USA data. Finally, a panel GMM exercise will be carried out to for a robust estimation of the parameters of the model.

3 Simple Model

We consider a two period OLG model where population in each generation is normalized to unity. We will consider an altruistic environment where parents care about how much they are contributing towards their child’s education. Parents enelastically supply 1 unit of time
to work. They decide whether their child will go to private or public school. If children go to public school, the expenditure is carried out by the government. Parents do not provide any educational supplement. If private school is chosen, parents bear the entire expenditure. Government finances the public education by a flat income tax. Children are not allowed to work when they are young. They only accumulate human capital by going to school. Human capital accumulation of the child depends not on the financial input, but also on the human capital of the parents. Households have initial income distribution given by $f(h)$ and $F(h)$ with support $\tilde{h}$ and $h$ such that $(\tilde{h}, h) \in [0, \infty]$. Parents are homogenous in ability but heterogeneous in income. Children are homogenous in ability. The aggregate human capital is given by:

$$H = \int_{\tilde{h}}^{h} h f(h) \, dh = E(h)$$

Goods are produced by using human capital only, such that

$$y_t = H_t$$

Following Epple and Romano (1996), the utility function of the parents look like

$$U(c_t, q_t) = \left[ \beta c_t^{-\rho} + (1 - \beta) D_t^\rho \right]^{-1/\rho}$$

where $c_t$ is the consumption of the parents and $D_t$ is the quality of education received by the children where $D_t = q_t$ if children attend private school and $D_t = E_t$ if children attend public school where $q_t$ is the out-of-pocket expenditure of the parents and $E_t$ is the per-pupil government expenditure on public education. $E_t$ is the government constraint which is defined as follows:

$$E_t = \frac{k \tau H_t}{N_t}$$

where $\tau$ is the exogenously fixed flat income tax rate, $k$ is an indicator for public education quality, $H_t$ is the aggregate human capital (aggregate income) and $N_t$ is the fraction of population going to public school. $N_t$ is defined as follows:

$$N = \int_{0}^{\tilde{h}} f(h) \, dh$$

where $\tilde{h}$ is the threshold income level below which all the parents send their kids to public school and vice versa.

The human capital technology is defined as follows; for children attending public school,

$$h_{t+1}^{PR} = \theta q_t^{\gamma} h_t^\delta$$

and for children attending private school,

$$h_{t+1}^{PR} = \theta E_t^{\gamma} h_t^\delta$$
Here $\theta$ is the productivity parameter, $\gamma$ and $\delta$ indicates the elasticity of $h_{t+1}$ with respect to $q_t$ (or $E_t$ ) and $h_t$.  
Parents who send their kids to private school choose $c_t$ and $q_t$, $\tau$ and $h_t$, to maximize

$$
\left[ \beta c_t^{-\rho} + (1 - \beta)q_t^{-\rho} \right]^{-1/\rho}
$$

subject to

$$
c_t + q_t = (1 - \tau)h_t
$$

Parents who send their kids to public school choose $c_t$, given $\tau$, $h_t$, $E_t$, to maximize:

$$
\left[ \beta c_t^{-\rho} + (1 - \beta)E_t^{-\rho} \right]^{-1/\rho}
$$

subject to

$$
c_t = (1 - \tau)h_t
$$

Then the optimal choice for the parents who send their kids to private school looks like,

$$
c_t = \frac{(1 - \tau)}{1 + \left( \frac{\beta}{1 - \beta} \right)^{1+\rho}} h_t
$$

$$
q_t = \left[ \frac{\left( \frac{\beta}{1 - \beta} \right)^{1+\rho}}{1 + \left( \frac{\beta}{1 - \beta} \right)^{1+\rho}} \right] (1 - \tau) h_t
$$

The indirect utility of the parents sending their kids to private school looks like:

$$
V^{PR}(h_t; \tau) = \beta \left\{ \frac{(1 - \tau)}{1 + \left( \frac{\beta}{1 - \beta} \right)^{1+\rho}} h_t \right\}^{-\rho} + (1 - \beta) \left\{ \frac{\left( \frac{\beta}{1 - \beta} \right)^{1+\rho}}{1 + \left( \frac{\beta}{1 - \beta} \right)^{1+\rho}} \right\}^{-\rho} \left[ (1 - \tau) h_t \right]^{-1/\rho}
$$

$$
\Rightarrow V^{PR}(h_t; \tau) = \beta \left\{ \frac{1}{1 + \left( \frac{\beta}{1 - \beta} \right)^{1+\rho}} \right\}^{-\rho} + (1 - \beta) \left\{ \frac{\left( \frac{\beta}{1 - \beta} \right)^{1+\rho}}{1 + \left( \frac{\beta}{1 - \beta} \right)^{1+\rho}} \right\}^{-\rho} \left[ (1 - \tau) h_{tt} \right]^{-1/\rho}
$$


Now for parents who send their kids to public schools, their optimal choice looks like,

\[ c_t = (1 - \tau)h_t \]  \hspace{2cm} (15)

where,

\[ E_t = \frac{k\tau H_t}{N_t} \]  \hspace{2cm} (16)

Also, the indirect utility of the parents who send their kids to public schools look like,

\[ V^{PB}(h_t; \tau) = \left[ \beta \{(1 - \tau)h_t\}^\rho + (1 - \beta) \left\{\frac{k\tau H_t}{N_t}\right\}^{-\rho}\right]^{-1/\rho} \]  \hspace{2cm} (17)

Similar to the linear case, the threshold level of income will be found by equating the indirect utility from public and private school, namely equating equation(32) and (34). The threshold level of income is defined as follows,

\[ h_t^* = \frac{E_t H_t}{N_t} \]  \hspace{2cm} (35)

where,

\[ F = \left\{ \left[ \frac{1-\beta}{D(1-\tau)} \right]^{1/\rho} \right\}^{-1/\rho} k.\tau \]  \hspace{2cm} (35)

Where,

\[ D = \left[ \beta \left\{ 1 + \left( \frac{\beta}{1-\beta} \right)^{1+\rho} \right\}^\rho + (1 - \beta) \left\{ \frac{1+\left( \frac{\beta}{1-\beta} \right)^{1+\rho}}{(1-\beta)^{1+\rho}} \right\}^\rho \right]^{-1/\rho} \]  \hspace{2cm} (36)

4 Definition of Competitive Equilibrium

A competitive equilibrium for the economy is a sequence of \( \{c_{it}, q_{it}, h_{it+1}\}_{i=0}^\infty \), \( E_t \), \( y_t \), \( H_t \) and \( H_{t+1} \) such that

a) Given \( \tau \) and \( h_t \), parents in the private education regime choose \( c_t \) and \( q_t \) to maximize(4) subject to (5),

b) Given \( E_t \), parents in the public school regime choose \( c_t \) to maximize(4) subject to (5),

c) There exists a threshold level of income \( h^*_t \) such that below which parents send their kids to public school and above which parents send their kids to private school.

d) Given \( N_t \) defined by (5) and \( H_t \) defined by (1), government balances its budget defined by (4).

e) Goods market clears, \( c_t = y_t \)

f) Human capital market clears,

\[ H = \int_{h}^{h^*_t} hf(h) \, dh \]  \hspace{2cm} (12)

Solving the private regime model yields the equilibrium allocation:

\[ c_t + q_t = \frac{(1 - \tau)h_t}{2} \]  \hspace{2cm} (13)
The indirect utility of the parents who send their kids to private school is defined as follows:

\[ V^{PR}(h_t, \tau) = 2 \ln \left\{ \frac{(1 - \tau)h_t}{2} \right\} \tag{14} \]

Finally, the Human capital of the children going to private school is defined as follows:

\[ H_{t+1}^{PR} = \theta \left\{ \frac{(1 - \tau)}{2} \right\}^\gamma h_t^{\gamma + \delta} \tag{15} \]

The indirect utility of the parents who send their kids to private school is defined as follows:

\[ V^{PB}(h_t, E_t, \tau) = \ln \left\{ (1 - \tau)h_t \frac{k\tau H_t}{N_t} \right\} \tag{16} \]

Finally, the Human capital of the children going to public school is given by

\[ H_{t+1}^{PB} = \theta \left\{ \frac{k\tau H_t}{N_t} \right\}^\gamma h_t^\delta \tag{17} \]

**Proposition 1** There exists a unique threshold level of income \( h^* \) such that below which parents send their kids to public school and above which parents send their kids to private school.

**Proof.** The threshold income would be derived by identifying the parents who are just indifferent between sending their kids to private or public school. These parents derive the same indirect utility by sending their kids to private or public school. By equating (14) and (16), we get

\[ 2 \ln \left\{ \frac{(1 - \tau)h_t}{2} \right\} = \ln \left\{ (1 - \tau)h_t \frac{k\tau H_t}{N_t} \right\} \implies h_t^* = \left( \frac{4k\tau}{(1 - \tau)} \right) \left( \frac{H_t}{N_t} \right) \tag{18} \]

\[ \implies h_t^* = \left( \frac{4k\tau}{(1 - \tau)} \right) \left( \frac{H_t}{N_t} \right) \]

It is clear from (18) that the value of \( h^* \) is unique. Also note that

\[ \text{For any } h_t < h_t^*, 2 \ln \left\{ \frac{(1 - \tau)h_t}{2} \right\} < \ln \left\{ (1 - \tau)h_t \frac{k\tau H_t}{N_t} \right\} \tag{19} \]

So these parents would send their kids to public school because of higher indirect utility. A similar thing happened when the inequality is reversed and parents then send their kids to private school. ■

In order to probe further into the analysis, we will rearrange (18) as follows:

Assume \( \left( \frac{4k\tau}{(1 - \tau)} \right) = C \). Then subbing (5) into (18) and reorganizing after eliminating the time subscript,

\[ CH = hN = h \int_0^h f(h) dh \tag{20} \]

Now since \( h \sim LN(\mu, \sigma^2) \), then \( Lnh \sim N(\mu, \sigma^2) \) and \( Lnh^* \sim N(\mu, \sigma^2) \). Again since, \( H = E(h) \), we can write,

\[ H = E(h) = e^{\mu + \sigma^2} \tag{21} \]
Furthermore,

\[ N = \int_0^{*h} f(h) \, dh = E \left( 1 \left( h \leq \*h \right) \right) = \text{Pr}(h \leq \*h) = \text{Pr}(Lnh \leq L\*h) \]

\[ = \text{Pr} \left( \frac{Lnh - \mu}{\sigma} \leq \frac{\*h - \mu}{\sigma} \right) = \Phi \left( \frac{\*h - \mu}{\sigma} \right) \]

Define, \( m^* = \frac{Lnh - \mu}{\sigma} \) and \( m = \frac{Lnh - h\mu}{\sigma} \). Then the above expression can be written as

\[ N = \Phi \left( \frac{\*m}{m} \right) \quad (22) \]

Where the right hand side is a \( cdf \) of a standard normal distribution with argument as \( \frac{\*m}{m} \).

Furthermore, assume \( \ln \*h = \*Z \). Then \( \*m = \frac{Z - \mu}{\sigma} \) and \( \*h = e^\*Z \). Finally subbing (20), (22) and (24) into (19):

\[ Ce^{\*m^2 - \*m} = \Phi \left( \frac{\*m}{m} \right) \quad (23) \]

Equation (22) will be our main equation for analyzing various comparative statics issues.

**Proposition 2** For a given \( \sigma \), an increase in \( \mu \) leaves \( N_t \) unchanged but increases \( \*h \).

**Proof.** If we rearrange equation (7), we get the following expression:

\[ Ce^{\*m^2} = e^{\*m \cdot \sigma} \Phi \left( \frac{\*m}{m} \right) \quad (24) \]

Differentiate both side of (25) with respect to \( \mu \)

\[ 0 = \Phi \left( \frac{\*m}{m} \right) \cdot e^{\*m \cdot \sigma} \cdot \frac{\partial \*m}{\partial \mu} + e^{\*m \cdot \sigma} \cdot \Phi \left( \frac{\*m}{m} \right) \cdot \frac{\partial \*m}{\partial \mu} \]

\[ \Rightarrow \frac{\partial \*m}{\partial \mu} \left[ \Phi \left( \frac{\*m}{m} \right) \cdot e^{\*m \cdot \sigma} \cdot \sigma + e^{\*m \cdot \sigma} \cdot \Phi \left( \frac{\*m}{m} \right) \right] = 0 \quad (25) \]

The expression within the bracket is not equals to zero. Hence, \( \frac{\partial \*m}{\partial \mu} = 0 \). Again,

\[ \frac{\partial \Phi \left( \frac{\*m}{m} \right)}{\partial \mu} = \Phi \left( \frac{\*m}{m} \right) \cdot \frac{\partial \*m}{\partial \mu} \quad (26) \]

Substituting the value of \( \frac{\partial \*m}{\partial \mu} \) from (24)

\[ \frac{\partial \Phi \left( \frac{\*m}{m} \right)}{\partial \mu} = 0, \text{ which implies from (21) that } \frac{\partial N}{\partial \mu} = 0 \quad (27) \]

Differentiate the definition of \( \frac{\*m}{m} \) with respect to \( \mu \)

\[ \frac{\partial \frac{\*m}{m}}{\partial \mu} = \frac{1}{\sigma} \left( 1 \frac{\partial \*h}{\partial \mu} - 1 \right) \quad (28) \]
Substituting the value of $\frac{\partial \tilde{m}}{\partial \mu}$ from (24)

$$\frac{1}{\sigma} \left( \frac{1}{\tilde{h}} \frac{\partial \tilde{h}}{\partial \mu} - 1 \right) = 0$$

$$\Rightarrow \frac{\partial \tilde{h}}{\partial \mu} = \tilde{h} \succ 0$$

Proposition 3: For a given $\mu$, increasing $\sigma$ increases $N$ iff $\tilde{h} < e^{\sigma^2 + \mu}$. It also increase $\tilde{h}$ iff $\tilde{h} \ln \tilde{h} > q$ where $q = \sigma^2 \left\{ \frac{\Phi(\tilde{m})(\tilde{m} - \sigma)}{\sigma \Phi(\tilde{m}) + \Phi(\tilde{m})} \right\}$

Proof. Differentiate both side of (25) with respect to $\sigma$:

$$C \cdot e^{\sigma^2} \cdot \sigma = \Phi \left( \tilde{m} \right) \cdot e^{\sigma \tilde{m}} \cdot \left( \tilde{m} + \sigma \frac{\partial \tilde{m}}{\partial \sigma} \right) + e^{\sigma \tilde{m}} \cdot \Phi' \left( \tilde{m} \right) \cdot \frac{\partial \tilde{m}}{\partial \sigma}$$

Subbing value from (25) on the left hand side and canceling terms,

$$\sigma \Phi \left( \tilde{m} \right) = \Phi \left( \tilde{m} \right) \tilde{m} + \frac{\partial \tilde{m}}{\partial \sigma} \left( \sigma \Phi \left( \tilde{m} \right) \right) + \Phi' \left( \tilde{m} \right) \cdot \frac{\partial \tilde{m}}{\partial \sigma}$$

Collecting terms, we get,

$$\sigma \Phi \left( \tilde{m} \right) = \Phi \left( \tilde{m} \right) \tilde{m} + \frac{\partial \tilde{m}}{\partial \sigma} \left( \sigma \Phi \left( \tilde{m} \right) \right) + \Phi' \left( \tilde{m} \right) \cdot \frac{\partial \tilde{m}}{\partial \sigma}$$

Subbing the value from (29),

$$\frac{\partial \Phi \left( \tilde{m} \right)}{\partial \sigma} = \Phi' \left( \tilde{m} \right) \left( \frac{\sigma - \tilde{m}}{\frac{\sigma}{\Phi' \left( \tilde{m} \right)} + \frac{\Phi \left( \tilde{m} \right)}{\sigma \Phi' \left( \tilde{m} \right)}} \right)$$

From the above equation, we see,

$$\frac{\partial \Phi \left( \tilde{m} \right)}{\partial \sigma} \succ 0 \iff \left( \frac{\sigma - \tilde{m}}{\sigma} \right) \succ 0 \Rightarrow \sigma \succ \tilde{m}$$

Subbing the value of $\tilde{m}$ from (21),

$$\frac{\partial \Phi \left( \tilde{m} \right)}{\partial \sigma} \succ 0 \iff \tilde{h} < e^{\sigma^2 + \mu} \Rightarrow \frac{\partial N}{\partial \sigma} \succ 0 \iff \tilde{h} < e^{\sigma^2 + \mu}$$

Also, from (29) after substituting the value of $\tilde{m}$ from (21) into (29),

$$\frac{\partial \tilde{m}}{\partial \sigma} = \frac{\frac{\sigma}{\tilde{m}} + \tilde{h}}{\sigma^2} - \frac{\ln \tilde{h} \tilde{h}}{\sigma^2} = \frac{\Phi \left( \tilde{m} \right) \left( \sigma - \tilde{m} \right)}{\sigma \Phi \left( \tilde{m} \right) + \Phi' \left( \tilde{m} \right)}$$

$$\Rightarrow \frac{\partial \tilde{m}}{\partial \sigma} = \left[ \frac{\sigma}{\sigma^2} \left\{ \frac{\Phi \left( \tilde{m} \right) \left( \sigma - \tilde{m} \right)}{\sigma \Phi' \left( \tilde{m} \right) + \Phi' \left( \tilde{m} \right)} \right\} + \ln \tilde{h} \right] \cdot \tilde{h} \sigma \frac{\tilde{h}}{\sigma}$$

(32)
The above equation implies that

$$\frac{\partial h^*}{\partial \sigma} > 0 \text{ iff } \sigma \left\{ \frac{\Phi \left( m^* \right) \left( \sigma - m^* \right)}{\sigma \Phi \left( m^* \right) + \Phi \left( m^* \right)} \right\} + \frac{h^* \ln h^*}{\sigma} > 0$$

$$\Rightarrow h^* \ln h^* > \sigma^2 \left\{ \frac{\Phi \left( m^* \right) \left( \sigma - m^* \right)}{\sigma \Phi \left( m^* \right) + \Phi \left( m^* \right)} \right\}$$

$$\Rightarrow h^* \ln h^* > q$$
References


