Entry Cost, Strategic Quality Choice and Charter Schools

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Abstract

I use a model of quality choice to highlight the importance of variable entry cost in the competition between an incumbent public school and an entrant charter school. When there is a capacity constraint and when the public school accommodates the charter school, an increase in state and federal funding has no effect on the market share but raises quality of education. But when local funding increases, the effect on market share and education quality depends on the nature of the entry cost.

Keywords: School quality; Charter schools; School competition

JEL classification: I2; L3

1 Introduction

Proponents of Charter schools (Hoxby(2003), Sandström and Bergström(2005)) argue that competition between Charter and traditional public schools will improve the educational outcomes of both Charter and public schools. But the results so far have been mixed (Hanushek, Kain, Rivkin and Branch(2007), Howell and Peterson (2002), Krueger and Zhu (2003)). According to many, one of the reasons why charter school failed to generate significant competitive pressure was the absence of public funding for set-up costs for these schools. Anderson, Watkins and Cotton(2003) explored the relative costs of educating a child in a charter school and in a traditional public school in Michigan. Their study made three important conclusions. First, charter schools receive less operational funding than the neighboring public schools although they are entitled equal amount by law. Second, Charter schools receive $1,036 less per student, on average, than the traditional public schools. Third, charter schools receive no capital funding for setting up new school. School officials have to borrow these fund from private sectors. For example, Educational Facilities Financing Center(2007) reports that NCB, a private loaning company, has loaned about $129 million to charter schools since its initiatives began in 1997. The size of the charter school loans also varies according to their start-up size. For example, Providence Financial Company(2010) reports different size of start-up loans to charter schools which vary from $2.5 million to $28 million. Hoxby(2003) reported that in 2002-2003, the average charter school spent just 45 percent of what its local public school competitors spent. The author also pointed out that availability of low funds to start up a new charter school makes it difficult to generate sufficient competition with the regular public schools. Prince(1999) reported that charter schools

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1For a look at their track record, please go to: http://www.providencefinancialco.com/charter-school-track-record.asp
have to spend more money on start-up and operations than their traditional public school counterparts. For example, during 1995-96, Michigan charter schools spent an average of 57 percent of revenues on instruction and about 43 percent on support services, compared to 65 and 35 percent for comparable districts. According to U.S. Department of Education, in fiscal year 2007, median per student capital outlay (much of which is used for start-up) expenditure from public provided revenue was $432 for public schools compared to a mere $28 for charter schools. Therefore, most of the start-up funds for the charter schools were acquired from private sources.

The model in this paper explores the nature of quality competition between an incumbent public school and an entrant charter school when there is a variable entry cost. The model is a variant of Cardon (2003) and Rahman (2010). Similar to their models, public and charter schools compete in quality with price fixed. Quality is costly and demand for education is inelastic for public education. Schools have capacity constraint. But the main difference is that the entrant charter school faces a variable entry cost in the spirit of Donnenfeld and Weber (1995) and Noh and Moschini (2006). It is the public school’s unwillingness to match costly quality, rather than price, which allows the entrant to survive. On the other hand, the variable entry cost prevents charter school to capture a bigger share of the market and also effects the quality of education it can offer. Interaction between these two kinds of frictions determine the quality of education in the market and the relative market share of these two types of school.

The model has several interesting implications for policy making. It shows that with capacity constraint, there exits an equilibrium where the charter school enters with a low capacity and (possibly) a better quality and the public school accommodates while maintaining a (possibly) lower quality. Under the assumption that charter school only receives fund from the state and federal level, an increase in these funds will have no effect on their capacity but will increase the quality of education if and only if the public school fails to match the quality and accommodates the charter school. But if there is an increase in local funding, the charter school losses market share. Also, the effect on quality depends on the interaction between the willingness of public schools to match quality and the nature of the entry cost of the charter school.

2 The Model

In the model of strategic competition, the public school is the incumbent and the Charter school is the entrant. They compete in quality with prices being fixed. Here price p refers to per student expenditure paid either by public or local funding or both. There is one incumbent public school and one entrant Charter school. Entrant has a variable entry cost which depends on the output. This is designed to capture the capital and start-up funds that charter schools have to acquire to enter into market. The entrant and the incumbent engage in a game of capturing the market for education where the size of the market is normalized to one. In this game, the entrant always moves first and the incumbent moves second. The administrative of the school is a representative household who produces and supplies good (education) into the market and also derives utility from the quality of the education. That means the model assumes warm glow altruism because the administrative can be thought as the parent who is sending his children to school and also managing the school. Following Cardon (2003) and Rahman (2010), I assume that the public school receives funds from both public and local sources, $p_S + p_L$, while the Charter school only receives funds from public source, $p_S$. I focus only to the capacity constraint case. In the capacity constraint case, the schools have to simultaneously choose quality and capacity, K. The entrant moves first and chooses $(q^E_1, K)$ where $K \in [0, 1]$. The incumbent’s capacity is a sunk cost. But there are variable costs of output. Let $C(q, K)$ be the cost function that is used to produce

\(^2\)In Donnenfeld and Weber (1996), entry cost is fixed. In Noh and Moschini (2006), entry cost is fixed and marginal cost is quality dependent. In my model, entry cost is variable and depends only on capacity.
education by both types of school. Let $U(q)$ be the utility function for the administrator for both schools. Finally, let $D(K)$ be the entry cost incurred by the entrant. In order to simplify my analysis, I make several assumptions following Cardon(2003). First, consumers strictly prefer the incumbent when qualities are equal. If qualities are unequal, all consumers prefer the school with higher quality. Second, I assume that $C(q, K)$ is twice continuously differentiable and strictly concave in both its arguments. I also assume that $U(q)$ is twice continuously differentiable and strictly concave. The entry cost $D(K)$ is twice continuously differentiable but could be either strictly concave or convex, following Rothschild(1971). Finally, I assume that the cost function $C(q, K)$ is separable in $q$ and $K$.

The game is solved backward. In the second stage of the game, if the incumbent matches quality, he gets the entire market and the administrator’s objective function is,

$$V(\Pi, q^E_1) = p_S + p_L - C(q^E_1, 1) + U(q^E_1). \quad (1)$$

If, on the other hand, the incumbent chooses to accommodate, the objective function is,

$$V(\Pi, q^*_1, 1 - K) = p_S(1 - K) + p_L(1 - K) - C(q^*_1, 1) + U(q^*_1). \quad (2)$$

Therefore, the incumbent will accommodate iff $V(\Pi, q^*_1, 1 - K) \geq V(\pi, q^E_1)$, which by using equation (1) and (2) can be written as,

$$C(q^E_1, 1) - C(q^*_1, 1) \geq (p_S + p_L)K + U(q^E_1) - U(q^*_1) \quad (3)$$

Now given the incumbent accommodates, the the administrator will optimize$^3$:

$$\max_{q_1} V(\Pi, q_1) = (p_S + p_L)(1 - K) - C(q_1, 1 - K) - U(q_1)$$

The optimal quantity $q^*_1$ is determined by the first order condition,

$$\frac{\partial C(q^*_1, 1 - K)}{\partial q^*_1} = U'(q^*_1) \quad (4)$$

Notice that with separable cost function, equation(4) defines a unique quality level for the incumbent; $q_1(K) = q^*_1$. Now anticipating the incumbent’s move in the second stage, the entrant will take incumbent’s action into consideration and solves,

$$\max_{\Pi, q^E_1, K} \left( p_S K - C(q^E_1, K) - D(K) + U(q^E_1) \right)$$

such that, $C(q^E_1, 1) - C(q^*_1, 1 - K) \geq (p_S + p_L)K + U(q^E_1) - U(q^*_1)$

The Lagrangian for this problem looks like,

$$\mathcal{L} = p_S K - C(q^E_1, K) - D(K) + U(q^E_1) + \lambda \left[ C(q^E_1, 1) - C(q^*_1, 1 - K) - (p_S + p_L)K - U(q^E_1) + U(q^*_1) \right]$$

The first order conditions are as follows:

$$- \frac{\partial C(q^E_1, K)}{\partial q^E_1} + U'(q^E_1) + \lambda \left[ \frac{\partial C(q^E_1, 1)}{\partial q^E_1} - U'(q^E_1) \right] = 0 \quad (5)$$

$^3$ Notice the incumbent only makes decision about quality of education. It takes the market share decision(K) made by the entrant as given when it accommodates. However, the entrant has to make decisions about both quality and market share.
\[ p_S - D'(K) - \frac{\partial C(q_1^E, K)}{\partial K} + \lambda \left[ \frac{\partial C(q_1^*, 1 - K)}{\partial (1 - K)} - (p_S + p_L) \right] = 0 \]  
(6)

\[ C(q_1^E, 1) - C(q_1^*, 1 - K) = (p_S + p_L)K + U(q_1^E) - U(q_1^*) \]  
(7)

From (5),

\[ \Rightarrow \lambda = \frac{\frac{\partial C(q_1^E, K)}{\partial q_1^E} - U(q_1^E)}{\frac{\partial C(q_1^*, I)}{\partial q_1^*} - U(q_1^*)} \]  
(8)

With separable cost functions, equation (8) shows that \( \lambda = 1 \). Substituting the value of \( \lambda \) from (8) into (6), we get:

\[ \frac{\partial C(q_1^*, 1 - K)}{\partial (1 - K)} - D'(K) - \frac{\partial C(q_1^E, K)}{\partial K} - p_L = 0 \]  
(9)

Equations (4), (7) and (9) form a system of equations which could be used to derive equilibrium values for \( q_1^*, q_1^E \) and \( K \) if we know the functional forms for \( C(q, K), U(q) \) and \( D(K) \). Assuming equation (7) holds with equality, it can shown that \( \frac{\partial q_1^E}{\partial K} > 0 \), similar to Cardon(2003). That means to maintain accommodation, the entrant must raise quality in order to increase capacity.

### 3 Comparative Statics

For policy analysis, I will use the model to analyze the effect of increasing state and local level funding on output and quality. In case of comparative statics involving an increase in \( p_S \), all the terms related to \( D(K) \) drops out. Therefore, the result is identical to Cardon(2003) that an increase in \( p_S \) has no effect on \( K \) and identical to Rahman(2010) that it increases \( q_1^E \) iff \( \frac{\partial C(q_1^E, 1)}{\partial q_1^E} > \frac{\partial U(q_1^E)}{\partial q_1^E} \). The intuition is that when \( p_S \) increases, charter schools do not receive it. So they cannot expand their capacity. Also, quality will increase only when public school fails to match quality and accommodate. This will happen when the marginal cost of matching quality is greater than the marginal utility. But the effect of increasing \( p_L \) involves \( D(K) \) and I will present the result in the following proposition.

**Proposition 1** Assume there is an increase in \( p_L \) with no change in \( p_S \). Then the following results will hold for capacity and quality:

a) If \( D(K) \) is strictly convex, then

i) \( \frac{\partial K}{\partial p_L} < 0 \).

ii) \( \frac{\partial q_1^E}{\partial p_L} > 0 \) iff \( \frac{\partial C(q_1^E, K)}{\partial q_1^E} > \frac{\partial U(q_1^E)}{\partial q_1^E} \)

b) If \( D(K) \) is strictly concave and if \( \left| \frac{\partial^2 D(K)}{\partial K^2} \right| < \left| \frac{\partial^2 C(q_1^E, K)}{\partial K^2} + \frac{\partial^2 C(q_1^*, 1 - K)}{\partial (1 - K)^2} \right| \), then

i) \( \frac{\partial K}{\partial p_L} < 0 \).

ii) \( \frac{\partial q_1^E}{\partial p_L} > 0 \) iff \( \frac{\partial C(q_1^E, 1)}{\partial q_1^E} > \frac{\partial U(q_1^E)}{\partial q_1^E} \)

**Proof.** a) In order to carry out the comparative statics, I will differentiate equation (7) and (9) with respect to \( p_L \). After applying Implicit function theorem and Cramer’s rule, I get:

\[ \frac{\partial K}{\partial p_L} = -\frac{\frac{\partial^2 D(K)}{\partial K^2} + \frac{\partial^2 C(q_1^E, K)}{\partial K^2} + \frac{\partial^2 C(q_1^*, 1 - K)}{\partial (1 - K)^2}}{\frac{\partial C(q_1^*, 1 - K)}{\partial (1 - K)}} \]
We see that if $D(K)$ is strictly convex, then the denominator is positive. Hence, $\frac{\partial K}{\partial p_L} < 0$.

But if $D(K)$ is strictly concave, then

\[
\frac{\partial K}{\partial p_L} < 0 \quad \text{iff} \quad \frac{\partial^2 D(K)}{\partial K^2} > - \left[ \frac{\partial^2 C(q^*_E, K)}{\partial K^2} + \frac{\partial^2 C(q^*_1, 1 - K)}{\partial (1 - K)^2} \right] \quad \text{iff} \quad \left| \frac{\partial^2 D(K)}{\partial K^2} \right| < \left| \frac{\partial^2 C(q^*_E, K)}{\partial K^2} + \frac{\partial^2 C(q^*_1, 1 - K)}{\partial (1 - K)^2} \right|
\]

b) Using Crammer’s rule again, I get:

\[
\frac{\partial q^*_E}{\partial p_L} = \frac{1}{\left[ \frac{\partial C(q^*_E, 1 - K)}{\partial q^*_E} - \frac{\partial U(q^*_E)}{\partial q^*_E} \right]} \left[ K + \frac{\partial C(q^*_1, 1 - K)}{\partial (1 - K)^2} \left( 1 + \frac{\partial^2 D(K)}{\partial K^2} \right) \right]
\]

Notice that $\frac{\partial C(q^*_E, 1 - K)}{\partial (1 - K)^2} > 0$ by assumption. If $D(K)$ is strictly convex, then $\frac{\partial^2 D(K)}{\partial K^2} + \frac{\partial^2 C(q^*_E, K)}{\partial K^2} + \frac{\partial^2 C(q^*_1, 1 - K)}{\partial (1 - K)^2} > 0$. Then $\frac{\partial q^*_E}{\partial p_L} > 0$ iff $\frac{\partial C(q^*_E, 1)}{\partial q^*_E} > \frac{\partial U(q^*_E)}{\partial q^*_E}$ and $\left| \frac{\partial^2 D(K)}{\partial K^2} \right| < \left| \frac{\partial^2 C(q^*_E, K)}{\partial K^2} + \frac{\partial^2 C(q^*_1, 1 - K)}{\partial (1 - K)^2} \right|$. If on the other hand, $D(K)$ is strictly concave, then $\frac{\partial q^*_E}{\partial p_L} > 0$ iff $\frac{\partial C(q^*_E, 1)}{\partial q^*_E} > \frac{\partial U(q^*_E)}{\partial q^*_E}$ and $\left| \frac{\partial^2 D(K)}{\partial K^2} \right| < \left| \frac{\partial^2 C(q^*_E, K)}{\partial K^2} + \frac{\partial^2 C(q^*_1, 1 - K)}{\partial (1 - K)^2} \right|$.\]

4 Conclusion

In this paper, I have used a model of strategic competition to highlight the importance of funding gap between traditional public schools and charter schools. Results from the paper show that competitive pressure from the charter schools and the type of their entry cost could have significant effect on their relative market share and the overall quality. Therefore, appropriate policy making targeted towards bridging the funding gap could have significant effect on the schooling choices and the quality of education.

References


