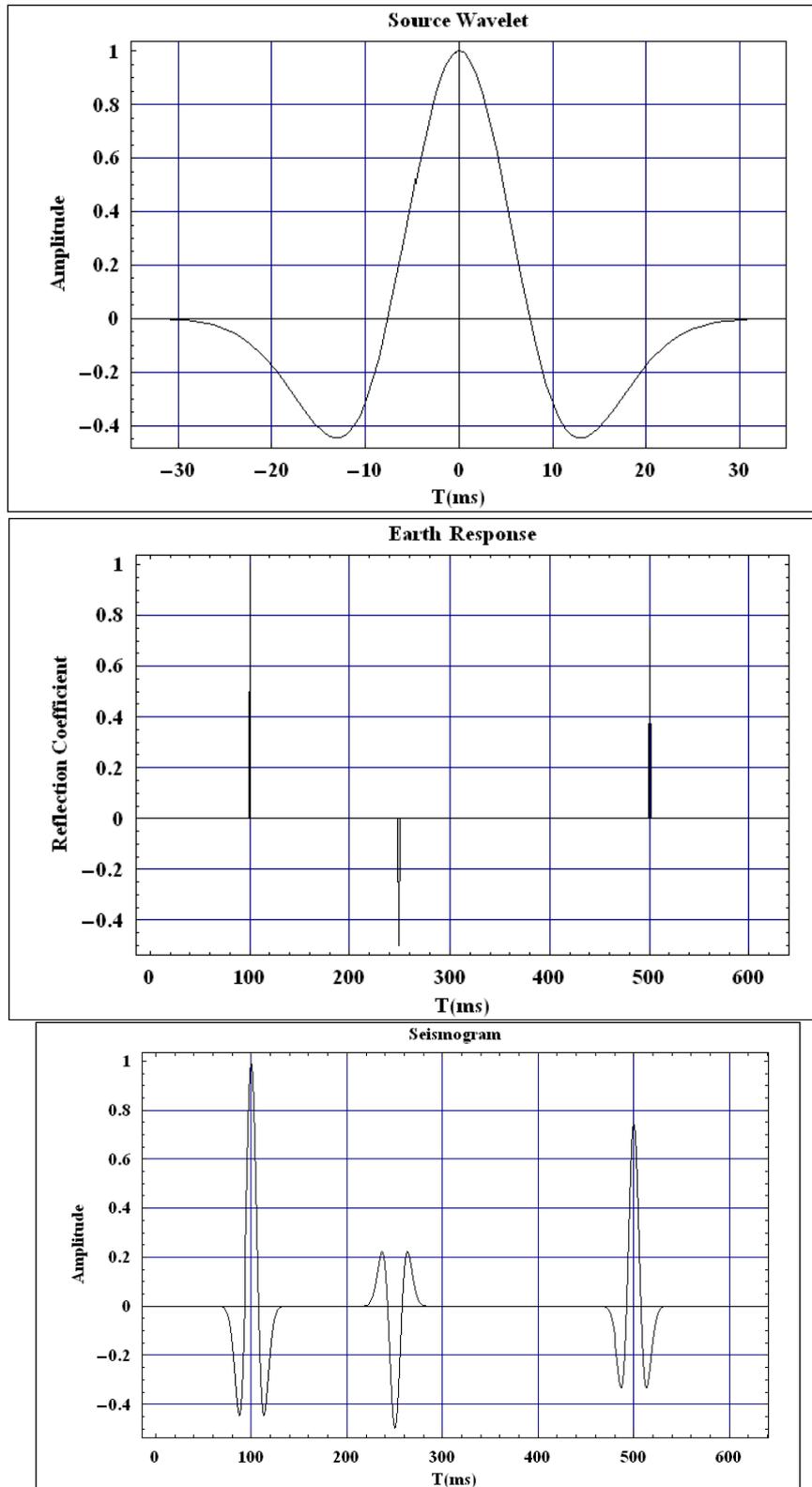
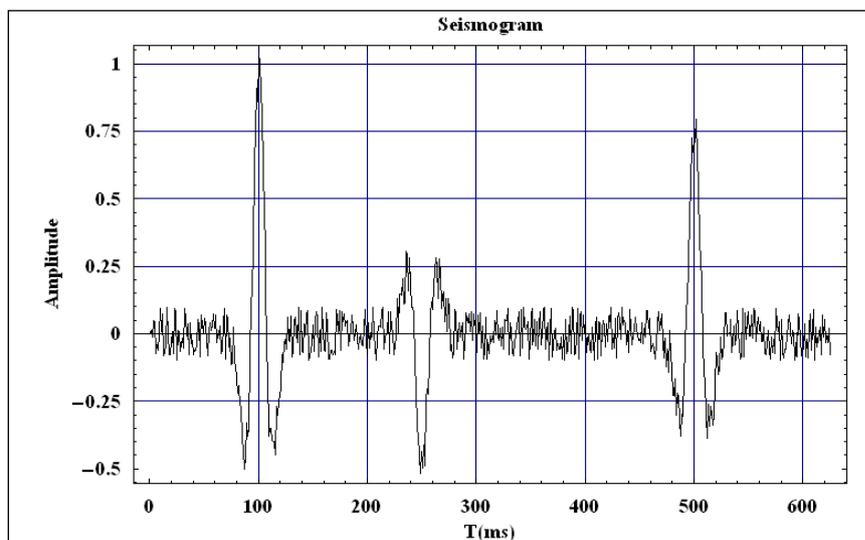
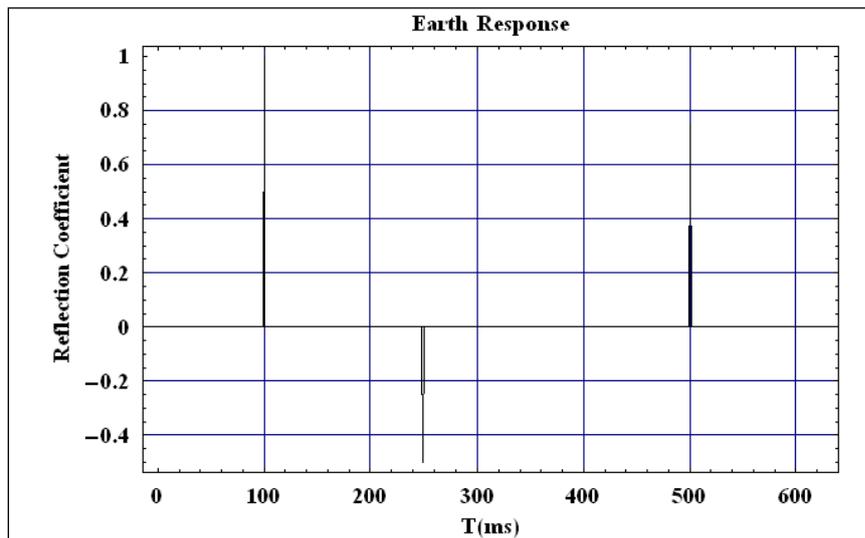
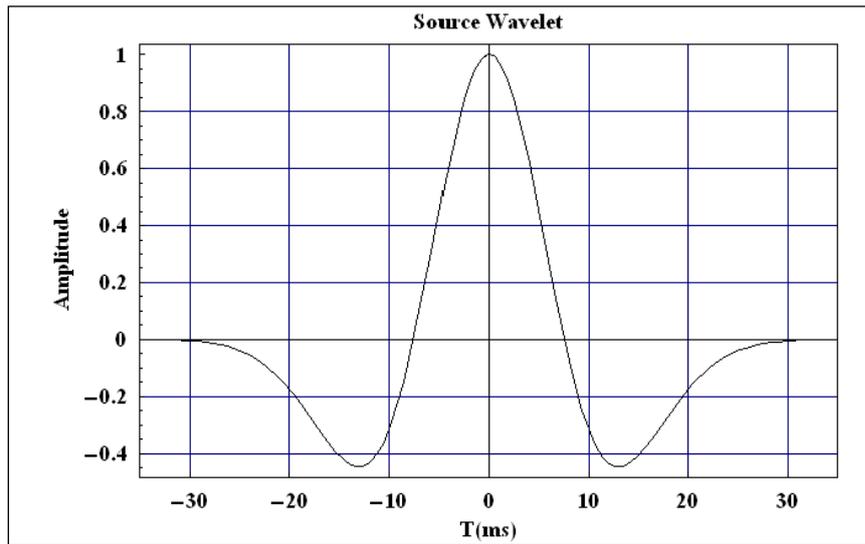


# Chapter 3 Figures

**Figure 3.1**  
**Noise-free**



### Noise-contaminated (sd = 0.1)



### Figure 3.2

#### Questions

Given the following wavelets:

(a)  $w(t) = (2, -1)$

(b)  $w(t) = (1, -1)$

(c)  $w(t) = (1, -2)$

(1) Find the inverse filters  $f_3(t) = (f_0, f_1, f_3)$  of wavelets (a), (b), and (c).

(2) Compute the actual output  $y(t)$  of wavelets (a), (b), and (c).

(3) Compute the error  $E$  between the desired and actual outputs of wavelets (a), (b), and (c).

(4) Which wavelet has the minimum error? Why?

#### Answers

(1) Inverse filters:

(a)  $W(z) = 2-z; F(z) = 1/W(z) = 1/(2-z) = (1/2)[1/(1-(1/2)z)] =$

$(1/2)[1+(1/2)z+(1/4)z^2+\dots] = 1/2+(1/4)z+(1/8)z^2+\dots; f_3(t) = (1/2, 1/4, 1/8);$

(b) Prove that  $f_3(t) = (1, 1, 1)$ .

(c) Prove that  $f_3(t) = (1, 2, 4)$

(2) Actual outputs:

(a)  $y(t) = w(t)*f_3(t) = (1, 0, 0, -1/8)$ .

(b) Prove that  $y(t) = (1, 0, 0, -1)$ .

(c) Prove that  $y(t) = (1, 0, 0, -8)$ .

(3) Errors:  $d(t) = \delta(t) = (1,0,0,0)$ :

(a)  $E = (1-1)^2 + (0-0)^2 + (0-0)^2 + (0+1/8)^2 = 1/64 = 0.015625$ .

(b) Prove that  $E = 1$ .

(c) Prove that  $E = 64$ .

(4) Wavelet (a) has the minimum error because it is a minimum-phase wavelet while wavelet

(b) is mixed-phase and (c) is a maximum-phase wavelet.

### **Figure 3.3**

Let the input be  $x(t)=(x_0,x_1) \dots$  known

And the desired output be  $d(t)=(d_0,d_1,d_2) \dots$  known

And the filter be  $f(t)=(f_0,f_1) \dots$  unknown

Therefore, the actual output will be  $y(t)=f(t)*x(t)=(x_0.f_0,x_0.f_1+x_1.f_0,x_1.f_1) \dots$   
unknown

The error between the desired and actual outputs is E given as:

$$E=(d_0-y_0)^2+(d_1-y_1)^2+(d_2-y_2)^2=(d_0-x_0.f_0)^2+(d_1-x_0.f_1-x_1.f_0)^2+(d_2-x_1.f_1)^2$$

To minimize E w.r.t.  $f_0$ , we do the following:

$$\frac{\partial E}{\partial f_0}=0$$

$$\Rightarrow -2x_0(d_0-x_0.f_0)-2x_1(d_1-x_0.f_1-x_1.f_0)=0$$

$$\Rightarrow (x_0^2+x_1^2)f_0+x_0.x_1.f_1=x_0.d_0+x_1.d_1$$

$$\Rightarrow r_0.f_0+r_1.f_1=g_0, \quad (1)$$

$$\text{where: } r(t)=x(t)\otimes x(t) \text{ and } g(t)=d(t)\otimes x(t)$$

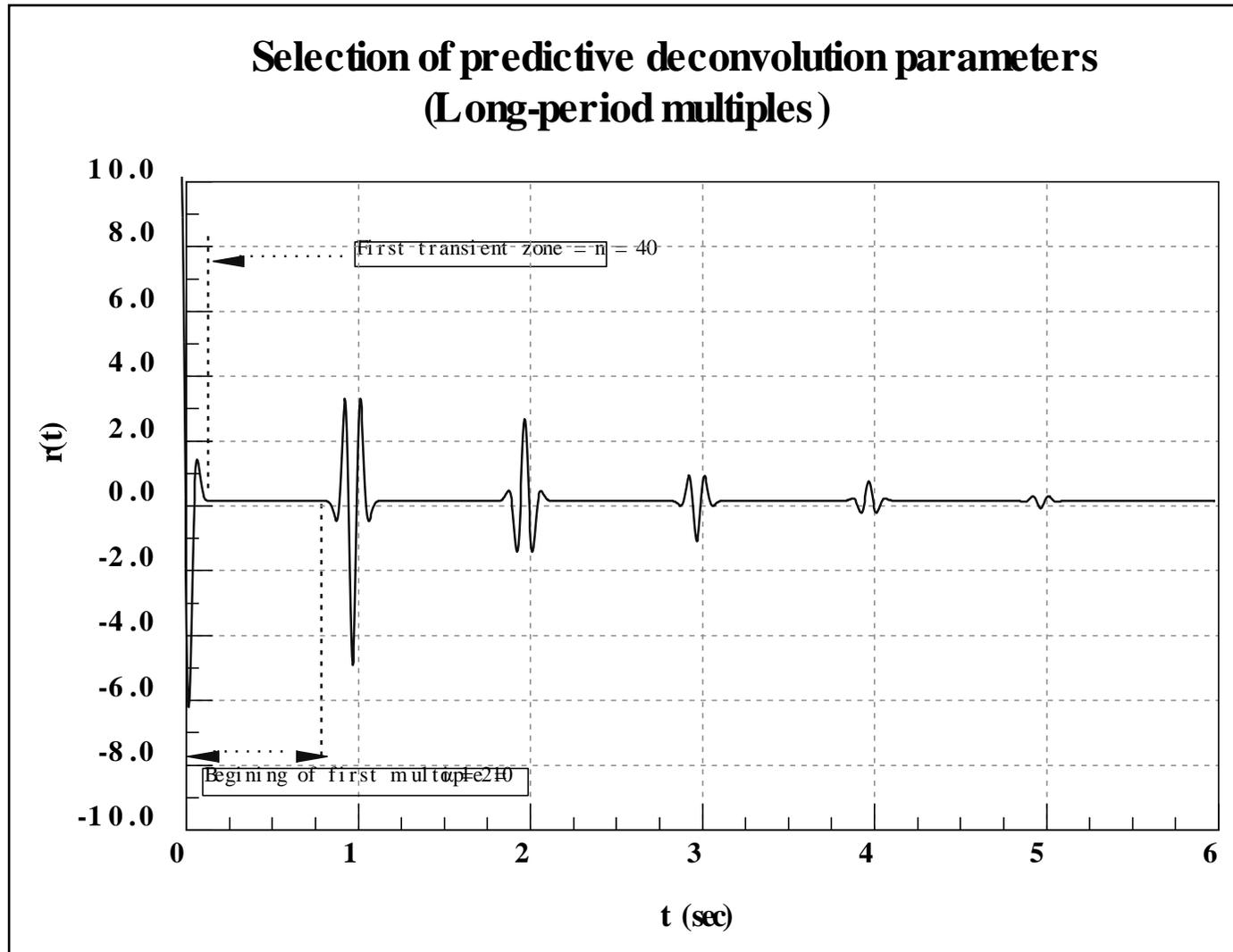
To minimize E w.r.t.  $f_1$ , we follow a similar approach and get:

$$r_1.f_0+r_0.f_1=g_1, \quad (2)$$

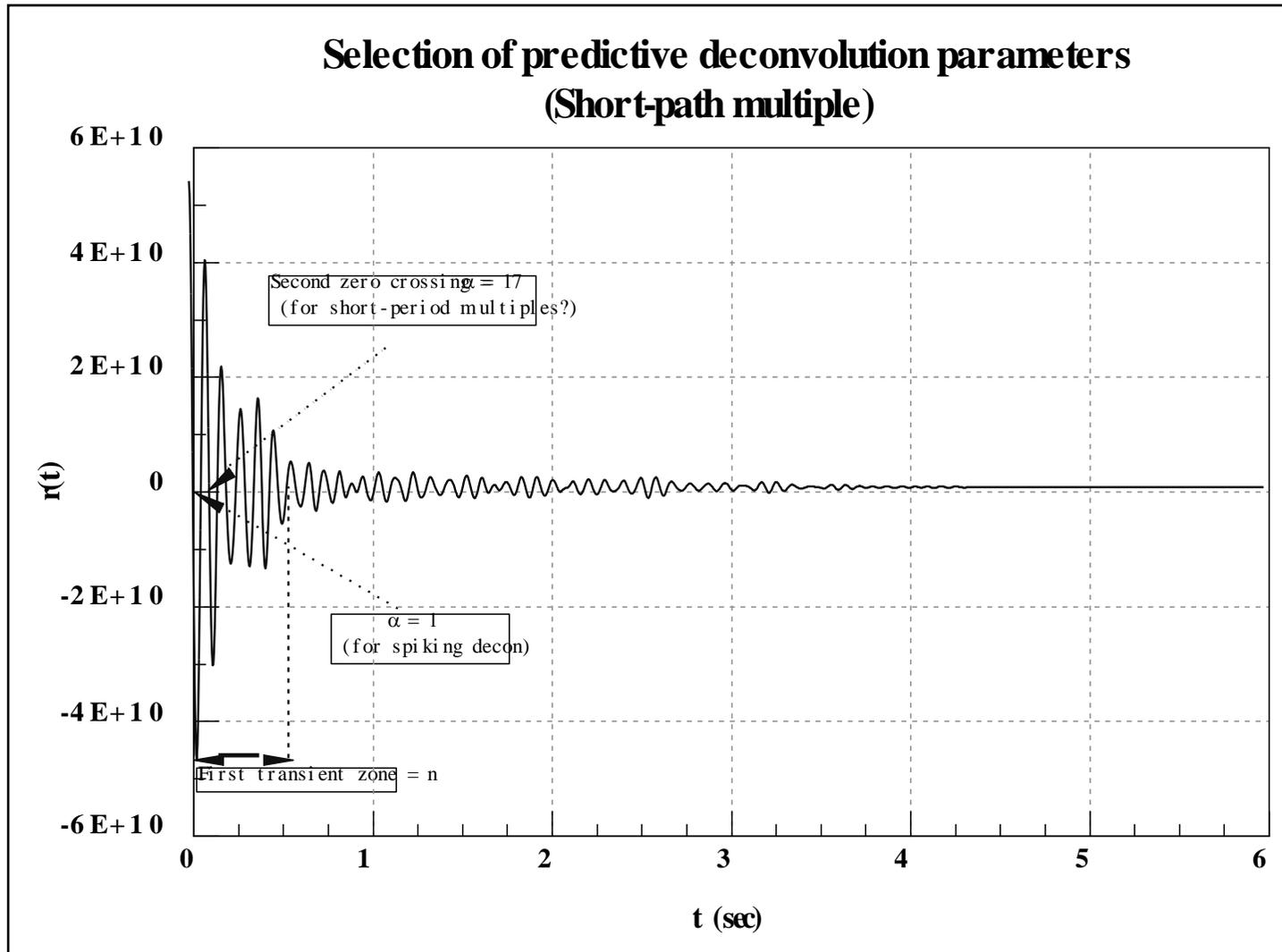
Solving equations (1) and (2) simultaneously, we get the unknown filter coefficients  $f_0$  and  $f_1$ .

Equations (1) and (2) are called the normal equations.

Figure 3.4A



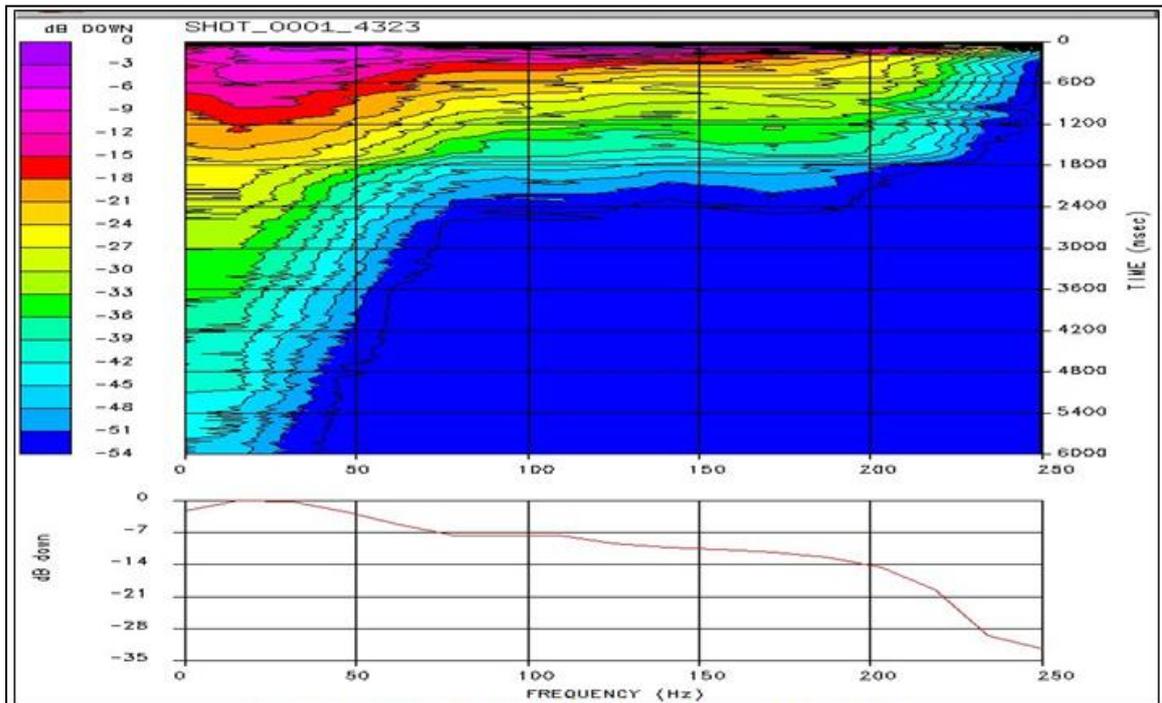
**Figure 3.4B**



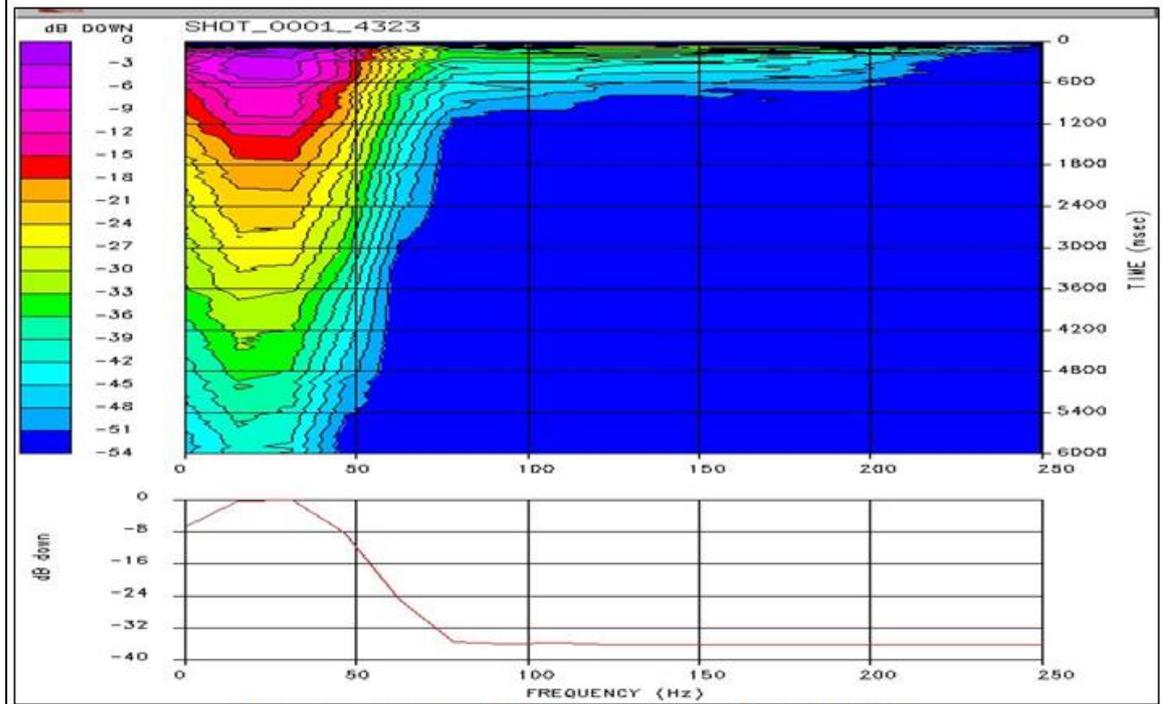
**Figure 3.5**

Parameter	Spiking deconvolution	Multiple suppression	
		Short-period (e.g., ghost)	Long-period (e.g., water-bottom)
<b>Autocorrelation Window (w)</b>	<ul style="list-style-type: none"> <li>➤ As long as possible.</li> <li>➤ Should be in a section with the highest S/N ratio.</li> <li>➤ Should be greater than eight times the longest operator length = <math>8n_{max}</math>.</li> </ul>		
<b>Operator length (n)</b>	<ul style="list-style-type: none"> <li>➤ As long as possible.</li> <li>➤ Should include the first transient zone.</li> </ul>		
<b>Prediction lag (<math>\alpha</math>)</b>	1	Second zero crossing	Beginning of first multiple
<b>Prewhitening (<math>\epsilon</math>) (%)</b>	0.1		

**Figure 3.6**



**Time-Frequency Analysis of a Raw Shot Gather**



**Time-Frequency Analysis of Filtered Shot Gather**

**Figure 3.7**

- G: Airgun (source), H: Hydrophone (receiver), sampling interval =  $\Delta t$ , and  $c = T_0/\Delta t$ .
- $dT \ll T_0$  is selected optimally such that the upgoing and downgoing waves add up in phase and a magnified downgoing wave is always recorded by the receivers.

