Introduction

- When a wave encounters an interface, part of its energy is reflected while the other part is transmitted (refracted) to the other medium.

- At the interface, a wave of a specific mode (e.g., P-wave) may produce other modes (e.g., SV-wave).

- A P- or SV-wave incident on an interface between two solids will produce the following four modes:
  - Reflected P- & SV-waves.
  - Transmitted P- & SV-waves.

- A SH-wave incident on an interface between two solids will only produce reflected and transmitted SH-waves.
Introduction

- Snell’s Law describes what happens to the angles of various wave modes as functions of their respective velocities.

- Zoeppritz’ equations describe what happens to the amplitudes of various wave modes as functions of their respective:
  - P-wave velocities.
  - S-wave velocities.
  - Densities.
  - Angles of incidence, reflection, and transmission.

- Reflection (transmission) coefficient is defined as the ratio between the amplitudes of the reflected (transmitted) and incident waves.
Introduction

$A_0$: amplitude of incident P-wave
$A_1$: amplitude of reflected P-wave
$B_1$: amplitude of reflected S-wave
$A_2$: amplitude of transmitted P-wave
$B_2$: amplitude of transmitted S-wave

$VP_1$: P-wave velocity in incident medium
$VS_1$: S-wave velocity in incident medium
$VP_2$: P-wave velocity in refraction medium
$VS_2$: S-wave velocity in refraction medium
$p$: ray parameter

$$\sin i_1 = \frac{\sin i_1}{VP_1} = \frac{\sin i_2}{VP_2} = \frac{\sin j_1}{VS_1} = \frac{\sin j_2}{VS_2} = p$$
The following boundary conditions have to be satisfied at the (solid-solid) interface:

1. The tangential displacement is continuous
2. The normal displacement is continuous
3. The tangential stress is continuous
4. The normal stress is continuous

Application of these boundary conditions results in the following Zoeppritz’ equations:

\[
\frac{A_1}{A_0} - 1 \right) \cos i_1 - \left( \frac{B_1}{A_0} \right) \sin j_1 = -\left( \frac{A_2}{A_0} \right) \cos i_2 - \left( \frac{B_2}{A_0} \right) \sin j_2
\]

\[
\left( \frac{A_1}{A_0} + 1 \right) \sin i_1 + \left( \frac{B_1}{A_0} \right) \cos j_1 = \left( \frac{A_2}{A_0} \right) \sin i_2 - \left( \frac{B_2}{A_0} \right) \cos j_2
\]

\[
\left( \frac{A_1}{A_0} + 1 \right) d_1 V_P V_1 \cos j_1 - \left( \frac{B_1}{A_0} \right) d_1 V_S V_1 \sin j_1 = \left( \frac{A_2}{A_0} \right) d_2 V_P V_2 \cos j_2 + \left( \frac{B_2}{A_0} \right) d_2 V_S V_2 \sin j_2
\]

\[
\left( \frac{A_1}{A_0} - 1 \right) \left( \frac{V_S}{V_P} \right) d_1 V_S V_1 \sin j_1 + \left( \frac{B_1}{A_0} \right) d_1 V_S V_1 \cos j_1 = -\left( \frac{A_2}{A_0} \right) \left( \frac{V_S}{V_P} \right) d_2 V_S V_2 \sin j_2 + \left( \frac{B_2}{A_0} \right) d_2 V_S V_2 \cos j_2
\]

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Zoeppritz’ Equations

- The reflection and transmission coefficients resulting from an incident P-wave are:
  - \( RPP = \frac{A_1}{A_0} \) is the reflection coefficient of the reflected P-wave
  - \( RPS = \frac{B_1}{A_0} \) is the reflection coefficient of the reflected S-wave
  - \( TPP = \frac{A_2}{A_0} \) is the transmission coefficient of the transmitted P-wave
  - \( TPS = \frac{B_2}{A_0} \) is the transmission coefficient of the transmitted S-wave

- Zoeppritz’ equations can be solved simultaneously to give these four coefficients.

- The resulting formulas are very algebraically complicated.
The most relevant cases in seismic exploration are the:

1. RPP: This case is important for AVO studies.
2. RPS: This case is important for converted-wave (multi-component) seismic studies.

We will only study the RPP case, its approximations and applications.
The full Zoeppritz’ equation of the RPP is shown next.

It can also be seen that RPP reduces to the more familiar simple form for normal incidence (i.e., $\theta = i_1 = 0^\circ$).

However, under certain assumptions, the RPP Zoeppritz’ equation can be approximated with a good accuracy.

We will study here the most widely used approximation by Shuey (1985).
Exact RPP from Zoeppritz-Knott Equations

\[
\begin{align*}
\text{RPP} & - \\
\sqrt{\frac{2 \sin(\theta)}{a_i}} a_i & - \sqrt{\frac{2 \sin(\theta)}{a_i}} a_i \\
\sum_{i=1}^{n} b_i & - \frac{2 \sin(\theta)}{a_i} b_i \left( \sin(\theta) b_i - b_i \right) \\
2 \sqrt{1 + \cos(\theta)} & \sin(\theta) a_i b_i \\
\sum_{i=1}^{n} c_i & - \frac{2 \sin(\theta)}{a_i} c_i \left( \sin(\theta) c_i - c_i \right) \\
\end{align*}
\]
Shuey’s AVO Approximation

Starting with the following approximation of Aki and Richards (1980):

\[ RPP(i) \approx \left( \frac{1}{2} \right) \left[ 1 - 4 \left( \frac{V_{S}^2}{V_{P}^2} \right) \sin^2 i \left( \frac{\Delta d}{d} \right) + \left( \frac{1}{2} \right) \left( \frac{\Delta V_{P}}{V_{P}} \right) \sec^2 i - 4 \left( \frac{V_{S}^2}{V_{P}^2} \right) \left( \frac{\Delta V_{S}}{V_{S}} \right) \sin^2 i \right] \]

- Aki and Richards (1980) assumed the following when deriving the above approximation of Zoeppritz’ equations:
  - Angles are not near 90°.
  - \( \Delta d/d \ll 1 \), \( \Delta V_{P}/V_{P} \ll 1 \), and \( \Delta V_{S}/V_{S} \ll 1 \), where:
    - \( \Delta d=d_2-d_1 \), \( \Delta V_{P}=V_{P2}-V_{P1} \), and \( \Delta V_{S}=V_{S2}-V_{S1} \) are the differences in the density, P-wave velocity, and S-wave velocity across the interface.
    - \( i=(i_2+i_1)/2 \) is the average of the P-wave incidence and transmission angles.
    - \( d=(d_2+d_1)/2 \), \( V_{P}=(V_{P2}+V_{P1})/2 \), and \( V_{S}=(V_{S2}+V_{S1})/2 \) are the average density, P-wave velocity, and S-wave velocity across the interface.

2. He expressed \( \frac{\Delta V_{S}}{V_{S}} \) with Poisson’s ratio (PR) using the relation:

\[ \frac{\Delta V_{S}}{V_{S}} = \frac{\Delta V_{P}}{V_{P}} + \left( \frac{\Delta P R}{2} \right) \left( \frac{1}{1 - P R} - \frac{2}{1 - 2 P R} \right) \]
Shuey’s AVO Approximation

This led to the following AVO expression:

\[ \text{RPP}(i) = A + B \sin^2 i + C(Tan^2 i - Sin^2 i) \]

where:

- \( A = R(0) = \frac{d_2 VP_2-d_1 VP_1}{d_2 VP_2+d_1 VP_1} \)
- \( B = R(0) \left[ BB - \frac{2(1+BB)(1-2PR)}{1-PR} \right] + \frac{\Delta PR}{(1-PR)^2} \); \( BB = \frac{1}{1-(d_1-d_2)/(d_2-d_1)} \) \( \frac{1-(VP_2-VP_1)/(VP_2-VP_1)}{1-(VP_2-VP_1)/(VP_2-VP_1)} \)
- \( C = \frac{\Delta VP}{2VP} \)

In general:

- \( A \) dominates at small offsets or incidence angles \((i \approx 0^\circ - 15^\circ)\).
- \( B \) becomes important at intermediate offsets or incidence angles \((i \approx 15^\circ - 30^\circ)\).
- \( C \) becomes important at large offsets or incidence angles \((i \approx 30^\circ - 45^\circ)\).

4. Most seismic exploration cases involve intermediate incidence angles, leading to the expression:

\[ \text{RPP}(i) = A + B \sin^2 i \]

- \( A \) and \( B \) are called the AVO intercept and gradient, respectively.
Shuey’s AVO Approximation
We usually measure the wave amplitude (RPP), $\theta$, and $\alpha$ from surface seismic data and we want to estimate $\Delta PR$ because it is mainly a function of pore fluids. See next how $\theta$ is measured.

We do this by analyzing the AVO gradient because it is mainly a function of $\Delta PR$ in typical sedimentary rocks.

However, before we do this, we need $\rho$ and $\beta$.

We can either use $\rho$ and $\beta$ from well logs (if available) or estimate them from $\alpha$ using empirical relations.
Shuey’s AVO Approximation

To transform from constant offset to constant angle, we need to know the relationship between $X$ and $\theta$. For a complete solution, a full ray tracing must be done. However, a good approximation is to use straight rays. In this case we find that:

$$\tan \theta = \frac{X}{2Z} \quad (26)$$

where:
- $\theta =$ angle of incidence
- $X =$ offset
- $Z =$ depth

If we know the velocity down to the layer of interest, we can write:

$$Z = \frac{V t_0}{2} \quad (27)$$

where:
- $V =$ velocity (RMS or average)
- $t_0 =$ total zero-offset travelt ime

Substituting equation (27) into (26) gives:

$$\tan \theta = \frac{X}{V t_0} \quad (28)$$

which gives us the mapping from offset to angle. By inverting equation (28), we get the mapping from angle to offset.
Empirical Relations

- **$\alpha - \beta$**
  - The following Castagna’s relations are commonly used (where velocities are in m/s):
    - Limestones: $\beta = \frac{\alpha}{1.9}$ (for $\beta > 1500$ m/s)
      $$\beta = -0.0551 \alpha^2 + 1.017 \alpha - 1031$$ (for $\beta < 1500$ m/s)
    - Sandstones: $\beta = 0.804 \alpha - 856$
    - Shales: $\beta = 0.77 \alpha - 867$
    - Dolomites: $\beta = \frac{\alpha}{1.8}$

- **$\alpha - \rho$**
  - The following Gardner’s rule is commonly used (where velocity is in m/s and density is in kg/m$^3$):
    - $\rho = 310 \alpha^{0.25}$
Processing AVO Data

- **Proposed Flow**
  1. **Surface-consistent amplitude gain** that preserves amplitude variation with offset and azimuth
  2. **Surface-consistent deconvolution**
  3. **Standard time corrections without stacking including:**
     1. NMO correction
     2. Noise attenuation
     3. CMP sorting
     4. Prestack migration
Processing AVO Data

- Overburden Effects
  - Waves suffer from any effects above the target reflection (i.e., overburden effects). Therefore, overburden effects (if present) must be accounted for.

  - There are many ways to do this including:
    1. Normalizing the target reflection by a reference reflection. If the AVO effect disappears; it must be related to overburden effects.
    2. Perform AVO analysis on key overburden layers to estimate any overburden AVO effects and account for their effects on the target reflection (i.e., layer-stripping).
Analyzing AVO Data

Typical Workflow

1. Offset is transformed to $\sin^2 \theta$ within each CMP.

2. The maximum (or RMS) amplitudes within a window around the target reflection are picked within each CMP.

3. The picked amplitudes are plotted against $\sin^2 \theta$ within each CMP.

4. A best-fit line is fitted to the amplitude-$\sin^2 \theta$ dataset and the intercept (A) and slope (B) are calculated for each CMP.

5. Relating A and B to changes in lithology and/or pore fluids using forward modeling and synthetic seismograms.
Applications

- **Gas Sand Detection (AVO Classes)**
  - These classes describe AVO at the interface separating a sandstone overlain by shale.
  - These AVO effects are more successful in soft reservoir rocks such as the Gulf of Mexico and North Sea.

![AVO behavior and A-B crossplot diagram](image)

**Fig. 26.7** AVO classification scheme based on A-B crossplotting. (Redrawn from [6]).
(A) Reflection coefficients as a function of incidence angle showing sandstone Classes I–IV. Type III is the classic response associated with a gas sand in the Gulf of Mexico.
(B) AVO crossplot of intercept against gradient. The four AVO classes plot to separate areas on this diagram making it a useful tool for discriminating sandstone targets. Quadrants of the plot are labeled Q1–Q4.

**Table 26.6** Summary of the connection between impedance contrast and AVO behavior. (From [39]). The quadrants refer to Figure 26.7B.
Applications

- Fracture Detection (AVOA)
  - Vertical aligned fractures generate azimuthal anisotropy in amplitudes.
  - Measurement of the AVO gradient along several azimuths can be used to detect azimuthal anisotropy and estimate fracture parameters such as orientation and intensity.
Applications

- Fracture Detection (AVOA)

Figure 4.1.2.1: Validation of AVOA with FMI log from Well-1. Each colored cell in the first panel (from left) represents fracture orientation obtained using AVOA (for a single CDP supergather). The area from which panel 1 is extracted is a localized area around the horizontal well. The middle panel represents the open fracture orientation in Arab-D encountered by Well-1 (obtained from the FMI which extends for 2.5 km). The third panel is a superimposition of the first and middle panels to better demonstrate the correlation between fracture orientations of AVOA and FMI logs (the 3 dominant fracture orientations are encircled). Paying a closer look to the third panel shows how the AVOA fracture orientations are closely following the dominant fracture orientation changes obtained using the FMI. The top and bottom histograms demonstrate the fracture orientations distribution obtained by AVOA and FMI, respectively. It is evident that the dominant fracture orientations match well from both histograms. Such match confirms the reliability of the AVOA method for detecting fractures. Note: Well head is at the upper right corner.

Balhareth (2009)
Applications

- Fracture Detection (AVOA)

Figure 4.1.2.2: Validations of AVOA with FMI log from Well-2: Each colored cell in the first panel (from left) represents fracture orientation obtained using AVOA (for a single CDP supergather). The area from which panel 1 is extracted is a localized area around the horizontal well. The middle panel represents the open fracture orientation in Arab-D encountered by Well-2 (obtained from the FMI log which extends for 2 km). The third panel is a superimposition of the first and middle panels to better demonstrate the correlation between fracture orientations of AVOA and FMI logs. The top and bottom histograms demonstrate the fracture orientations distribution obtained by AVOA and FMI, respectively. Both methods (AVOA and FMI) demonstrate a dominant East-West orientation of fractures.
Applications

- Fracture Detection (AVOA)

Figure 4.2.1: AVOA ellipticity (fracture intensity) map. Generally, the backlimb portion of the study exhibits lower fracture intensity, with localities with relatively higher intensity. For example, the red ellipse encircled a high intensity anomaly on the backlimb where horizontal wells encountered loss of circulation confirming fracturing. The hinge zone and forelimb part of the structure exhibit the highest fracture intensity. Focusing on the intensity parallel to the anticline structure (between the dashed black curves), it is evident that the intensity of AVOA is dissipating northward as expected from the structural geology of the field. The red curve along the hinge line is an estimated oil-water contact (i.e., west of the drawn red curve Arab-D is brine-saturated and oil toward the east).
Applications

- Fracture Detection (AVOA)

Figure 12. On the left is the predicted FoI (four-month cumulative production from the Lance formation), output of the neural network using all attributes except those from AVAZ. High production is indicated by hot colors and low by cool colors. In the center the new well, Riverside 4-10, is added to the FoI prediction. On the right is the FoI is predicted using the AVAZ attributes. The Riverside 4-10 well completed in August 2002 is highlighted as the large white circle in the upper left of each of the displays.

Continued on Page 47
Applications

- More reading
  - Shuey (1985)
  - Rurtherfors and Williams (1989)
  - Foster et al. (2010)