A Noniterative Formulation for 2-D Optical Waveguide Discontinuity Problems Based on Padé Approximants

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***Abstract*—In this letter, we propose a simple noniterative formulation for the analysis of optical waveguide discontinuity problems. The formulation, which is based on rotated branch cut Padé approximation scheme, is both accurate and efficient. It is computationally fast due to its noniterative nature. The effectiveness of the proposed approach is demonstrated by modeling an optical waveguide air facet. Calculations show good agreement with previously published results.**

***Index Terms*—Optical waveguide discontinuities, Padé approximants, waveguide facet.**

I. Introduction

PADÉ approximants have been established as efficient rational approximations for modeling optical waveguide structures exhibiting longitudinal discontinuities. This is due to their ability to account for highly evanescent modes excited at waveguide interfaces using complex Padé primes or the branch cut rotation technique. This approach has been widely applied in the calculation of the reflected and transmitted fields of two-dimensional (2-D) optical waveguide structures exhibiting both, single [1]–[5] and multiple longitudinal discontinuities [6]–[9]. Very recently, Padé approximants have also been applied to three-dimensional (3-D) full-vectorial optical waveguide facet problems [10], showing the flexibility and versatility of the method. However, the previously reported algorithms are generally iterative in nature, which require iterative solvers to obtain the numerical solution. In general, these iterative methods require a relatively large number of iterations before convergence is attained. The use of preconditioners [3]–[5] results in a reduced number of iterations and thus enhances the numerical efficiency of these methods.

In this letter, we present a simple and efficient noniterative scheme based on Padé approximants, in the context of 2-D space, for the simulation of optical waveguide structures exhibiting an abrupt longitudinal discontinuity. The algorithm utilizes Padé approximants with branch cut rotation. To the best of our knowledge, such formulation for the facet problem has not been reported in the literature. Since the scheme is noniterative, it significantly reduces the computational time to obtain the numerical solution and it eliminates the need for a preconditioner.

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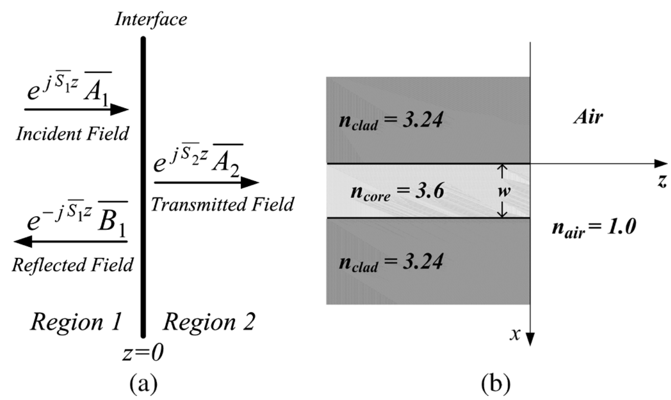

Fig. 1. (a) Abrupt interface between two longitudinally homogeneous regions, located at $z = 0$ and (b) optical waveguide-air facet.

II. Background

Consider the 2-D longitudinal discontinuity shown in Fig. 1(a). Discretization of the 2-D wave equation in the transverse direction x into M sample points, leads to an ordinary matrix differential equation

$$\frac{d^2\overline{\Psi}(z)}{dz^2} + \overline{S}^2\overline{\Psi}(z) = 0 \quad (1)$$

whose general solution is given by [1]–[5]

$$\overline{\Psi}(z) = e^{j\overline{S}z}\overline{A} + e^{-j\overline{S}z}\overline{B} \quad (2)$$

where the column vector $\overline{\Psi}(z)$ contains the discretized electric field E_y for transverse-electric (TE) waves or the discretized magnetic field H_y for transverse-magnetic (TM) waves. The matrix $\overline{S}^2 = \overline{D}_{xx}^2 + k_0^2\overline{N}^2$, where the matrices $\overline{D}_{xx}^2$ and $\overline{N}^2$ represent the transverse second-order derivative operator and the square of the refractive index, respectively, and k_o is the free space wave number. The first and second terms on the right side of (2) represent the forward and backward fields, respectively.

Continuity of the tangential field components at the interface $z = 0$ leads to the following relations [1]–[5]:

$$(\overline{S_1} + \overline{U}\,\overline{S_2})\overline{B_1} = (\overline{S_1} - \overline{U}\,\overline{S_2})\overline{A_1} \quad (3)$$

$$(\overline{S_1} + \overline{U}\,\overline{S_2})\overline{A_2} = 2\overline{S_1}\,\overline{A_1} \quad (4)$$

where the matrix $\overline{U} = \overline{I}$ for the TE waves, $\overline{I}$ is the identity matrix and $\overline{U} = \overline{N}_1^2/\overline{N}_2^2$ for the TM waves. Equations (3) and (4) relate the reflected field $\overline{B_1}$ and the transmitted field $\overline{A_2}$ to the known incident field $\overline{A_1}$, respectively.

III. Formulation

In the following, (3) will be used to derive the proposed noniterative formulation for the reflected part of the field. The square root matrix operator $\overline{S}$ appearing in (3) is first approximated by use of Padé approximants of the pth order and a rotated branch cut complex scalar γ. This yields the following matrix relation:

$$\gamma^{\frac{-1}{2}}\left(\left[\prod_{k=1}^{p}\frac{\overline{I}+a_k^{(p)}\overline{X_1}}{\overline{I}+b_k^{(p)}\overline{X_1}}\right]+\overline{U}\left[\prod_{k=1}^{p}\frac{\overline{I}+a_k^{(p)}\overline{X_2}}{\overline{I}+b_k^{(p)}\overline{X_2}}\right]\right)\overline{B_1} = \gamma^{\frac{-1}{2}}\left(\left[\prod_{k=1}^{p}\frac{\overline{I}+a_k^{(p)}\overline{X_1}}{\overline{I}+b_k^{(p)}\overline{X_1}}\right]-\overline{U}\left[\prod_{k=1}^{p}\frac{\overline{I}+a_k^{(p)}\overline{X_2}}{\overline{I}+b_k^{(p)}\overline{X_2}}\right]\right)\overline{A_1} \quad (5)$$

where the square matrix $\overline{X} \equiv \gamma\overline{S} - \overline{I}$ and the coefficients $a_k^{(p)}$, $b_k^{(p)}$ are Padé primes of the square root operator [9]. Rearranging (5) leads to the following form:

$$\left[\prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_1}\right)^{-1}\prod_{k=1}^{p}\left(\overline{I}+a_k^{(p)}\overline{X_1}\right) + \overline{U}\prod_{k=1}^{p}\left(\overline{I}+a_k^{(p)}\overline{X_2}\right)\prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_2}\right)^{-1}\right]\overline{B_1} = \left[\prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_1}\right)^{-1}\prod_{k=1}^{p}\left(\overline{I}+a_k^{(p)}\overline{X_1}\right) - \overline{U}\prod_{k=1}^{p}\left(\overline{I}+a_k^{(p)}\overline{X_2}\right)\prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_2}\right)^{-1}\right]\overline{A_1}. \quad (6)$$

By extracting the matrix product $\prod_{k=1}^{p}(\overline{I}+b_k^{(p)}\overline{X_2})^{-1}$ and left-multiplying by the matrix product $\prod_{k=1}^{p}(\overline{I}+b_k^{(p)}\overline{X_1})$, (6) takes the following form:

$$\left[\prod_{k=1}^{p}\left(\overline{I}+a_k^{(p)}\overline{X_1}\right)\prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_2}\right) + \prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_1}\right)\overline{U}\prod_{k=1}^{p}\left(\overline{I}+a_k^{(p)}\overline{X_2}\right)\right] \times \left[\prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_2}\right)^{-1}\right]\overline{B_1}$$

$$= \left[\prod_{k=1}^{p}\left(\overline{I}+a_k^{(p)}\overline{X_1}\right)\prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_2}\right) - \prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_1}\right)\overline{U}\prod_{k=1}^{p}\left(\overline{I}+a_k^{(p)}\overline{X_2}\right)\right] \times \left[\prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_2}\right)^{-1}\right]\overline{A_1} \quad (7)$$

from which the following expression may be obtained:

$$\overline{B_1} = \left[\prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_2}\right)\right] \times \left[\prod_{k=1}^{p}\left(\overline{I}+a_k^{(p)}\overline{X_1}\right)\prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_2}\right) + \prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_1}\right)\overline{U}\prod_{k=1}^{p}\left(\overline{I}+a_k^{(p)}\overline{X_2}\right)\right]^{-1} \times \left[\prod_{k=1}^{p}\left(\overline{I}+a_k^{(p)}\overline{X_1}\right)\prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_2}\right) - \prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_1}\right)\overline{U}\prod_{k=1}^{p}\left(\overline{I}+a_k^{(p)}\overline{X_2}\right)\right] \times \left[\prod_{k=1}^{p}\left(\overline{I}+b_k^{(p)}\overline{X_2}\right)^{-1}\right]\overline{A_1}. \quad (8)$$

Equation (8) represents an explicit expression that can be used to directly calculate the reflected field $\overline{B_1}$, based on Padé order p. The transmitted field $\overline{A_2}$ may then be obtained using $\overline{A_2} = \overline{A_1} + \overline{B_1}$. It is noteworthy to mention that for Padé order p, the present formulation requires merely $p+1$ sparse matrix-vector divisions, resulting in much improved computational efficiency when compared to available iterative algorithms [1]–[8] that typically require $2p$ sparse matrix-vector divisions for *each* iteration of the left hand side of (5). An additional advantage of the present formulation is that it requires no preconditioner. This leads to further improvement of the computational efficiency and reduction in the complexity of the formulation.

IV. Numerical Results

The formulation developed in the previous section will be demonstrated for both accuracy and efficiency by applying it to the waveguide facet problem shown in Fig. 1(b). This same waveguide facet problem has been analyzed previously [1]. The waveguide core index $n_{\mathrm{core}} = 3.6$, cladding index $n_{\mathrm{clad}} = 3.24$, and the waveguide facet is terminated by air. The operating wavelength $\lambda = 0.86\ \mu$m. A perfectly matched absorbing layer has been used on either side of the computational window in order to absorb the radiative field [11]. Fig. 2 shows the calculated fundamental TE and TM mode reflectivities as a function of the core width w, for Padé orders 1, 2, and 3. It can be seen that, for Padé order 3, the calculated results converge and are in good agreement with the results reported in [1]. It should be noted that, for lower values of Padé orders ($p = 1, 2$, the present formulation overestimates the modal reflectivity values for TE

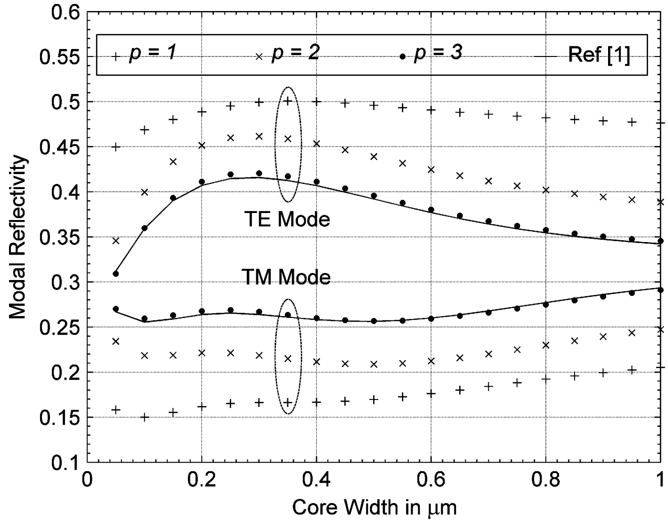

Fig. 2. Calculated fundamental TE and TM mode facet reflectivities corresponding to the waveguide air facet shown in Fig. 1(b), for Padé approximants of different orders. The operating wavelength $\lambda = 0.86\ \mu$m.

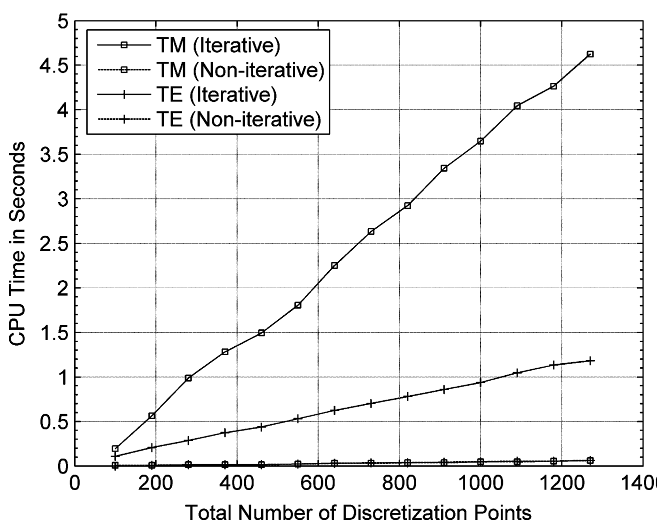

Fig. 3. CPU time as a function of the total number of discretization points corresponding to the waveguide air facet shown in Fig. 1(b). The core width and the Padé order are fixed at $w = 1\ \mu$m and $p = 3$, respectively.

polarized waves and underestimates the reflectivity values for TM polarized waves. This is due to the relatively poor approximation of the square root matrix $\overline{S}$ when Padé approximants of orders $p < 3$ are used.

The computational efficiency of the current approach is next demonstrated. This is done by comparing the CPU time required to solve (5) iteratively with the CPU time requirement of the present approach. For this purpose, we utilize an efficient biconjugate stabilized (Bi-CGSTAB) solver to iteratively solve (5) in the same manner used in [8] which *does not* utilize a preconditioner. However, the use of a preconditioner in the iterative approach generally results in a reduced number of iterations with an overhead due to the resulting increase in the computational time per iteration. Convergence of the iterative approach utilizing the Bi-CGSTAB solver is assumed to be achieved when the residual error becomes less than 10^{-7}. For the purpose of comparison, the waveguide core width is fixed at $w = 1\ \mu$m and the Padé order is set to $p = 3$. Fig. 3 shows the total CPU time required (to calculate the modal reflectivity) as a function of M (the total number of discretization points along the transverse direction x). It is clearly seen that the proposed noniterative scheme requires much less CPU time compared with the iterative scheme, using the Bi-CGSTAB solver. For instance, consider $M = 1270$. For the TE waves, the iterative algorithm requires 12 iterations to converge, which correspond to a CPU runtime of 1.2 s using a 1.8-GHz processor with 512-MB memory. For the TM waves, 48 iterations are required by the iterative scheme which correspond to a CPU runtime of 4.63 s. For the TM waves, the Bi-CGSTAB solver requires a relatively large number of iterations to converge, which is probably due to the TM longitudinal boundary condition and the use of a nonpreconditioned system of equations. The corresponding CPU time requirement of the noniterative scheme is approximately 0.06 s for both TE and TM polarized waves. The rate of increase of the CPU time requirement with respect to M is much lower for the noniterative scheme when compared with the rate associated with the iterative scheme.

V. Conclusion

An efficient noniterative formulation based on Padé approximants for calculating the reflected and transmitted fields at an abrupt waveguide discontinuity has been presented. The accuracy of the present method is assessed, showing good agreement with previously published results. Moreover, the CPU runtime requirement of the present scheme is much less than the corresponding runtime required by the conventional iterative scheme.

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