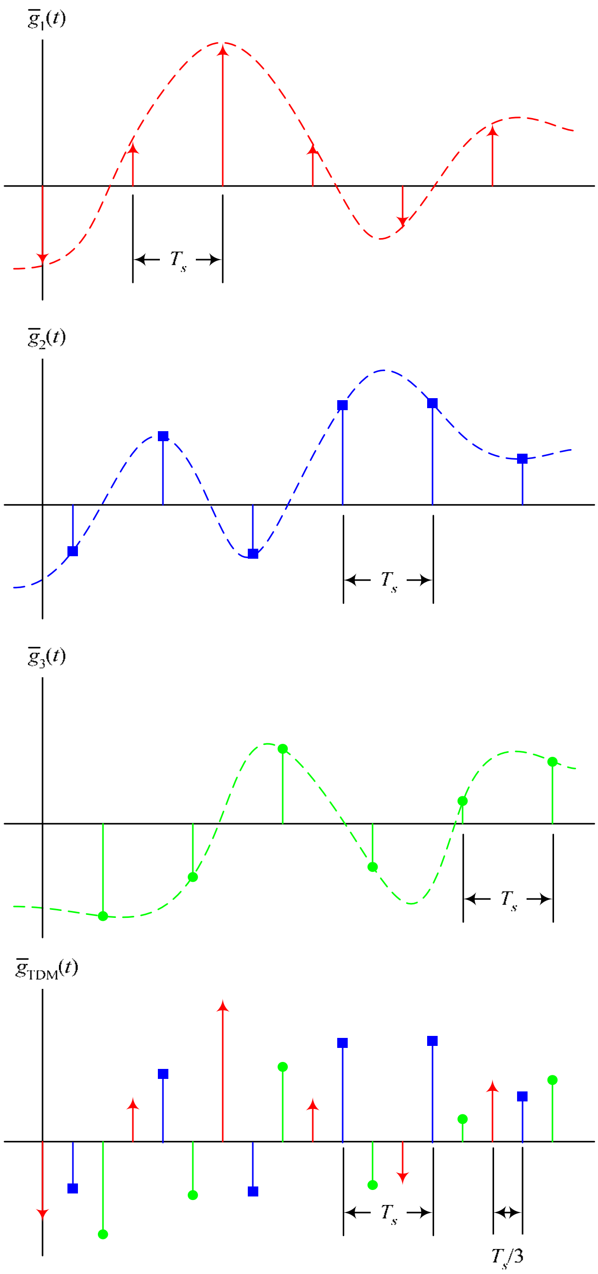
Application of Sampling in TDM

In many cases, we would like to transmit multiple signals over the same communication channels without modulating the signals first. Therefore, we have to use time–division multiplexing (TDM). TDM is a process in which different signals that have the same frequency are transmitted over the same channel. These signals instead of being multiplexed in frequency, they are multiplexed in time. One method for performing TDM is to sample the different signals at the same rate but at different time instants and the samples of the different signals are interleaved (placed in a sequence). Consider for example the three signals represent by the dashed lines shown below.

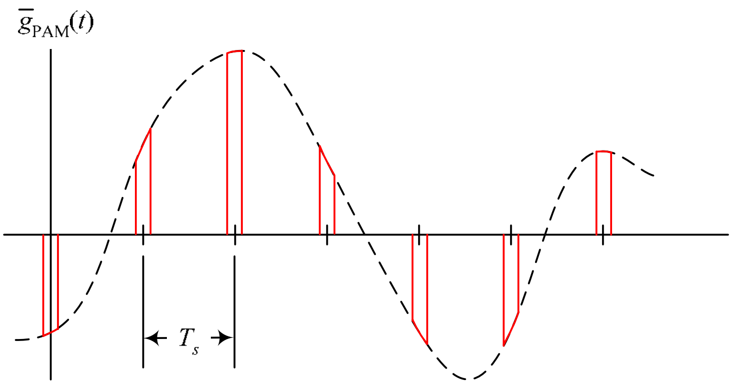


The signal containing the samples of the different original signals is a TDM signal. This signal can be transmitted over a channel and the received samples can be DE–INTERLEAVED (samples are separated to create the original signals). It is clear that TDM cannot be performed for continuous time signals.

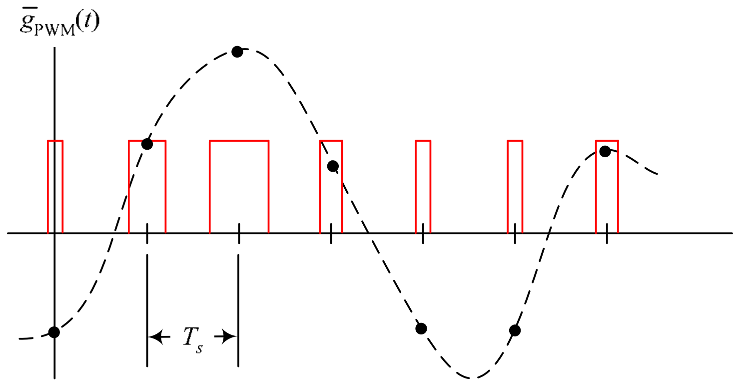
Pulse Modulated Signals

Since ideal delta function cannot be implemented in practice, representing samples of signals in terms of delta functions is only theoretical. Therefore, one practical method for representing samples is using pulses (rect functions) instead of impulses (delta functions). There are three main types using which we represent the information carried by a sequence of samples (three types of pulse modulations). Notice that the term "modulation" here is not used in the sense of modulation that we used in the previous chapters, which the frequency of a signal is shifted to a higher frequency for transmission. The term modulation here is used to specify the process in which the information signal modifies some parameter of a sequence of pulses. This parameter is used to transmit the desired information.

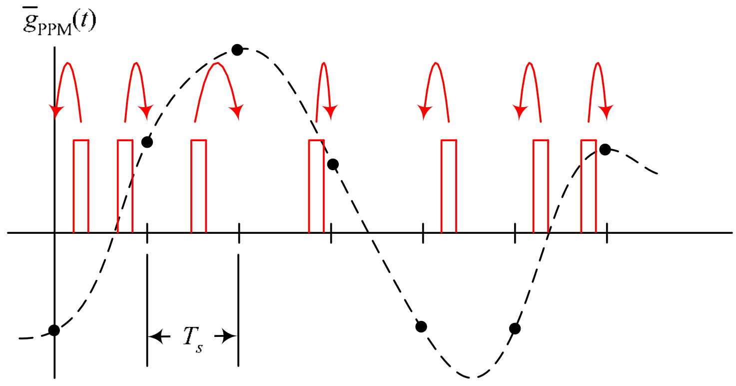
<u>Pulse Amplitude Modulation (PAM)</u>: in this modulation scheme, the information is carrier in the amplitude (or height) of the pulses. This is the most logical pulse modulation method. The following shows an example of PAM. Notice that the width of the different pulses is exactly the same and that the pulses are always centered at the sampling instants (or may start at the sampling instants), but there centers are always separated by the sampling period T_s.



<u>Pulse–Width Modulation (PWM)</u>: in this modulation, the information is carrier in the width (or duration) of the pulses. The following shows an example of PWM. Notice that the height (amplitude) of the different pulses is exactly the same and that the pulses are always centered at the sampling instants and separated by the sampling period T_s.



Pulse–Position Modulation (PPM): in this modulation, the information is carrier in the position of the pulses. The following shows an example of PPM. Notice that the height (amplitude) and width of the different pulses is exactly the same. Here the pulses are not centered at sampling instants.



Comment: Each of the above pulse modulation methods has advantages and disadvantages. For example, the advantage of PPM and PWD over PAM is that they have constant amplitude. For transmissions over channels that change with time (called time–varying channels) the gain of the channels may change, and therefore the height of the pulses may change. If the transmitted pulses originally had constant height as it is the case for PPM and PWM, even if the received pulses had varying amplitudes, the height amplitude has no effect on the detector of the information. This is generally not possible if PAM was used.

Pulse Code Modulation (PCM)

The modulation methods PAM, PWM, and PPM discussed in the previous lecture still represent analog communication signals since the height, width, and position of the PAM, PWM, and PPM, respectively, can take any value in a range of values. Digital communication systems require the transmission of a digital for of the samples of the information signal. Therefore, a device that converts the analog samples of the message signal to digital form would be required. Analog to Digital Converters (ADC) are such devices. ADCs sample the input signal and then apply a process called quantization. The quantized forms of the samples are then converted to binary digits and are outputted in the form of 1's and 0's. The sequence of 1's and 0's outputted by the ADC is called a PCM signal (Pulses have been coded to 1's and 0's).

Example: A color scanner is scanning a picture of height 11 inches and width 8.5 inches (Letter size paper). The resolution of the scanner is 600 dots per inch (dpi) in each dimension and the picture will be quantized using 256 levels per each color. Find the time it would require to transmit this picture using a modem of speed 56 k bits per second (kbps).

We need to find the total number of bits that will represent the picture. We know that 256 quantization levels require 8 bits to represent each quantization level.

Number of bits = 11 inches (height) * 8.5 inches (width) * 600 dots / inch (height)
* 600 dots / inch (width) * 3 colors (red, green, blue)
* 8 bits / color = 807,840,000 bits

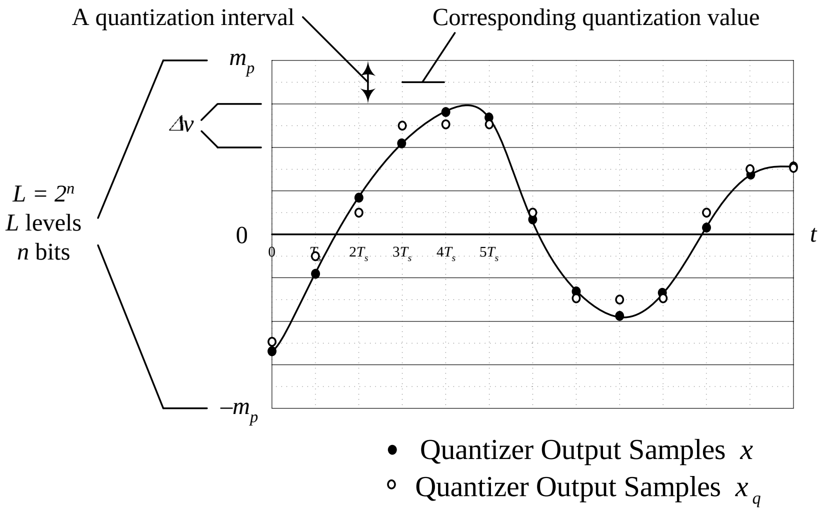
Using a 56 kbps modem would require 807,840,000 / 56,000 = 14426 seconds of transmission time = 4 hours.

For this reason, compression techniques are generally used to store and transmit pictures over slow transmission channels.

Quantization

The process of quantizing a signal is the first part of converting an sequence of analog samples to a PCM code. In quantization, an analog sample with an amplitude that may take value in a specific range is converted to a digital sample with an amplitude that takes one of a specific pre–defined set of quantization values. This is performed by dividing the range of possible values of the analog samples into L different levels, and assigning the center value of each level to any sample that falls in that quantization interval. The problem with this process is that it approximates the value of an analog sample with the nearest of the quantization values. So, for almost all samples, the quantized samples will differ from the original samples by a small amount. This amount is called the quantization error. To get some idea on the effect of this quantization error, quantizing audio signals results in a hissing noise similar to what you would hear when play a random signal.

Assume that a signal with power P_s is to be quantized using a quantizer with $L = 2^n$ levels ranging in voltage from $-m_p$ to m_p as shown in the figure below.

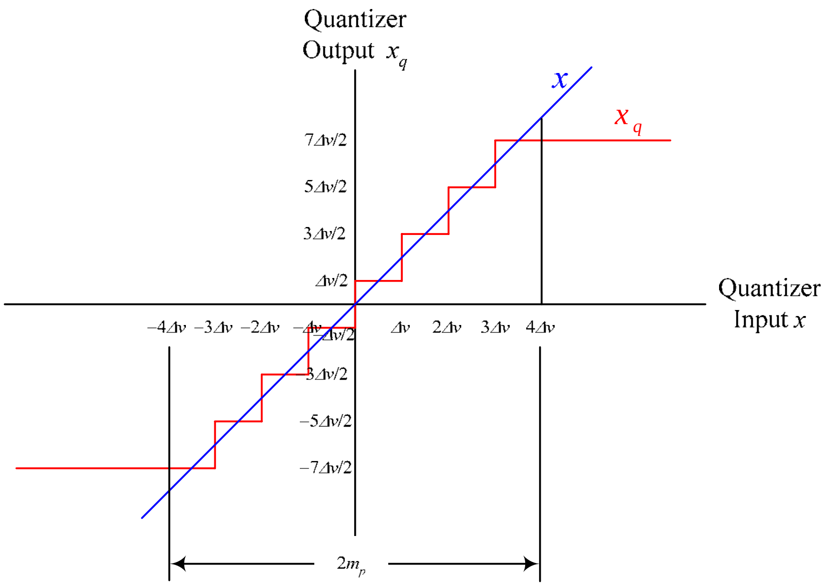


We can define the variable Δv to be the height of the each of the L levels of the quantizer as shown above. This gives a value of Δv equal to

$$\Delta v = \frac{2m_p}{L}.$$

Therefore, for a set of quantizers with the same m_p, the larger the number of levels of a quantizer, the smaller the size of each quantization interval, and for a set of quantizers with the same number of quantization intervals, the larger m_p is the larger the quantization interval length to accommodate all the quantization range.

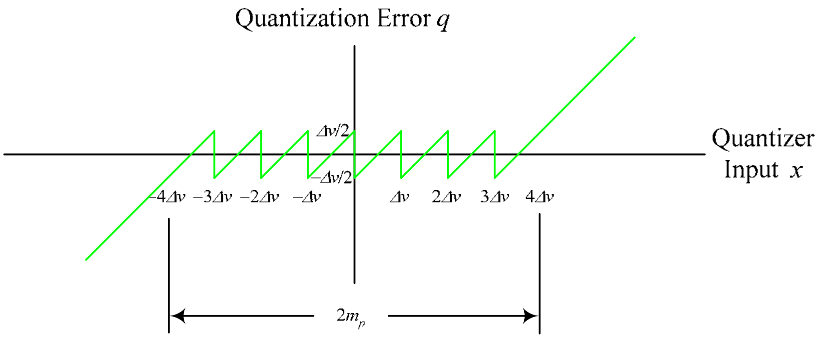
Now if we look at the input output characteristics of the quantizer, it will be similar to the red line in the following figure. Note that as long as the input is within the quantization range of the quantizer, the output of the quantizer represented by the red line follows the input of the quantizer. When the input of the quantizer exceeds the range of $-m_p$ to m_p, the output of the quantizer starts to deviate from the input and the quantization error (difference between an input and the corresponding output sample) increases significantly.



Now let us define the quantization error represented by the difference between the input sample and the corresponding output sample to be q, or

$$q = x - x_q \,.$$

Plotting this quantization error versus the input signal of a quantizer is seen next. Notice that the plot of the quantization error is obtained by taking the difference between the blow and red lines in the above figure.



It is seen from this figure that the quantization error of any sample is restricted between $-\Delta v/2$ and $\Delta v/2$ except when the input signal exceeds the range of quantization of $-m_p$ to m_p.

It can be shown that the power of the quantization noise is equal to

$$P_q = \frac{\left(2m_p / L\right)^2}{12} = \frac{m_p^2}{3L^2},$$

As predicted, the power of the noise decreases as the number of levels L increases, and increases as the edge of the quantization range m_p increases.

Now let us define the Signal to Noise Ratio (SNR) as the ratio of the power of the input signal of the quantizer to the power of the noise introduced by the quantizer (note that the SNR has many other definitions used in communication systems depending on the applications)

$$\boxed{\begin{aligned} SNR &= \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{P_s}{P_q} \\ &= \frac{3L^2}{m_p^2} P_s. \end{aligned}}$$

In general the values of the SNR are either much greater than 1 or much less than 1. A more useful representation of the SNR can be obtained by using logarithmic scale or dB. We know that L of a quantizer is always a power of two or $L = 2^n$. Therefore,

$$\boxed{SNR_{Linear} = \frac{3L^2}{m_p^2} P_s ,}$$

$$\begin{aligned} SNR_{dB} &= 10 \cdot \log_{10}\left(\frac{3L^2}{m_p^2} P_s \right) = 10 \cdot \log_{10}\left(\frac{3}{m_p^2} P_s \right) + 10 \cdot \log_{10}\left(2^{2n}\right) \\ &= \underbrace{10 \cdot \log_{10}\left(\frac{3}{m_p^2} P_s \right)}_{\alpha} + 20n \cdot \log_{10}(2) \\ &= \alpha + 6n \qquad \text{dB.} \end{aligned}$$

Note that α shown in the above representation of the SNR is a constant for a specific signal when different quantizers with the same m_p are used.

It is clear that the SNR of a quantizer in dB increases linearly by 6 dB as we increase the number of bits that the quantizer uses by 1 bit. The cost for increasing the SNR of a quantizer is that more bits are generated and therefore either a higher bandwidth or a longer time period is required to transmit the PCM signal.

Generation of the PCM Signal

Now, once the signal has been quantized by the quantizer, the quantizer converts it to bits (1's and 0's) and outputs these bits. Looking at the figure in the previous lecture, which shown here for convenience. We see that each of the levels of the quantizer is assigned a code from 000…000 for the lowest quantization interval to 111…111 for the highest quantization interval as shown in the column to the left of the figure. The PCM signal is obtained by outputting the bits of the different samples one bit after the other and one sample after the other.