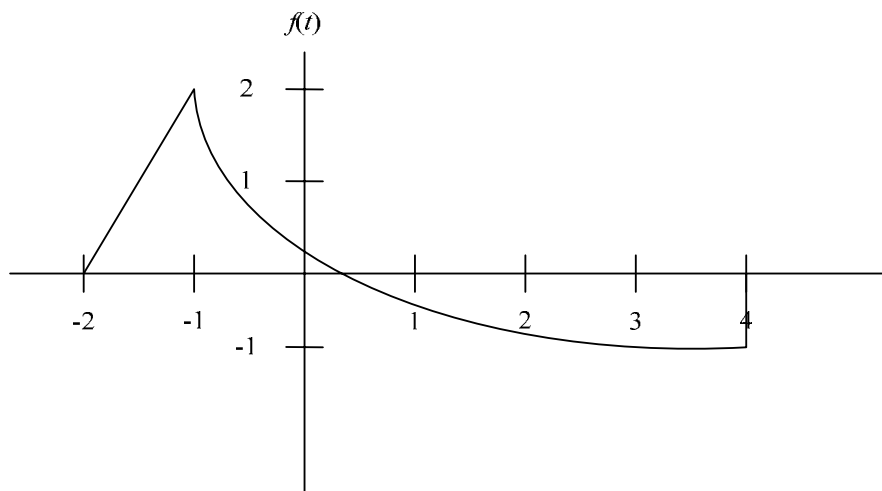


Basic Signal Operations

- A) *Time Shifting*: given the signal $f(t)$, the signal $f(t-t_0)$ is a time-shifted version of $f(t)$ that is shifted to the LEFT if t_0 is positive and to the RIGHT if t_0 is negative.
- B) *Magnitude Shifting*: given the signal $f(t)$, the signal $c + f(t)$ is a magnitude-shifted version of $f(t)$ that is shifted UP if c is positive and shifted DOWN if c is negative.
- C) *Time Scaling and Time Inversion*: given $f(t)$, the signal $f(a \cdot t)$ is a time-scaled version of $f(t)$, where a is a constant, such that $f(a \cdot t)$ is an EXPANDED version of $f(t)$ if $0 < |a| < 1$, and $f(a \cdot t)$ is a COMPRESSED version of $f(t)$ if $|a| > 1$. If a is negative (i.e. $a < 0$), the signal $f(a \cdot t)$ is also a time-inverted version of $f(t)$.
- D) *Magnitude Scaling and Magnitude Inversion*: given $f(t)$, the signal $b \cdot f(t)$ is a magnitude-scaled version of $f(t)$, where b is a constant, such that $b \cdot f(t)$ is an ATENUATED version of $f(t)$ if $0 < |b| < 1$, and $b \cdot f(t)$ is an AMPLIFIED version of $f(t)$ if $|b| > 1$. If b is negative (i.e. $b < 0$), the signal $b \cdot f(t)$ is also a magnitude-flipped version of $f(t)$.

Example 1: given the signal $f(t)$ shown below, sketch the signal $4 - 3f(-2t - 6)$



Example 2: assume the function shown above is $-3g(-t/2+1)$, sketch $g(t)$.

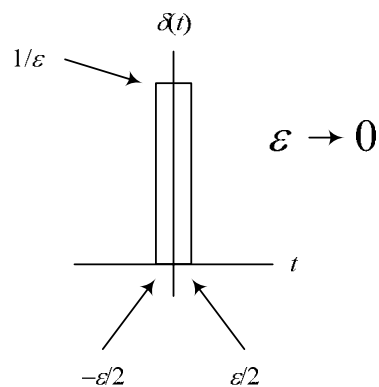
Unit Impulse Function

The unit impulse function is one of the most important functions that we will be using extensively in this course. Yet, it is one of the most difficult functions to understand and use.

A couple of definitions exist for the unit impulse function $\delta(t)$, which is sometimes also called the Dirac delta function. The following are two definitions:

Graphical Definition:

The rectangular pulse shape shown below approaches the unit impulse function as ε approaches 0 (notice that the area under the curve is always equal to 1).



Mathematical Definition:

The unit impulse function $\delta(t)$ satisfies the following conditions:

1. $\delta(t) = 0$ if $t \neq 0$, (therefore it is non-zero only at $t = 0$).
2. $\int_{-\infty}^{\infty} \delta(t) dt = 1$. (so, all of the area under it is concentrated at $t = 0$)

Properties of the Unit Impulse Function:

- a) Multiplication of a function by the unit impulse response:

$$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$$

- b) Sampling of a function using the unit impulse response:

$$\int_{-\infty}^{\infty} f(t)\delta(t-T)dt = \int_{-\infty}^{\infty} f(T)\delta(t-T)dt = f(T) \int_{-\infty}^{\infty} \delta(t-T)dt = f(T)$$

- c) Obtaining the unit step function from the unit impulse function

$$\int_{-\infty}^t \delta(\tau) d\tau = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases} \quad \rightarrow \quad \frac{du(t)}{dt} = \delta(t) \\ = u(t)$$