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CONTROL DESIGN WITH TRANSFER FUNCTIONS ASSOCIATED TO HIGHER ORDER VOLTERRA KERNELS

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Abstract

The Volterra series provides a convolution-oriented method for representing the input/output behavior of a nonlinear system. For the case of constant system parameters, such a representation is naturally suited to control design with transfer functions: zeroth order, first order, second order, and so forth. In 1979, Peczkowski, Sain, and Leake [1] introduced a Total Synthesis Problem (TSP) approach to linear feedback synthesis; and in 1981, Peczkowski and Sain [2] demonstrated how to schedule TSP into a nonlinear controller. For plants with one input and one output, Al-Baiyat and Sain [3] extended TSP to higher order transfer functions for the class of linear analytic systems. In this paper, we complete the extension by treating multiple inputs and multiple outputs. The method is illustrated by designing a control system for a DC to AC converter.

Introduction

Most realistic control systems are nonlinear to some extent. The typical approach to the design of control systems for these nonlinear systems begins with a local linearization in a neighborhood of a nominal operating point. A linear model, valid in that neighborhood, is obtained and then a convenient linear feedback design method is applied. In general, however, nonlinear processes can only be adequately characterized by nonlinear models.

In recent years, considerable progress has been made in representing the behavior of nonlinear systems by Volterra series [4-8]. In the case of multiple input, multiple output, time-invariant systems such a representation has the form

$$y(t) = \int_0^t p_1(\tau_1)u(t-\tau_1)d\tau_1$$

$$+ \int_0^t\int_0^{\tau_1} p_2(\tau_1,\tau_2)u(t-\tau_1) \otimes u(t-\tau_2)d\tau_2 d\tau_1$$

$$+ \dots \quad t \geq \tau_1 \geq \dots \geq \tau_j \geq 0, \tag{1}$$

where p_j, $j = 1,2,\dots$ is called the jth Volterra kernel. Using operator notation equation (1) can also be written as

$$y(t) = P[u](t) = \sum_{j=1}^{\infty} P_j[u](t), \tag{2}$$

in which the homogeneous operator P_j is defined by

$$P_j[u](t) = \int_0^t\int_0^{\tau_1} \dots \int_0^{\tau_{j-1}} p_j(\tau_1,\dots,t_j)u(t-\tau_1)$$

$$\otimes \dots \otimes u(t-\tau_j)d\tau_j \dots d\tau_1. \tag{3}$$

The goal of this paper is to use the Volterra series in the design of nonlinear control systems. The approach adopted in this work is based upon the concept of the Total Synthesis Problem, which addresses the ability to design simultaneously for a specific output response and reasonable control signal. Mathematically, the Total Synthesis Problem can be divided into two sub-problems. The first is that of Nominal Design which depends on plant characteristics only. The second sub-problem is that of Feedback Synthesis which depends on the controller structure.

The Total Synthesis Problem approach to design of nonlinear feedback systems was introduced for a single point of operation by Peczkowski, Sain, and Leake in 1979 [1], and extended later to general operating regions by Peczkowski and Sain [2,9,10,11,12]. This paper contributes an extension to the 1979 TSP work, which was phrased in terms of transfer functions. The TSP definition itself is extended by means of Volterra representation for input-output behavior of a nonlinear system. Using operators based upon Volterra convolutions, TSP is extended initially in the time domain; using transforms of Volterra kernels, TSP is then extended also in the transfer function domain.

Problem Definition

Let R, U, and Y denote the spaces of requests to the system, controls to the plant, and responses from the plant, respectively. With these spaces defined, let

$$P : U \to Y, \tag{4}$$

denote a nonlinear plant. In similar fashion, define the desired plant response to request by the operator

$$T : R \to Y; \tag{5}$$

and finally, define the controls needed to generate such response by the operator

$$M : R \to U. \tag{6}$$

The operators P, T, and M are assumed to have local Volterra representations. For the illustration of these operators see Figure 1. This leads to the first of two sub-problems to be studied.

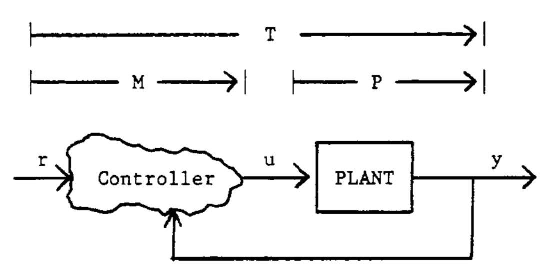

Figure 1.

The Nominal Design Problem (NDP) is to find a pair (M, T), for a given plant P, such that the diagram of Figure 2 commutes, that is

$$T = PM \,. \tag{7}$$

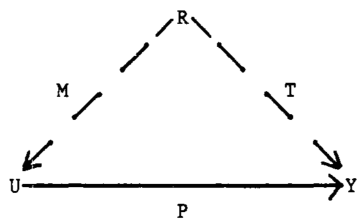

Figure 2.

Consider next the question of synthesizing the pair (M, T) in a specific feedback structure. But before that, let us define the operator

$$E : R \to Z, \tag{8}$$

which represents the mapping of the requests to the errors. Let us also define the operator

$$G : Z \to U, \tag{9}$$

which describes the mapping of errors to the plant controls action. Now we are in a position to state the second sub-problem.

The Feedback Synthesis Problem (FSP) is to find a controller G which synthesizes the pair (M, T) according to the structure of Figure 3.

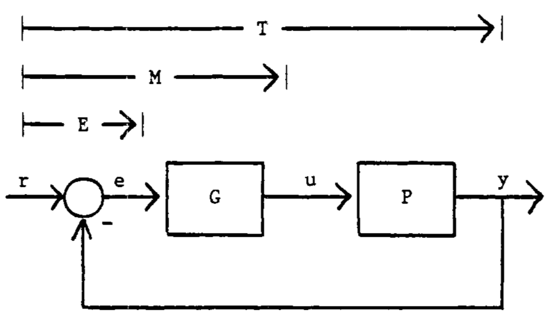

Figure 3.

TSP with Volterra Operators

In this section, TSP will be studied using Volterra operators. As we mentioned in the preceding section, TSP can be divided into two sub-problems, the NDP and the FSP. We will start our study by considering the NDP first. By NDP we mean a pair (M, T) will be found for a given P, where the oeprators, P, M, and T admit Volterra representation. (M, T) can be found by the selection of the homogeneous operators that are associated with the operators M and T from the knowledge of the homogeneous operators associated with the operator P.

Recall that the plant in the input-output sense can be written as, see Figure 1,

$$y = P[u] = \sum_{i=1}^{\infty} P_i[u]. \tag{10}$$

Clearly, it is also possible to write

$$y = T[r] = \sum_{j=1}^{\infty} T_j[r], \tag{11}$$

and

$$u = M[r] = \sum_{k=1}^{\infty} M_k[r]. \tag{12}$$

Note that the time argument in (10-12) has been dropped for reasons of simplicity, and we will continue to do so in the rest of this section. In order to develop a relation between the operators T_j, M_k, and P_i, let us substitute for u, obtained from (12), in (10) and equate the result to (11). That will give

$$\sum_{j=1}^{\infty} T_j[r] = \sum_{i=1}^{\infty} P_i[\sum_{j=1}^{\infty} M_j[r]]. \tag{13}$$

But the Volterra operators are multilinear operators [13-15]. Hence, if the request signal r is replaced by cr in (13), where c is an arbitrary constant, then

$$\sum_{j=1}^{\infty} c^j T_j[r] = \sum_{i=1}^{\infty} P_i[\sum_{j=1}^{\infty} c^j M_j[r]]. \tag{14}$$

In order to make use of relation (14), it is necessary to simplify the expression in the right hand side. A simple computation gives

$$P_i[\sum_{j=1}^{\infty} c^j u_j] = \sum_{j_1=1}^{\infty} \cdots \sum_{j_i=1}^{\infty} c^{j_1+\dots+j_i} \cdot$$

$$P_i(M_{j_1}\times\dots\times M_{j_i})[r], \tag{15}$$

where we defined $u_j = M_j[r]$ and

$$(M_{j_1}\times\dots\times M_{j_i})[r] = (M_{j_1}[r],\dots,M_{j_i}[r]), \tag{16}$$

and the multilinear operator

$$P_i(M_{j_1}[r],\dots,M_{j_i}[r]) = \int_0^t \cdots \int_0^{\tau_{i-1}} p_i(\tau_1,\dots,\tau_i)$$

$$\{M_{j_1}[r](t-\tau_1) \otimes \dots \otimes M_{j_i}[r](t-\tau_i)\}d\tau_i \dots d\tau_1. \tag{17}$$

Substituting (15) in (13) and dropping the argument [r] gives

$$\sum_{j=1}^{\infty} c^j T_j = \sum_{i=1}^{\infty} \sum_{j_1=1}^{\infty} \cdots \sum_{j_i=1}^{\infty} c^{j_1+\dots+j_i} P_i(M_{j_1}\times\dots\times M_{j_i}). \tag{18}$$

Equating like powers of c in both sides of (18) gives

$$T_1 = P_1M_1, \tag{19}$$

$$T_2 = P_1M_2 + P_2(M_1\times M_1), \tag{20}$$

$$T_3 = P_1M_3 + P_2(M_1\times M_2) + P_2(M_2\times M_1) + P_3(M_1\times M_1\times M_1), \tag{21}$$

$$\vdots$$

$$T_i = P_1M_i + \sum_{j=2}^{i} \{ \sum_{k_1=1}^{i-j+1} \sum_{k_2=1}^{i-j+2-k_1} \cdots \sum_{k_{j-1}=1}^{i-k_1-k_2-\dots-k_{j-2}-1}$$

$$P_j(M_{k_1}\times M_{k_2}\times\dots\times M_{i-k_1-k_2-\dots-k_{j-1}})\}. \tag{22}$$

The preceding equations (19-22) are convolutional type equations, so it is extremely difficult to design T and M without going to the transform domain. But before we go to the transform domain, we will solve the FSP in the time domain.

Consider next the second sub-problem of the TSP, that is, the Feedback Synthesis Problem. In this problem, FSP, a controller G will be found which realizes the pair (M, T) according to the structure of Figure 3. Again, that can be done by finding the components of the operator G from the knowledge of the components of the operators M and T. To begin this process, let us find a relation between the Volterra operators G, M, and E. Clearly from Figure 3, this relation is given by

$$M = GE. \tag{23}$$

But equation (23) is strikingly similar to the Nominal Design equation (7). Hence we can write, by inspection, the relations between the components of the operators M, G, and E. The first three of these relations are

$$M_1 = G_1E_1, \tag{24}$$

$$M_2 = G_1E_2 + G_2(E_1 \times E_1), \tag{25}$$

$$M_3 = G_1E_3 + G_2(E_1 \times E_2) + G_2(E_2 \times E_1) + G_3(E_1 \times E_1 \times E_1), \tag{26}$$

and, in general we have

$$M_i = G_1E_i + \sum_{j=2}^{i} \{ \sum_{k_1=1}^{i-j+1} \sum_{k_2=1}^{i-j+2-k_1} \cdots \sum_{k_{j-1}=1}^{i-k_1-k_2-\cdots-k_{j-2}-1} G_j(E_{k_1} \times E_{k_2} \times \cdots \times E_{i-k_1-k_2-\cdots-k_{j-1}})\}. \tag{27}$$

Next, let us find a relation between the operators E and T. Again from Figure 3 we have

$$e = r - y. \tag{28}$$

Now we can write

$$e = E[r] = \sum_{i=1}^{\infty} E_i[r], \tag{29}$$

and

$$y = T[r] = \sum_{i=1}^{\infty} T_i[r]. \tag{30}$$

So equation (28) can be written as

$$\sum_{i=1}^{\infty} E_i[r] = I[r] - \sum_{i=1}^{\infty} T_i[r] \tag{31}$$

where I is the identity operator. Again, replacing r by cr in (31) and equating like powers of c gives

$$E_1 = I - T_1, \tag{32}$$

$$E_2 = -T_2, \tag{33}$$

$$E_3 = -T_3, \tag{34}$$

and in general, for $i > 1$,

$$E_i = -T_i. \tag{35}$$

Clearly, from equations (24-27) and from (32-35) we can find a direct relation between the components of the operators T, M, and G.

Nonlinear Total Synthesis: The Transform Domain

In the preceding section, the TSP was formulated using Volterra operators. In order to make use of this formulation we will work in the transform domain, where the design method will be greatly simplified. In this section the TSP will therefore be studied in the transform domain. To do this, let us define the multidimensional Laplace transform of the ith order Volterra kernel as [16,17]

$$P_i(s_1,\ldots,s_i) = \int_0^{\infty} \cdots \int_0^{\infty} p_i(\tau_1,\ldots,\tau_i) \exp\{-(s_1\tau_1+\ldots+s_i\tau_i)\}d\tau_1 \ldots d\tau_i. \tag{36}$$

Recall that the first step in the TSP is the process of selecting a pair (M, T) for a given P, such that the diagram of Figure 2 commutes. The pair (M, T) can be found by determining the kernels that are associated with the operators M and T from knowledge of the kernels associated with the operator P. In the preceding section we have found relations between T_i, M_i, and P_i, the homogeneous operators associated with T, M, and P. These relations, (19-22), are convolutional type equations. By applying definition (36), and after some algebraic manipulation, these equations, (19-22), are converted into the following multiplicative equations

$$T_1(s) = P_1(s)M_1(s), \tag{37}$$

$$T_2(s_1,s_2) = P_1(s_1+s_2)M_2(s_1,s_2) + P_2(s_1,s_2)(M_1(s_1) \otimes M_1(s_2)), \tag{38}$$

$$T_3(s_1,s_2,s_3) = P_1(s_1+s_2+s_3)M_3(s_1,s_2,s_3) + P_2(s_1,s_2+s_3)(M_1(s_1) \otimes M_2(s_2,s_3)) + P_2(s_1+s_2,s_3)(M_2(s_1,s_2) \otimes M_1(s_3)) + P_3(s_1,s_2,s_3)(M_1(s_1) \otimes M_1(s_2) \otimes M_1(s_3)), \tag{39}$$

and in general

$$T_i(s_1,\ldots,s_i) = P_1(s_1+\ldots+s_i)M_i(s_1,\ldots,s_i) + \sum_{j=2}^{i} \{ \sum_{k_1=1}^{i-j+1} \sum_{k_2=1}^{i-j+2-k_1} \cdots \sum_{k_{j-1}=1}^{i-k_1-\cdots-k_{j-2}-1} \cdot P_j(s_1+\ldots+s_{k_1}, s_{k_1+1}+\ldots+s_{k_1+k_2}, \ldots, s_{k_1+\ldots+k_{j-1}+1}+\ldots s_i) \cdot (M_{k_1}(s_1,\ldots,s_{k_1}) \otimes M_{k_2}(s_{k_1+1},\ldots,s_{k_1+k_2}) \otimes \cdots \otimes M_{i-k_1-\ldots-k_{j-1}}(s_{k_1+k_2+\ldots+k_{j-1}+1},\ldots,s_i))\}. \tag{40}$$

Recall that our goal is to design the kernels of T and M from the knowledge of P. This task becomes very simple now and can be performed recursively starting with equation (37). But equation (37) is the designing equation of the linear NDP, which is studied by Peczkowski, et. al., [1]. So clearly we can select the pair $(T_1(s), M_1(s))$ for a given $P_1(s)$. Having selected $T_1(s)$ and $M_1(s)$ we can proceed to (38) where now the only unknowns are $T_2(s_1,s_2)$ and $M_2(s_1,s_2)$. To clarify this point let us rewrite (38) as

$$T_2(s_1,s_2) - P_1(s_1+s_2)M_2(s_1,s_2) = A_2(s_1,s_2), \tag{41}$$

where $A_2(s_1,s_2)$ is a known quantity given by

$$A_2(s_1,s_2) = P_2(s_1,s_2)(M_1(s) \otimes M_1(s_2)). \tag{42}$$

So again, from (42) we can select the pair $(T_2(s_1,s_2), M_2(s_1,s_2))$ and we can proceed to the next step. In general we will have

$$T_i(s_1,\ldots,s_i) - P_1(s_1+\ldots+s_i)M_i(s_1,\ldots,s_i) = A_i(s_1,\ldots,s_i), \tag{43}$$

where $A_i(s_1,\ldots,s_i)$ is a known quantity which depends on terms involving M_L and P_{L+1} for $L \leq (i-1)$. Hence from (43) we can select the general pair $(T_i(s_1,\ldots,s_i), M_i(s_1,\ldots,s_i))$.

To complete the design, we will look into the second sub-problem of the TSP which is the Feedback Synthesis Problem in the transform domain. Recall that FSP deals with the problem of finding a controller G which synthesizes the pair (T, M) according to the structure of Figure 3. Here, the goal is to find the kernels of G in the transform domain. This can be done by taking the transform of the convolutional relations (24-27) and (32-35). To begin this process let us look into equations (24-27), since those equations have the same form as the NDP equations (19-22). In the transform domain, equations (24-27) become

$$M_1(s) = G_1(s)E_1(s) \tag{44}$$

$$M_2(s_1,s_2) = G_1(s_1+s_2)E_1(s_1,s_2) + G_2(s_1,s_2)(E_1(s) \otimes E_1(s_2)) \tag{45}$$

$$\begin{aligned} M_3(s_1,s_2,s_3) &= G_1(s_1+s_2+s_3)E_3(s_1,s_2,s_3) \\ &+ G_2(s_1,s_2+s_3)(E_1(s_1) \otimes E_2(s_2,s_3)) \\ &+ G_2(s_1+s_2,s_3)(E_2(s_1,s_2) \otimes E_1(s_3)) \\ &+ G_3(s_1,s_2,s_3)(E_1(s_1) \otimes E_1(s_2) \otimes E_1(s_3)), \end{aligned} \tag{46}$$

and, in general we have

$$\begin{aligned} M_i(s_1,\dots,s_i) &= G_1(s_1+\dots+s_i)E_i(s_1,\dots,s_i) \\ &+ \sum_{j=2}^{i} \{ \sum_{k_1=1}^{i-j+1} \sum_{k_2=1}^{i-j+2-k_1} \cdots \sum_{k_{j-1}=1}^{i-k_1-\dots-k_{j-2}-1} \cdot \\ & G_j(s_1+\dots+s_{k_1}, s_{k_1+1}+\dots+s_{k_1+k_2}, s_{k_1+\dots+k_{j-1}+1}+\dots+s_i) \cdot \\ & (E_{k_1}(s_1,\dots,s_{k_1}) \otimes E_{k_2}(s_{k_1+1},\dots,s_{k_1+k_2}) \otimes \dots \otimes \\ & E_{i-k_1-\dots-k_{j-1}}(s_{k_1+\dots+k_{j-1}+1},\dots,s_i))\}, \end{aligned} \tag{47}$$

where

$$E_1(s) = I - T_1(s) \tag{48}$$

and

$$E_i(s_1,\dots,s_i) = -T_i(s_1,\dots,s_i), \qquad i > 1. \tag{49}$$

Clearly, from equations (44-47) and from (48-49), we find a direct relation between the kernels of the Volterra operators T, M, and G.

Feedback Linearization

An interesting special case of the feedback design we proposed in the preceding section is the following. Suppose we want to design a feedback system for a given nonlinear system such that

$$T_i(s_1,\dots,s_i) = 0 \tag{50}$$

for $i > 1$. This means we want to use the feedback design to linearize the system. Of course, this is an approximation linearization. In recent years feedback linearization, exact linearization, represents one of the most active areas of research in control systems [18-20]. Most of the work in this area follows the differential geometric approach.

Here, our goal is to design a controller G, that is to find the kernels of G, which achieve feedback linearization. This means we want the higher order kernels of T to be zero, kernels beyond the first order. But from (49) this means that

$$E_i(s_1,\dots,s_i) = 0, \qquad i > 1. \tag{51}$$

From this, and assuming $E_1(s)$ and $P_1(s)$ to be invertibles, it can be shown [21] that the kernels of the controller G are given by

$$G_1(s) = P_1^{-1}(s)T_1(s)(I-T_1(s))^{-1}, \tag{52}$$

$$G_2(s_1,s_2) = -P_1^{-1}(s_1+s_2)P_2(s_1,s_2)(G_1(s_1) \otimes G_1(s_2)), \tag{53}$$

$$\begin{aligned} G_3(s_1,s_2,s_3) = -P_1^{-1}(s_1+s_2+s_3)\{&P_2(s_1,s_2+s_3)(G_1(s_1) \otimes G_2(s_2,s_3)) \\ &+ P_2(s_1+s_2,s_3)(G_2(s_1,s_2) \otimes G_1(s_3)) \\ &+ P_3(s_1,s_2,s_3)(G_1(s_1) \otimes G_1(s_2) \otimes G_1(s_3))\}, \end{aligned} \tag{54}$$

and in general:

$$\begin{aligned} G_i(s_1,\dots,s_i) &= -P_1^{-1}(s_1+\dots+s_i) \\ &+ \sum_{j=2}^{i} \{ \sum_{k_1=1}^{i-j+1} \sum_{k_2=1}^{i-j+2-k_1} \cdots \sum_{k_{j-1}=1}^{i-k_1-\dots-k_{j-2}-1} \cdot \\ & P_j(s_1+\dots+s_{k_1}, s_{k_1+1}+\dots+s_{k_1+k_2}, s_{k_1+\dots+k_{j-1}+1}+\dots+s_i) \cdot \\ & (G_{k_1}(s_1,\dots,s_{k_1}) \otimes G_{k_2}(s_{k_1+1},\dots,s_{k_1+k_2}) \otimes \dots \otimes \\ & G_{i-k_1-\dots-k_{j-1}}(s_{k_1+\dots+k_{j-1}+1},\dots,s_i))\}, \quad i > 1. \end{aligned} \tag{55}$$

Example

To illustrate the foregoing ideas, consider the problem of designing a nonlinear feedback control system for a simplified model of a DC to AC converter as shown in Figure 4.

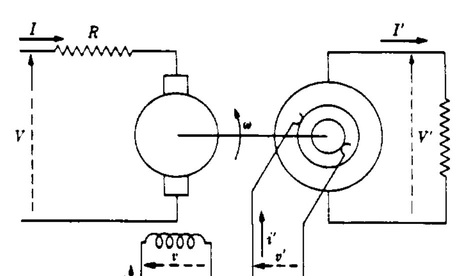

Figure 4.

We will specifically study the influence of the inputs ν' and V on the outputs ω and V', in the neighborhood of an operating point, while holding ν as a constant. The differential equation model for the DC to AC converter is given by [22]

$$\frac{d\omega}{dt} = \frac{1}{J}\left\{k\frac{\nu}{r}\frac{1}{R}\left[V-k\frac{\nu}{r}\omega\right] - \frac{V'}{\omega}\frac{V'}{R_\ell}\right\} \tag{56}$$

$$\frac{dV'}{dt} = \frac{V'}{\omega}\frac{d\omega}{dt} - \frac{r'V'}{\ell'} + \frac{k'}{\ell'}\omega\nu' \tag{57}$$

where

J = moment of inertia

k = emf constant

k' = torque constant

r = field resistance of the motor

r' = field resistance of the generator

ℓ' = field inductance of the generator .

To simplify our equations let us assume that $k = k' = 1$, $r = r' = 1$, $R = .2$, $R_\ell = 1$, $\ell' = .5$, and $\nu = 1$. So equations (56) and (57) become

$$\frac{d\omega}{dt} = 5[V-\omega] - \frac{(V')^2}{\omega}, \tag{58}$$

$$\frac{dV'}{dt} = \frac{V'}{\omega}\frac{d\omega}{dt} - 2V' + 2\omega\nu'. \tag{59}$$

Suppose we make the following shift of coordinate

$$\frac{d(\omega-\bar{\omega})}{dt} = 5[(V-\bar{V}) + \bar{V} - (\omega-\bar{\omega}) - \bar{\omega}] - \frac{[V'-\bar{V}'+\bar{V}']^2}{\omega-\bar{\omega}+\bar{\omega}}, \tag{60}$$

$$\frac{d(V'-\bar{V}')}{dt} = \frac{(V'-\bar{V}')+\bar{V}'}{\omega-\bar{\omega}+\bar{\omega}}\frac{d(\omega-\bar{\omega})}{dt} + 2[((\omega-\bar{\omega}) + \bar{\omega})((\nu'-\bar{\nu}') + \bar{\nu}') - (V'-\bar{V}') - \bar{V}'], \tag{61}$$

where $\bar{\omega}$, $\bar{V}$, $\bar{V}'$ and $\bar{\nu}'$ are steady state values of ω, V, V' and ν' respectively. Let us now define

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \omega-\bar{\omega} \\ V'-\bar{V}' \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} V-\bar{V} \\ \nu'-\bar{\nu}' \end{bmatrix}.$$

Equations (60) and (61) become

$$\dot{x}_1 = -5(x_1+\bar{\omega}) - \frac{[x_2+\bar{V}']^2}{x_1+\bar{\omega}} + 5u_1 + 5\bar{V}, \tag{62.a}$$

$$\dot{x}_2 = \frac{x_2+\bar{V}'}{x_1+\bar{\omega}}\dot{x}_1 + 2x_1\bar{\nu}' + 2\bar{\omega}\bar{\nu}' - 2x_2 - 2\bar{V}' + 2\bar{\omega}u_2 + 2x_1u_2. \tag{62.b}$$

The system described by equations (62.a) and (62.b) is called a linear analytic system with the output given by

$$y_1 = x_1 \tag{62.c}$$

$$y_2 = x_2. \tag{62.d}$$

Suppose now we want to design a nonlinear feedback control system for the DC to AC converter such that

$$T_1(s) = \begin{bmatrix} \frac{1}{(\alpha_1 s+1)(\alpha_2 s+1)} & 0 \\ 0 & \frac{1}{(\alpha_3 s+1)(\alpha_4 s+1)} \end{bmatrix}, \tag{63}$$

and

$$T_2(s_1,s_2) = [0]. \tag{64}$$

The first step in the design method is to calculate the kernels of the plant in the transform domain. This can be done for this particular system, (62), by using the works in [4,15,21]. Having done that, we can now find the first and the second order controllers by using equations (51) and (52). Due to the space limitation we have not shown the calculation of the plant kernels and the controllers. The interested reader can find those calculations in [21], or they are available from the authors on request.

In order to check our design a computer simulation for the closed loop system has been done. Figure 5 shows responses for request changes of .1 in the voltage and .1 in the speed for the operating condition of 1.3, and .3, voltage and speed respectively; and the desired time constants were $\alpha_1 = \alpha_3 = .001$, and $\alpha_2 = \alpha_4 = .005$. In this figure, 5, a comparison between our design and a linear design, obtained by linearizing the plant first and then using linear TSP, to the desired response shows that our design has a little improvement over the linear design. However, by increasing our request change in the voltage to .2 and the speed to .2, for the same operating condition in the last step we find a much better performance from our design than the linear design, see Figure 6. The improvement is not so much in the speed, Figure 6.a, but in the voltage, where the linear design shows an effect similar to that of a right-half plane zero.

Conclusion

This paper has presented a procedure for designing nonlinear feedback controls for nonlinear systems. By developing this procedure we have achieved our main goal which, as we pointed out, is to utilize the Volterra series in the design problem. The design technique was formulated in terms of kernels of the Volterra series in the transform domain. The kernels play the same role as the transfer function in the design of linear systems. A pleasant feature of our designing method is that it is relatively easy to understand; moreover, it is a systematic approach. Hence, it can be programmed easily. We have demonstrated the method in the example of the preceding section. This example has illustrated the potential advantage of designing higher order controllers for nonlinear systems. This work may be regarded as an extension of the paper by Peczkowski, Sain, and Leake [1] in 1979.

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