Resource Allocation Scheme for MIMO-OFDMA Systems with Proportional Fairness Constraints

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Introduction

Resource Allocation Problem:

- Scheduling available resources amongst users efficiently.
- Challenging for wireless systems that are power, bandwidth, and complexity limited.
- Scheme assigns the subcarriers to the users and the power to the subcarriers based on the instantaneous CSI - maximize overall systems throughput.
- We consider a multi-user downlink MIMO-OFDMA system with the objective of dynamic resource allocation across the channels which are created in space and frequency.

Literature Survey

- Algorithms generally found in literature can be broadly classified as:
  - Margin Adaptive.
  - Rate Adaptive.
- The optimization problem in margin adaptive allocation schemes is formulated with the objective of minimizing the total transmit power.
- The rate adaptive schemes objective is to maximize the systems total data rate with constraint on the total transmit power.

Rate adaptive resource allocation schemes for OFDMA systems found in literature:

- **W. Rhee and J.M. Cioffi** [1], proposed a scheme that allocates subcarriers with priority to users having min. rate assuring all users equal datarates.
- The scheme assumes flat power distr., FSC are not utilized fully although it was able to achieve acceptable fairness among users.

Literature Survey

- **Z. Shen et. Al** [2], modified the above scheme by giving priority to users having least Prop' datarate- introducing concept of prop. Constraints for user datarates.
- User power optimization problem was also formulated using Lagrange multipliers method, & assumptions were made to linearize the problem.
- The power obtained by each user was distributed over assigned subcarriers – water filling scheme.
- **C. Mohanram, and S. Bhashyam** [3], proposed a scheme that performs subcarrier and power allocation simultaneously.
- With each subcarrier a preset share of transmit power is also allocated to respective user.
- The power obtained by each user was distributed over assigned subcarriers – water filling scheme.
- The scheme achieves high datarates but compromises with fairness of users in extreme conditions.

Literature Survey

For multiuser MIMO-OFDM system there are very less algorithms proposed that enhance proportional fairness among users.

- **P. Uthansakul and M.E Bialkowski** in [4], proposed a novel scheme where the scheduler adaptively assigns the 3-dimensional slots i.e. space, frequency and time, among users depending on the instantaneous channel state information (CSI).

- **Lo et al.** [5], formulated the resource allocation problem as a cross-layer optimization framework, the system was investigated with and without the need for fairness among users, where fairness was modeled as maximum number of allowable channel assignments per user.

- Moreover, the optimal water level for power distribution was obtained with the help of bisection method.

Optimization Problem

- Problem formulation for resource allocation in MIMO-OFDMA system is similar to that of OFDMA but is more challenging due to multiple antennas.

- We consider the downlink of a multi user MIMO-OFDMA system with one BS and *K* geographically dispersed mobile users, where the BS is equipped with M_t transmit antennas, and the kth user is equipped with M_r receive antennas.

- As signals in a scattering environment appears to be uncorrelated, it is assumed that elements of MIMO channel are i.i.d complex Gaussian random variable with zero-mean and unit variance.

- In a MIMO-OFDMA channel various users have varying channel conditions with respect to the base station, exhibiting frequency selective nature over subcarriers.

Optimization Problem

Assign sub carrier allocations in order to maximize the systems overall capacity:

$$\max_{p_{k,n_s}} \sum_{k=1}^{K} \sum_{n_s \in \Omega_k} \frac{1}{N} \log_2 \left(\det \left(I_{M_R} + \frac{p_{k,n_s}}{M_t N_0} \bar{\boldsymbol{H}}_{k,n_s} \bar{\boldsymbol{H}}_{k,n_s}^H \right) \right) \quad (1)$$

- While satisfying the total power constraint

$$\sum_{k=1}^{K} \sum_{n_s \in \Omega_k} p_{k,n_s} \leq P_{total} \text{ and } p_{k,n_s} \geq 0 \quad (2)$$

- Sub carrier allocations made for different users must be mutually exclusive and disjoint.

- Also satisfy proportional datarate constraints,

$$R_1 / \gamma_1 = R_2 / \gamma_2 \ldots.. = R_K / \gamma_K;$$

R_K is K^{th} user datarate

given by $R_k = \frac{1}{N} \log_2 \left\{ det \left(I_{M_r} + \frac{p_{k,n}}{M_t N_0} \bar{\boldsymbol{H}}_{k,n_s} \bar{\boldsymbol{H}}_{k,n_s}^H \right) \right\}$ (3)

Mathematical Analysis

- Using Lagrange multipliers technique with corresponding constraints we can solve the optimization problem by formulating a cost function, which yields the following fn. upon diff. and equating it to zero.

$$\frac{1}{\gamma_1}\frac{T_1}{N}\left(\log_2\left(1+\ \lambda_{1,1}\ \frac{P_{1,total}\ -A_1}{N_1}\right)+\log_2 B_1\right) = \frac{1}{\gamma_k}\frac{T_k}{N}\left(\log_2\left(1+\ \lambda_{k,1}\frac{P_{k,total}\ -A_k}{N_k}\right)+\log_2 B_k\right) \quad (5)$$

$$\mathrm{A_k} = \sum_{n=2}^{T_k}\frac{\lambda_{k,n}-\lambda_{k,1}}{\lambda_{k,n}\,\lambda_{k,1}},\ \ B_k = \left(\prod_{n=2}^{T_k}\frac{\lambda_{k,n}}{\lambda_{k,1}}\right)^{\frac{1}{T_k}} \ \text{and}\ \ P_{k,total} = \sum_{n=1}^{T_k} p_{k.n} \quad (6)$$

- The cost function assumes that $\lambda_{k,1} \leq \lambda_{k,2} \leq \ldots \leq \lambda_{k,T_k}$

  i.e The no. of elements in the set Ω_k is equal to T_k and $A_k \geq 0$.

Mathematical Analysis

- To solve the (K-1) nonlinear equations of the power optimization problem, obtained from eqn (5) by defining a new parameter X_k,

$$X_k = \frac{\left[(X_j B_j)^{\gamma_k T_j / \gamma_j T_k} \right]}{B_k}, \forall j, k \in \{1,2, \dots . K\} \quad (10)$$

- To solve for X_j we use the above equation and invoke the total power constraint defined in eqn (2), deriving

$$\sum_{k=1}^{K} \left\{ A_k + \frac{T_k}{\lambda_{k,1}(i)} . \left[\frac{[(X_j B_j)^{\gamma_k T_j / \gamma_j T_k}]}{B_k} - 1 \right] \right\} - P_{total} = 0 \quad (11)$$

- Solution for eqn(10) exists for K^{th} user, its sub-channel allocations, Ω_k, should be such that the corresponding A_k satisfies $\sum_{k=1}^{K} A_k \leq P_{total}$ .....(12)

  => X_k should be >1 always.

- Therefore, power allocation scheme should drop weak channels to ensure the existence of a valid solution for eqn (11), by satisfying the requirement in eqn(12) at first.

Proposed Scheme

- As MIMO channel matrix is resolved into independent parallel channels, thus for a given user its datarate is given by

$$R_k = \sum_{n=1}^{T} \frac{\rho_{k,n}}{N} . \log_2 \left(1 + \frac{\lambda_{k,n}\, p_{k,n}}{N_0} \right) \quad (13)$$

Subcarrier Allocation:

Initialization:

- ✓ $R_k = 0$, $\Omega_k = \emptyset$, for all k= 1,2,….K
- ✓ Set S = {1,2….T}.

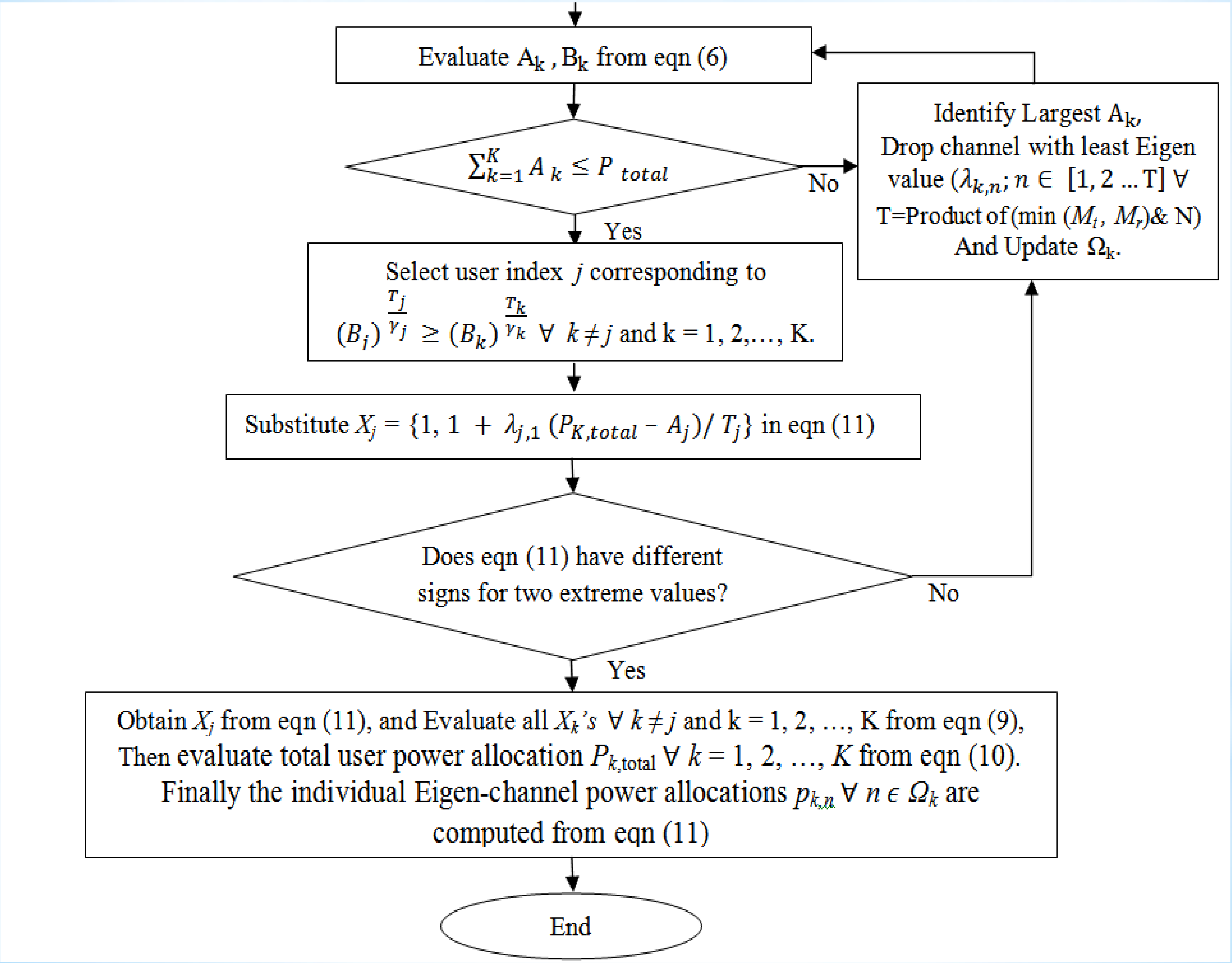
for k=1 to K (subchannels to users in round robin fashion)

- ✓ Find n satisfying $\lambda_{k,n} \geq \lambda_{k,v}$ for all v ϵ S.
- ✓ Let $\Omega_k = \Omega_k \cup \{n\}$, $S = S - \{n\}$, update Rk based on eqn (13).

While $S \neq \emptyset$ (priority to users with least proportional datarate)

- ✓ Find k such that it satisfies $R_k / \gamma_k \leq R_w / \gamma_w \;\; \forall\, 1 \leq w \leq K$.
- ✓ After computing k, find n satisfying $\lambda_{k,n} \geq \lambda_{k,v}$ $\forall$ v ϵ S.
- ✓ After computing n and K, Let $\Omega_k = \Omega_k \cup \{n\}$,$S = S - \{n\}$, update R_k based on eqn (13).

Power Allocation for Proposed Scheme



Simulation Results

- To evaluate performance of the proposed schemes we compare it against an algorithm where the subcarrier allocations are done in manner similar to proposed scheme and transmit power is distributed equally across all the Eigen-channels – referring it as Flat scheme.

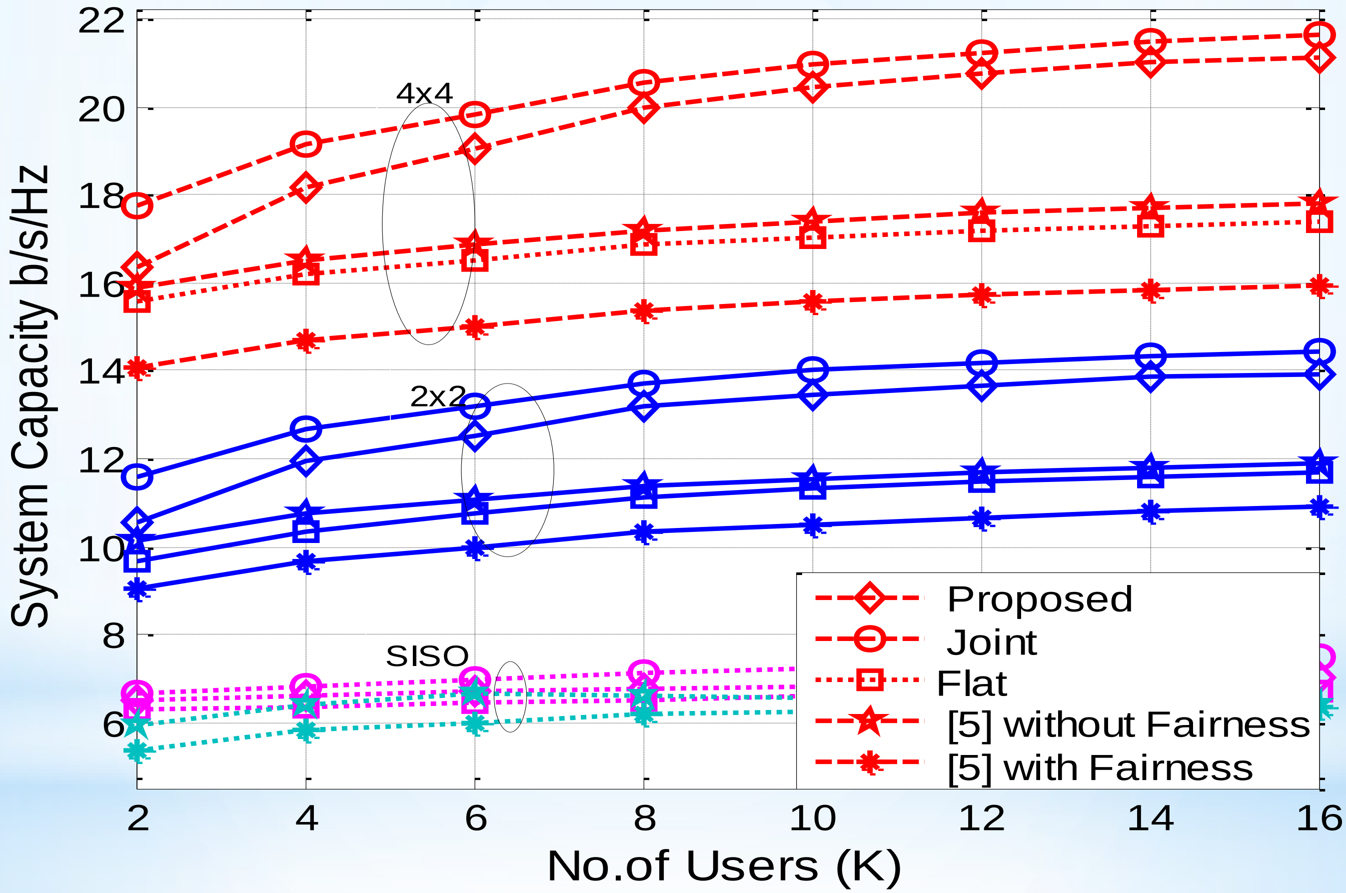
- We also compare the performance of the proposed scheme against a joint resource allocation scheme (Joint) where Eigen-channels and available power is distributed among users simultaneously as in [3] and a bisection based resource allocation scheme [5].

- Simulation Results are obtained for following set of parameters

Total Power	1Watt
Noise PSD	- 80 dBW/Hz
Number of Subcarriers	64
System Bandwidth	1MHz
Number of Users	Varying from 2-16.

Simulation Results

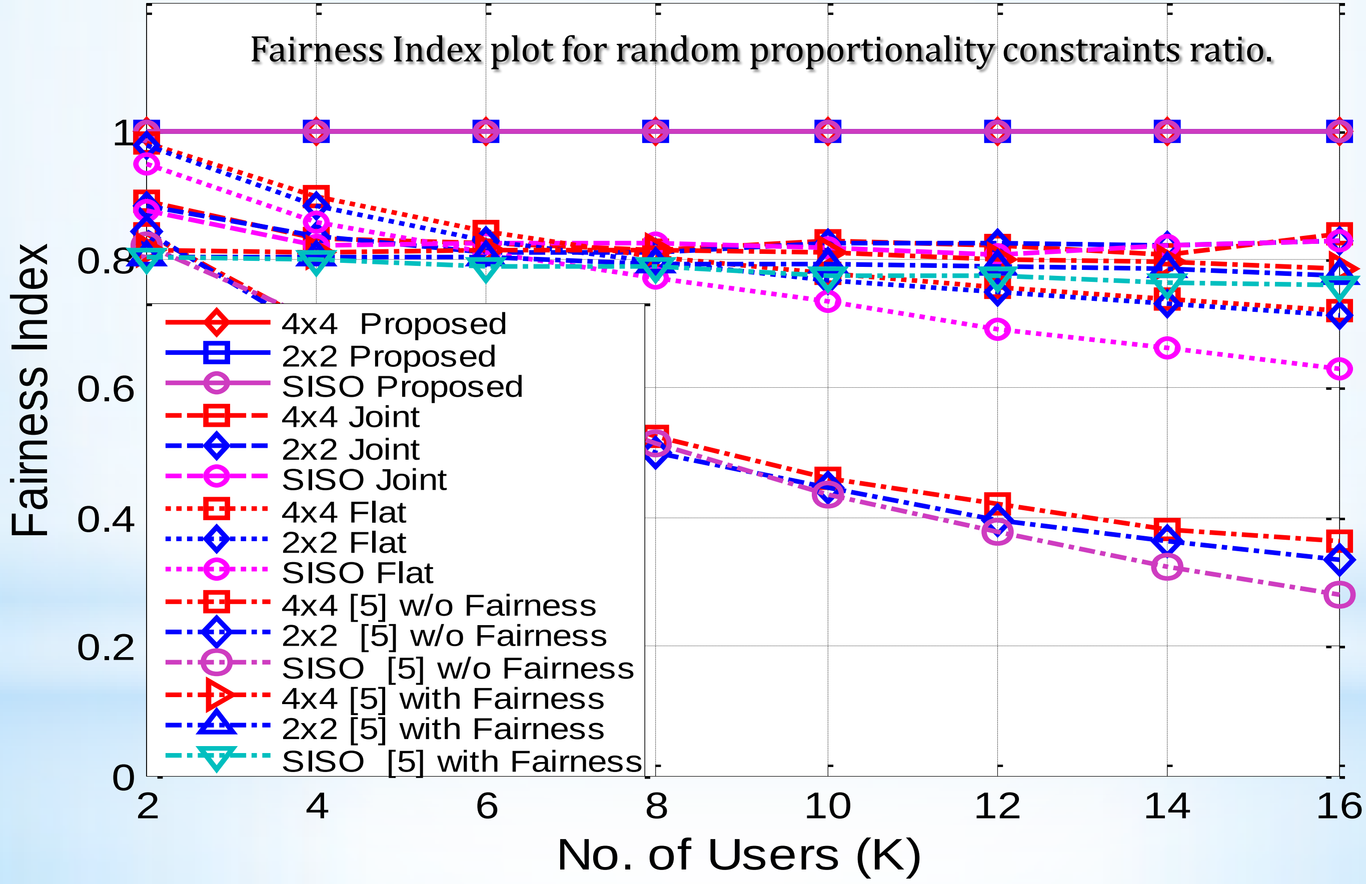
- System capacity for random proportionality constraints ratio.



Simulation Results

- To evaluate the systems performance Jain's Fairness index is used, defined as

$$\text{Fairness Index} = \frac{\left[\sum_{k=1}^{K} \Gamma_k\right]^2}{K\left[\sum_{k=1}^{K} \Gamma_k^2\right]} ; \quad \forall\, \Gamma_k = \frac{R_k}{\gamma_k}$$



Conclusion

- Proposed algorithm performs Eigen-channel allocation and optimal power allocation to maximize the overall system capacity, whilst achieving strict fairness levels among active users of the system.

- A comparison of simulation results of these schemes show that the proposed scheme has best performance in terms of fairness, although it negotiates to some extent with system total capacity.

- Similarly, the comparison of the existing schemes with the proposed scheme reveals that our power allocation routine can provide much better capacity gain while ensuring strict level of fairness among users.

References

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Mathematical Analysis

- To solve the power optimization problem in MIMO-OFDMA system we make use of Lagrange multipliers technique. Using this technique we can formulate a cost function, as follows.

$$L = \sum_{k=1}^{K} \sum_{n_s \in \Omega_k} \frac{1}{N} log_2 \left\{ \det \left(I_{M_r} + \frac{p_{k,n_s}}{M_t} \overline{\boldsymbol{H}}_{k,n_s} \overline{\boldsymbol{H}}_{k,n_s}^{H} \right) \right\} + \alpha_1 \left(\sum_{k=1}^{K} \sum_{n_s \in \Omega_k} p_{k,n_s} - P_{total} \right)$$
$$+ \sum_{k=2}^{K} \alpha_k \left(\sum_{n_s \in \Omega_k} \frac{1}{N} log_2 \det \left(I_{M_r} + \frac{p_{k,n_s}}{M_t} \overline{\boldsymbol{H}}_{k,n_s} \overline{\boldsymbol{H}}_{k,n_s}^{H} \right) - \frac{\gamma_1}{\gamma_k} \sum_{n_s \in \Omega_k} \frac{1}{N} log_2 \det \left(I_{M_r} + \frac{p_{k,n_s}}{M_t} \overline{\boldsymbol{H}}_{k,n_s} \overline{\boldsymbol{H}}_{k,n_s}^{H} \right) \right)$$

- MIMO channel matrix can be transformed into non-interfering parallel SISO channels through singular value decomposition of the channel matrix.

$$L = \sum_{k=1}^{K} \sum_{n=1}^{T} \frac{1}{N} \log_2(1 + \lambda_{k,n} p_{k,n}) + \alpha_1 \left(\sum_{k=1}^{K} \sum_{n=1}^{T} p_{k,n} - P_{total} \right)$$
$$+ \sum_{k=2}^{K} \alpha_k \left(\sum_{n=1}^{T} \frac{1}{N} \log_2(1 + \lambda_{k,n}\, p_{k,n}) - \frac{\gamma_1}{\gamma_k} \sum_{n=1}^{T} \frac{1}{N} \log_2(1 + \lambda_{k,n}\, p_{k,n}) \right)$$