Proakis Chapter 3 and 4
PASSBAND DATA TRANSMISSION

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EE571

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Content

- *Different methods of digital modulation, namely, phase-shift keying, quadrature- amplitude modulation, and frequency-shift keying, and their individual variants.*
- *Coherent detection of modulated signals in additive white Gaussian noise, which requires the receiver to be synchronized to the transmitter with respect to both carrier phase and bit timing.*
- *Noncoherent detection of modulated signals in additive white Gaussian noise, disregarding phase information in the received signal*

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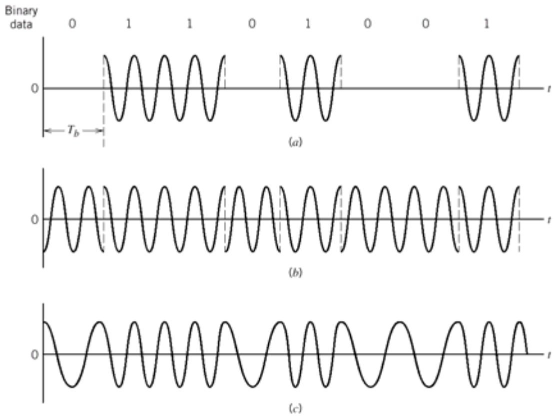
6.1 Introduction



- (*a*) Amplitude-shift keying.
- (*b*) Phase-shift keying.
- (*c*) Frequency-shift keying

Performance Metrics

- PROBABILITY OF ERROR
- POWER SPECTRA
  - Baseband power spectral density
- BANDWIDTH EFFICIENCY
  - *The ratio of the data rate in bits per second to the effectively utilized channel bandwidth*

$$\rho = \frac{R_b}{B} \text{ bits/s/Hz}$$

Power Spectra

$$s(t) = s_I(t)\cos(2\pi f_c T) - s_Q(t)\sin(2\pi f_c t)$$
$$= \mathrm{Re}[\tilde{s}(t)\exp(j2\pi f_c t)]$$

Complex envelope $\tilde{s}(t) = s_I(t) + js_Q(t)$ Baseband equivalent signal for the bandpass signal s(t)

$$\exp(j2\pi f_c T) = \cos(2\pi f_c t) + j\sin(2\pi f_c t)$$

Let **S_B(f)** be the baseband power spectral density, then

$$S_S(t) = \frac{1}{4}[S_B(f - f_c) + S_B(f + f_c)]$$

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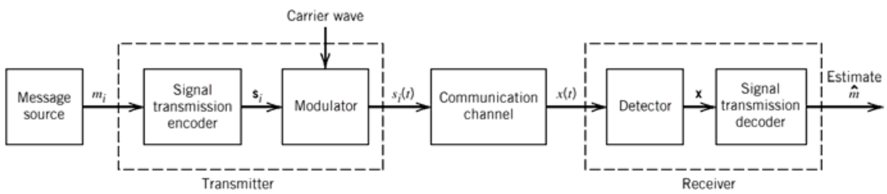
Passband Transmission Model



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Coherent Phase-Shift Keying

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

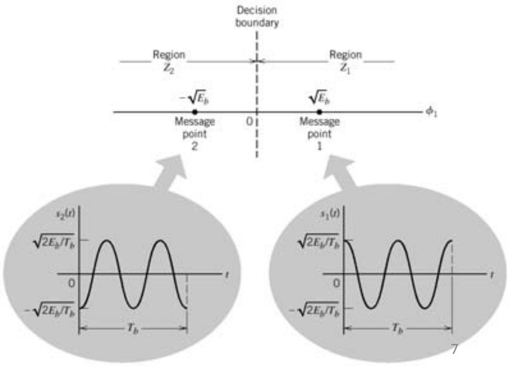
$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t), \qquad 0 \le t < T_b$$

$$s_1(t) = \sqrt{E_b}\,\phi_1(t), \qquad 0 \le t < T_b$$

$$s_2(t) = -\sqrt{E_b}\,\phi_1(t), \qquad 0 \le t < T_b$$

$$s_{11} = \int_0^{T_b} s_1(t)\phi_1(t)\,dt = +\sqrt{E_b}$$

$$s_{21} = \int_0^{T_b} s_2(t)\phi_1(t)\,dt = -\sqrt{E_b}$$



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Error Probability of BPSK

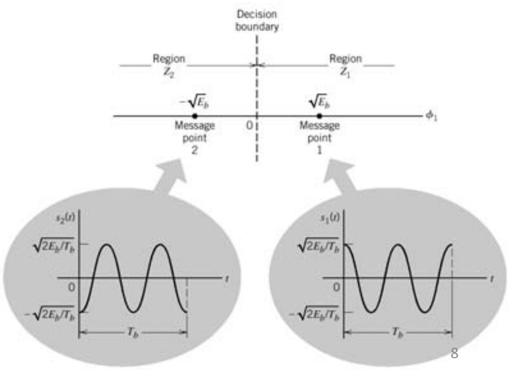
Using pairwise error probability relation as discussed in chapter 5

$$\Pr\{s_i \to s_k\} = \frac{1}{2}erfc\left(\frac{\|s_i - s_k\|}{2\sqrt{N_0}}\right) = Q\left(\frac{\|s_i - s_k\|}{\sqrt{2N_0}}\right)$$

Since the Euclidean Distance between the two symbols is

$$\|s_1 - s_2\| = d_{12} = 2\sqrt{E_b}$$

$$P_e = \frac{1}{2}erfc\left(\sqrt{\frac{E_b}{N_o}}\right)$$



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Error Probability of BPSK Detailed Analysis

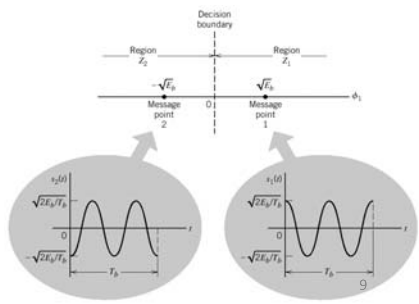
$$f_{X_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - s_{21})^2\right]$$
$$= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right]$$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$p_{10} = \int_0^\infty f_{X_1}(x_1|0)\, dx_1$$
$$= \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1$$

$$z = \frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b})$$

$$p_{10} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^\infty \exp(-z^2)\, dz$$
$$= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$



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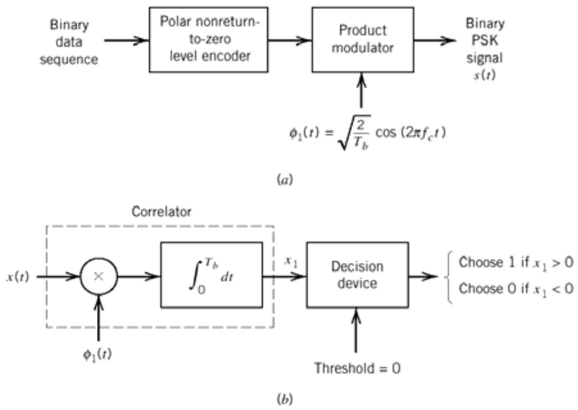
Generation and Detection of Coherent BPSK Signals



(a) binary PSK transmitter

(a) coherent binary PSK receiver.

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Power Spectra of BPSK Signals

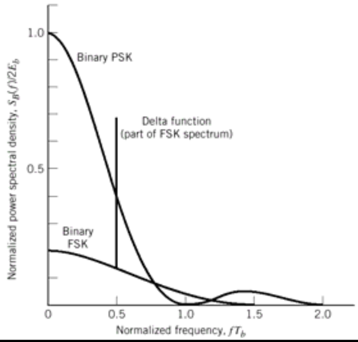
symbol shaping function

$$g(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}, & 0 \le t \le T_b \\ 0, & \text{otherwise} \end{cases}$$

he power spectral density of a random binary wave so described is equal to the energy spectral density of the symbol shaping function divided by the symbol duration. The energy spectral density of a Fourier transformable signal g*(t)* is defined as the squared magnitude of the signal's Fourier transform. (***see example 1.6 in textbook***)

$$S_X(f) = \frac{\mathscr{E}_g(f)}{T}$$

$$S_B(f) = \frac{2E_b \sin^2(\pi T_b f)}{(\pi T_b f)^2} = 2E_b \operatorname{sinc}^2(T_b f)$$



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QUADRIPHASE-SHIFT KEYING (QPSK)

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\dfrac{\pi}{4}\right], \\ 0, \end{cases}$$

$$\mathbf{s}_i = \begin{bmatrix} \sqrt{E}\cos\left((2i-1)\dfrac{\pi}{4}\right) \\ -\sqrt{E}\sin\left((2i-1)\dfrac{\pi}{4}\right) \end{bmatrix}, \qquad i = 1, 2, 3, 4$$

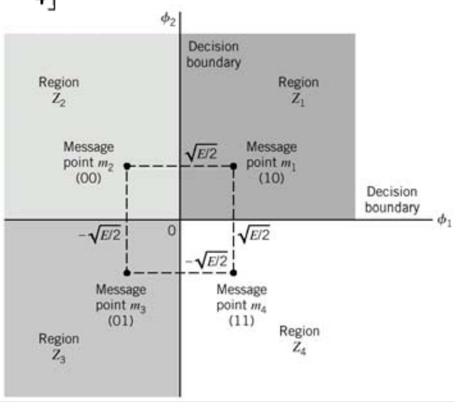
$$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left[(2i-1)\frac{\pi}{4}\right]\cos(2\pi f_c t) - \sqrt{\frac{2E}{T}}\sin\left[(2i-1)\frac{\pi}{4}\right]\sin(2\pi f_c t)$$

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t), \qquad 0 \le t \le T$$

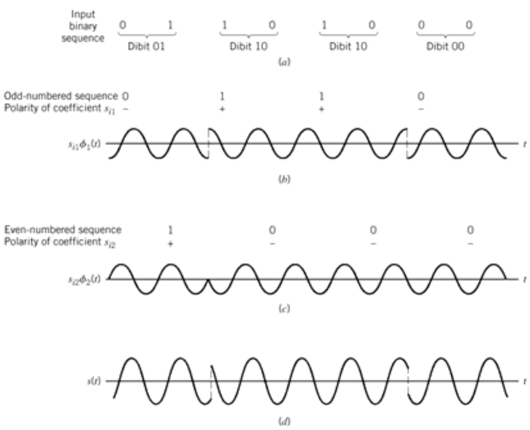
$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t), \qquad 0 \le t \le T$$

TABLE 6.1 *Signal-space characterization of QPSK*

Gray-encoded Input Dibit	*Phase of QPSK Signal (radians)*	*Coordinates of Message Points* s_{i1}	s_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$



Example of QPSK



(a) Input binary sequence.
(b) Odd-numbered bits of input sequence and associated binary PSK wave.
(c) Even-numbered bits of input sequence and associated binary PSK wave.
(d) QPSK waveform defined as
$s(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$.

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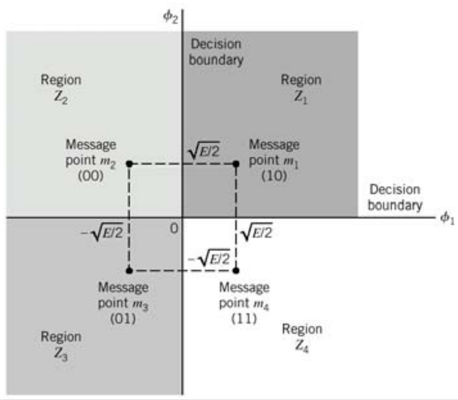
Error Probability of QPSK

Using the union bound as described in chapter 5

$$P_e \le erfc\left(\sqrt{\frac{E}{2N_0}}\right) + \frac{1}{2} erfc\left(\sqrt{\frac{E}{N_0}}\right)$$

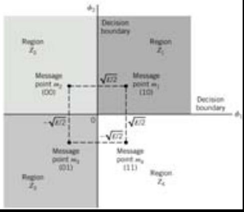
By reducing the overlap region, a tighter bound is

$$P_e \le erfc\left(\sqrt{\frac{E}{2N_0}}\right)$$



Exact Error Analysis

- To calculate the average probability of symbol error, we note from Equation that a coherent QPSK system is in fact equivalent to two coherent binary PSK systems working in parallel and using two carriers that are in phase quadrature.
- These two binary PSK systems may be characterized as follows:
  – The signal energy per bit is *E/2.*
  – The noise spectral density is $N_0/2$.
- Thus, the average probability of bit error in *each* channel of the coherent QPSK system is

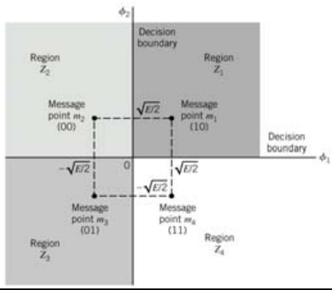


$$P' = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E/2}{N_0}}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

Exact Error Analysis

- Since the bit errors in the in-phase and quadrature channels of the coherent QPSK system are statistically independent.
- Then, the *average probability of a correct decision* resulting from the combined action of the two channels working together is

$$\begin{aligned} P_c &= (1 - P')^2 \\ &= \left[1 - \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)\right]^2 \\ &= 1 - \text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) + \frac{1}{4}\,\text{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right) \end{aligned}$$



- The average probability of symbol error

for coherent QPSK is therefore

$$\begin{aligned} P_e &= 1 - P_c \\ &= \text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) - \frac{1}{4}\,\text{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right) \end{aligned}$$

Error Analysis

$$P_e = 1 - P_c$$
$$= \text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) - \frac{1}{4}\,\text{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right)$$

- we may ignore the quadratic term on the right-hand side

$$P_e \simeq \text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) \longrightarrow$$ **Equals to the tight bound**

QPSK vs BPSK

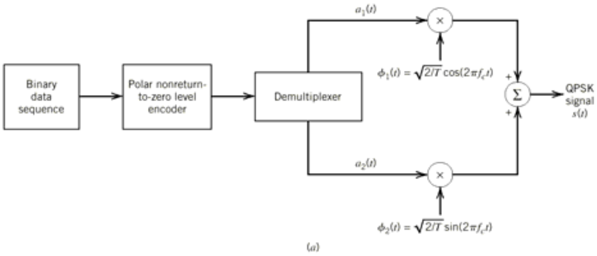
- In a QPSK system, we note that since there are two bits per symbol, the transmitted signal energy per symbol is twice the signal energy per bit $E=2E_b$
- Thus expressing the average probability of symbol error in terms of the ratio E_b/N_0, we may write

$$P_e \simeq \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

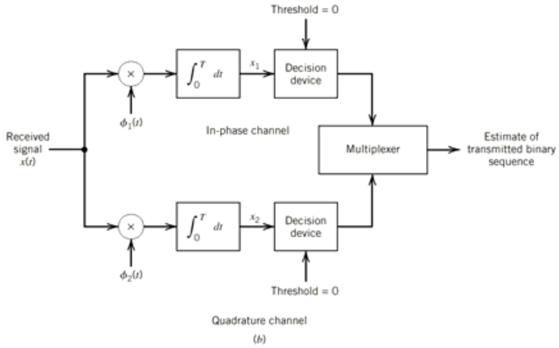
- With Gray encoding used for the incoming symbols, we find in chapter 5 that the *bit error rate* of QPSK is

$$\text{BER} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \longrightarrow$$ **Same as BPSK**

Generation and Detection of Coherent QPSK Signals



(a) QPSK transmitter



(a) coherent QPSK receiver.

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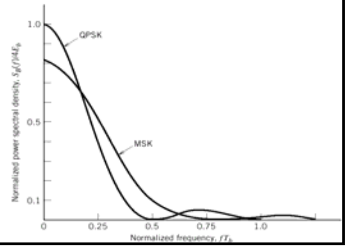
Power Spectra of QPSK Signals

the symbol shaping function
$$g(t) = \begin{cases} \sqrt{\dfrac{E}{T}}, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

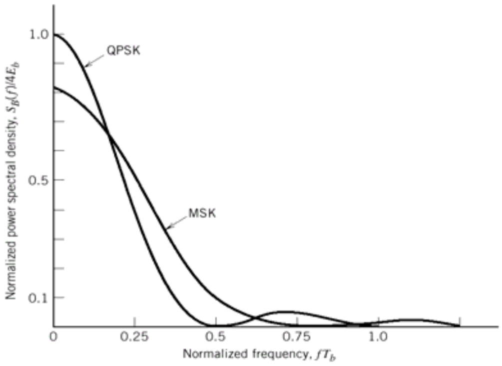
Hence, the in-phase and quadrature components have a common power spectral density, namely, $E\ \mathrm{sinc}^2(T/f)$

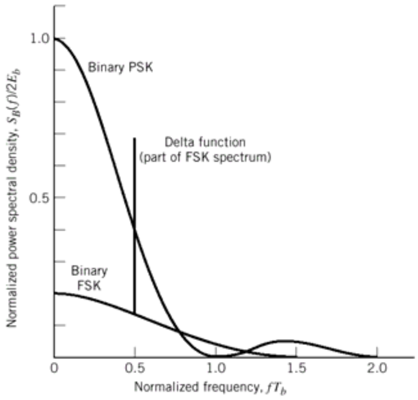
The in-phase and quadrature components are statistically independent. Accordingly, the baseband power spectral density of the QPSK signal equals the sum of the individual power spectral densities of the in-phase and quadrature components

$$\begin{aligned} S_B(f) &= 2E\ \mathrm{sinc}^2(Tf) \\ &= 4E_b\ \mathrm{sinc}^2(2T_b f) \end{aligned}$$



Power Spectra QPSK vs BPSK





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M-PSK

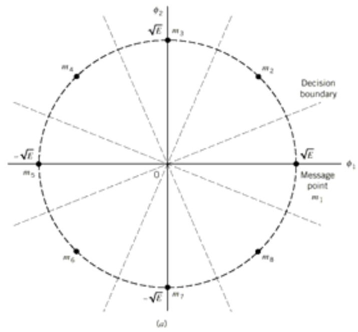
$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right), \qquad i = 1, 2, \ldots, M$$

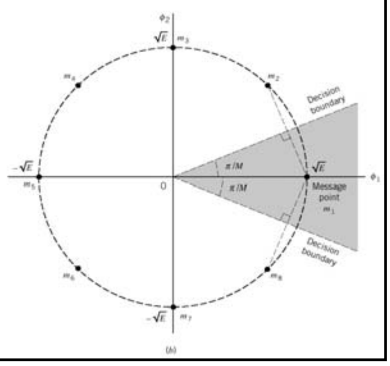
$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \qquad 0 \le t \le T$$

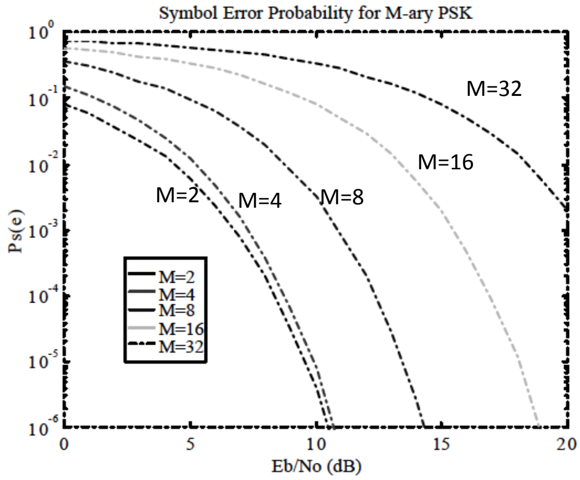
$$d_{12} = d_{18} = 2\sqrt{E} \sin\left(\frac{\pi}{M}\right)$$

$$P_e \simeq \operatorname{erfc}\left(\sqrt{\frac{E}{N_0}} \sin\left(\frac{\pi}{M}\right)\right)$$





M-PSK



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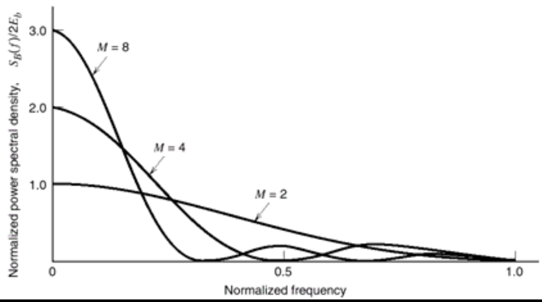
Power Spectral of M-PSK

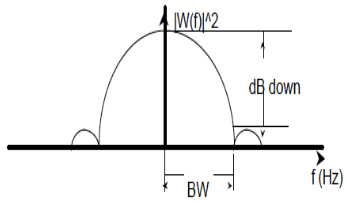
- The symbol duration of M-ary PSK is defined by $T=T_b \log_2 M$ where T_b is the bit duration.
- Proceeding in a manner similar to that described for a QPSK signal, we may show that the baseband power spectral density of an M-ary PSK signal is given by

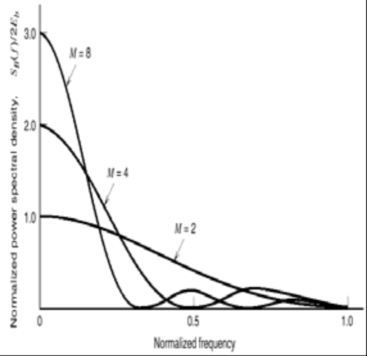
$$S_B(f) = 2E\,\mathrm{sinc}^2(Tf)$$
$$= 2E_b \log_2 M\,\mathrm{sinc}^2(T_b f \log_2 M)$$



BANDWIDTH EFFICIENCY OF M-ARY PSK SIGNALS

- The power spectra of M-ary PSK signals possess a *main lobe* bounded by well-defined *spectral nulls* (i.e., frequencies at which the power spectral density is zero).
- Accordingly the spectral width of the main lobe provides a simple and popular measure for the band width of M-ary PSK signals.
- This definition is referred to as the *null-to-null bandwidth.*
- With the null-to-null bandwidth encompassing the main lobe of the power spectrum of an M-ary signal, we find that it contains most of the signal power.





BANDWIDTH EFFICIENCY OF M-ARY PSK SIGNALS

- The channel bandwidth required to pass M-ary PSK signals (more precisely, the main spectral lobe of M-ary signals)

$$B = \frac{2}{T}$$

- Where T is the symbol duration. Recall that $T = T_b \log_2 M$
- Since the bit rate $R_b = 1/T_b$, then we have

$$B = \frac{2R_b}{\log_2 M}$$

- Therefore, the bandwidth efficiency of M-ary PSK signals will be:

$$\rho = \frac{R_b}{B} = \frac{\log_2 M}{2}$$

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BANDWIDTH EFFICIENCY OF M-ARY PSK SIGNALS

$$\rho = \frac{R_b}{B} = \frac{\log_2 M}{2}$$

TABLE 6.4 ***Bandwidth efficiency of M-ary PSK signals***

M	2	4	8	16	32	64
ρ (bits/s/Hz)	0.5	1	1.5	2	2.5	3

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Coherent Frequency-Shift Keying

BINARY FSK

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases}$$

$$f_i = \frac{n_c + i}{T_b} \quad \text{for some fixed integer } n_c \text{ and } i = 1, 2$$

we observe directly that the signals $s_1(t)$ and $s_2(t)$ are orthogonal, but not normalized to have unit energy. We therefore deduce that the most useful form for the set of orthonormal basis functions is

$$\phi_i(t) = \begin{cases} \sqrt{\dfrac{2}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases}$$

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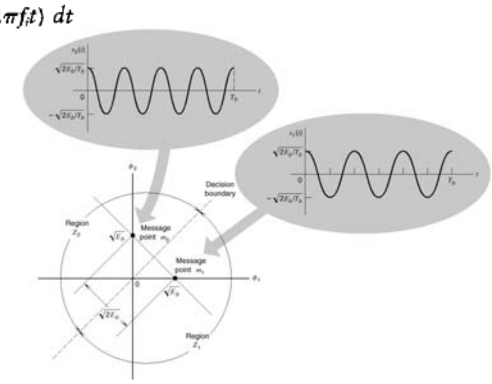
Coherent Frequency-Shift Keying

BINARY FSK

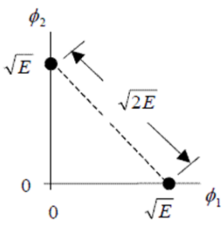
$$s_{ij} = \int_0^{T_b} s_i(t)\phi_j(t)\,dt$$
$$= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_i t)\sqrt{\frac{2}{T_b}}\cos(2\pi f_j t)\,dt$$
$$= \begin{cases} \sqrt{E_b}, & i = j \\ 0, & i \neq j \end{cases}$$

$$\mathbf{s}_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \qquad \mathbf{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$



Pairwise Error Probability: Binary FSK

- The signal constellation for binary FSK is:



E is the average signal Energy

The Euclidean distance between the two signals is:

$$d_{12} = \|s_1 - s_2\| = \sqrt{2E_b}$$

$$\Pr\{s_i \to s_j\} = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2}erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

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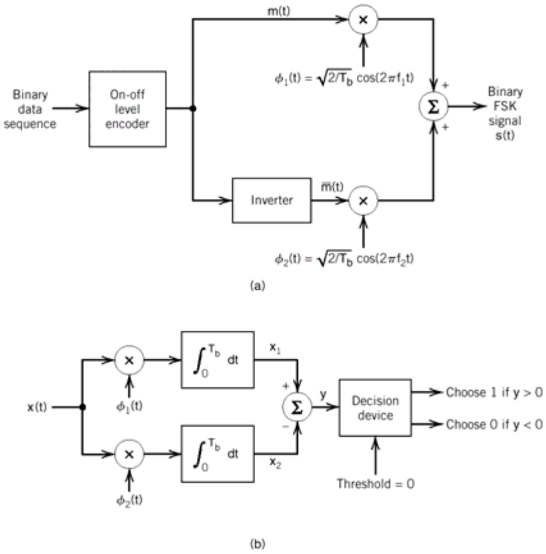
Block Diagram of Binary FSK



(a) binary FSK transmitter

(b) coherent binary FSK receiver.

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Power Spectra of Binary FSK Signals

Consider the case of Sunde's FSK, for which the two transmitted frequencies f_1 and f_2 differ by an amount equal to the bit rate $1/T_b$, and their arithmetic mean equals the nominal carrier frequency f_c ; phase continuity is always maintained, including inter-bit switching times.

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t \pm \frac{\pi t}{T_b}\right), \qquad 0 \le t \le T_b$$

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(\pm \frac{\pi t}{T_b}\right) \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}} \sin\left(\pm \frac{\pi t}{T_b}\right) \sin(2\pi f_c t)$$

$$= \sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{\pi t}{T_b}\right) \cos(2\pi f_c t) \mp \sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi t}{T_b}\right) \sin(2\pi f_c t)$$

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Power Spectra of Binary FSK Signals

$$s(t) = \sqrt{\frac{2E_b}{T_b}}\cos\left(\pm\frac{\pi t}{T_b}\right)\cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}}\sin\left(\pm\frac{\pi t}{T_b}\right)\sin(2\pi f_c t)$$
$$= \sqrt{\frac{2E_b}{T_b}}\cos\left(\frac{\pi t}{T_b}\right)\cos(2\pi f_c t) \mp \sqrt{\frac{2E_b}{T_b}}\sin\left(\frac{\pi t}{T_b}\right)\sin(2\pi f_c t)$$

1. The in-phase component is completely independent of the input binary wave. It equals $\sqrt{2E_b/T_b}\cos(\pi t/T_b)$ for all values of time *t.* The power spectral density of this component therefore consists of two delta functions, weighted by the fact $E_b / 2T_b$, and occurring at $f = \pm 1/2T_b$.
2. The quadrature component is directly related to the input binary wave. During the signaling interval $0 \le t \le T_b$, it equals -g*(t)* when we have symbol 1, and +g*(t)* when we have symbol 0. The symbol shaping function g*(t)* is defined by

$$g(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}}\sin\left(\frac{\pi t}{T_b}\right), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases}$$

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Power Spectra of Binary FSK Signals

The energy spectral density of this symbol shaping function equals

$$\Psi_g(f) = \frac{8E_bT_b\cos^2(\pi T_b f)}{\pi^2(4T_b^2f^2 - 1)^2}$$

The power spectral density of the quadrature component equals $\Psi_g(f)/T_b$. It is also apparent that the in-phase and quadrature components of the binary FSK signal are in-dependent of each other. Accordingly, the baseband power spectral density of Sunde's FSK signal equals the sum of the power spectral densities of these two components, as shown
by :

$$S_B(f) = \frac{E_b}{2T_b}\left[\delta\left(f - \frac{1}{2T_b}\right) + \delta\left(f + \frac{1}{2T_b}\right)\right] + \frac{8E_b\cos^2(\pi T_b f)}{\pi^2(4T_b^2f^2 - 1)^2}$$

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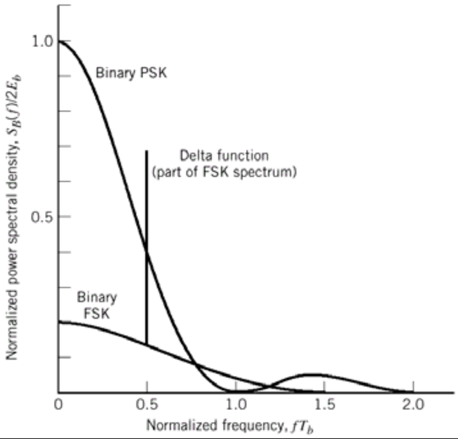
Power Spectra of Binary FSK Signals

$$S_B(f) = \frac{E_b}{2T_b}\left[\delta\left(f - \frac{1}{2T_b}\right) + \delta\left(f + \frac{1}{2T_b}\right)\right] + \frac{8E_b \cos^2(\pi T_b f)}{\pi^2(4T_b^2 f^2 - 1)^2}$$



For Binary PSK

$$S_B(f) = \frac{2E_b \sin^2(\pi T_b f)}{(\pi T_b f)^2} = 2E_b \operatorname{sinc}^2(T_b f)$$

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M-ARY FSK

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[\frac{\pi}{T}(n_c + i)t\right], \qquad 0 \le t \le T$$

where $i = 1, 2, \ldots, M$, and the carrier frequency $f_c = n_c / 2T$ for some fixed integer n_c. The transmitted symbols are of equal duration T and have equal energy E. Since the individual signal frequencies are separated by 1/2T Hz, the signals are orthogonal; that is

$$\int_0^T s_i(t)s_j(t)\,dt = 0, \qquad i \ne j$$

Complete orthonormal set of basis functions are :

$$\phi_i(t) = \frac{1}{\sqrt{E}} s_i(t), \qquad \begin{array}{l} 0 \le t \le T \\ i = 1, 2, \ldots, M \end{array}$$

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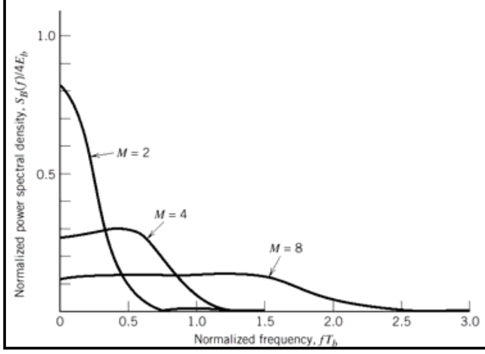
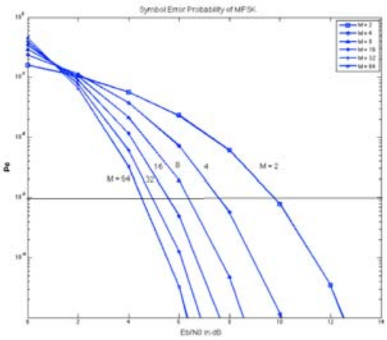
MFSK: Error Probability

Using the union bound to place an upper bound on the average probability of symbol error for M-ary FSK. Specifically, noting that the minimum distance d_{min} in M-ary FSK is $\sqrt{2E}$

$$P_e \le \frac{1}{2}(M-1)\,\mathrm{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

Power Spectra of* M-ary *FSK Signals





Bandwidth Efficiency of M-ary FSK *Signals*

When the orthogonal signals of an M-ary FSK signal are detected coherently, the adjacent signals need only be separated from each other by a frequency difference 1/2T so as to maintain orthogonality. Hence, we may define the channel bandwidth required to transmit M-ary FSK signals as

$$B = \frac{M}{2T}$$

Since $T = T_b \log_2 M$ and $R_b = 1/T_b$

$$B = \frac{R_b M}{2 \log_2 M}$$

The bandwidth efficiency of M-ary signals is therefore $\rho = \frac{R_b}{B}$

$$= \frac{2 \log_2 M}{M}$$

TABLE 6.6 ***Bandwidth efficiency of M-ary FSK signals***

M	2	4	8	16	32	64
ρ (bits/s/Hz)	1	1	0.75	0.5	0.3125	0.1875

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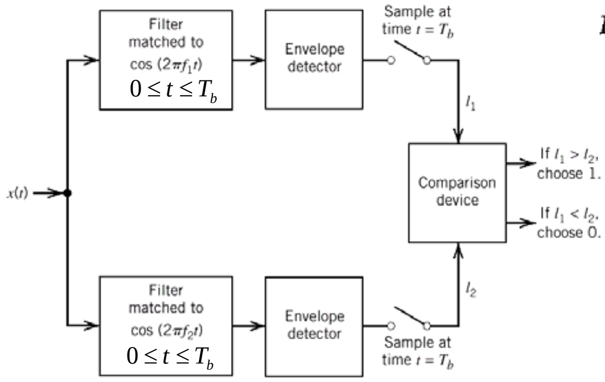
Noncoherent Binary FSK

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases}$$



$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{2N_0}\right)$$

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Differential Phase-Shift Keying

Differential phase-shift keying (DPSK) is the noncoherent version of PSK. It eliminates the need for a coherent reference signal at the receiver by combining two basic operations at the transmitter:
(1) differential encoding of the input binary wave
(2) phase-shift keying

The differential encoding process at the transmitter input starts with an arbitrary first bit, serving as reference. Let *[d_k]* denote the differentially encoded sequence with this added reference bit. We now introduce the following definitions in the generation of this sequence:

- If the incoming binary symbol b_k is 1, leave the symbol d_k unchanged with respect to the previous bit.
- If the incoming binary symbol b_k is 0, change the symbol d_k with respect to the previous bit.

TABLE 6.7 ***Illustrating the generation of DPSK signal***

$\{b_k\}$		1	0	0	1	0	0	1	1
$\{d_{k-1}\}$		1	1	0	1	1	0	1	1
Differentially encoded sequence $\{d_k\}$	1	1	0	1	1	0	1	1	1
Transmitted phase (radians)	0	0	π	0	0	π	0	0	0

Differential Phase-Shift Keying

The transmission of symbol 1 leaves the carrier phase unchanged

$$s_1(t) = \begin{cases} \sqrt{\dfrac{E_b}{2T_b}}\cos(2\pi f_c t), & 0 \le t \le T_b \\ \sqrt{\dfrac{E_b}{2T_b}}\cos(2\pi f_c t), & T_b \le t \le 2T_b \end{cases}$$

The transmission of 0 advances the carrier phase by 180 degrees

$$s_2(t) = \begin{cases} \sqrt{\dfrac{E_b}{2T_b}}\cos(2\pi f_c t), & 0 \le t \le T_b \\ \sqrt{\dfrac{E_b}{2T_b}}\cos(2\pi f_c t + \pi), & T_b \le t \le 2T_b \end{cases}$$

Bit error rate for DPSK is given by

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

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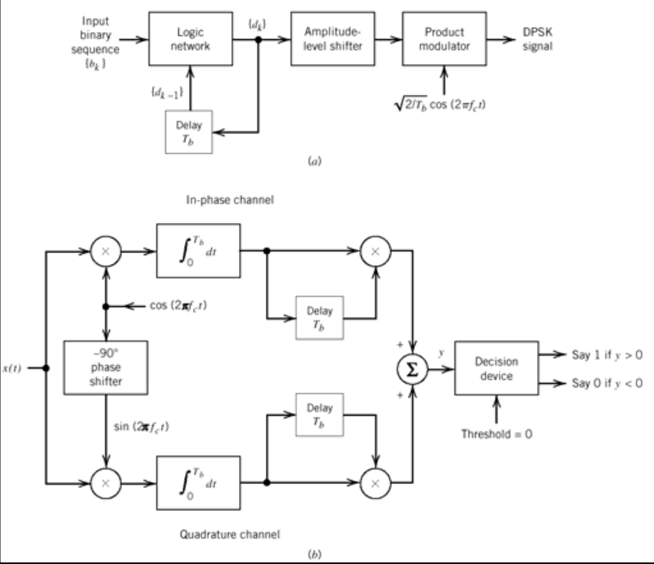
Differential Phase-Shift Keying



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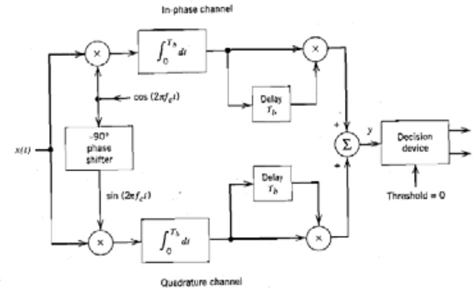
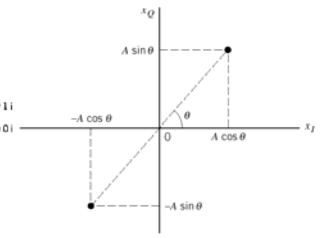
Differential Phase-Shift Keying: Receiver





The receiver measures the coordinates *(x_{Io}, x_{Qo})* at time $t = T_b$ and *(x_{I1}, x_{Q1})* at time $t = 2T_b$. The issue to be resolved is whether these two points map to the same signal point or different ones.

Recognizing that the two vectors $\mathbf{x_0}$ and $\mathbf{x_1}$, are pointed roughly in the same direction if their inner product is positive.

$$x_{I_0}x_{I_1} + x_{Q_0}x_{Q_1} \underset{\text{say } 0}{\overset{\text{say } 1}{\gtrless}} 0$$

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M-ARY Quadrature Amplitude Modulation

$$s_k(t) = \sqrt{\frac{2E_0}{T}}\, a_k \cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}}\, b_k \sin(2\pi f_c t), \qquad \begin{array}{l} 0 \le t \le T \\ k = 0, \pm 1, \pm 2, \ldots \end{array}$$

Where E_o is the energy of the signal with the lowest amplitude

Let d_{min} be the minimum distance between two points, thus $\frac{d_{min}}{2} = \sqrt{E_o}$

Thus , the i[th] message s_i will be $\left(a_i d_{min}/2\ ,\ b_i d_{min}/2\right)$

The orthonormal basis functions are:

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \qquad 0 \le t \le T$$

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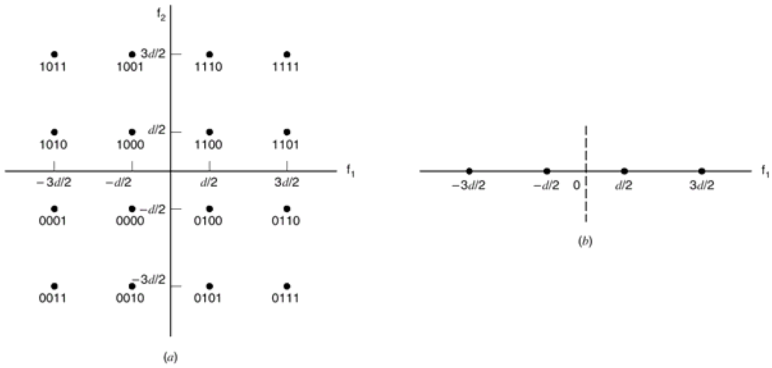
M-ARY Quadrature Amplitude Modulation



Symbol error probability for M-ary QAM is

$$P_e \simeq 2\left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right)$$

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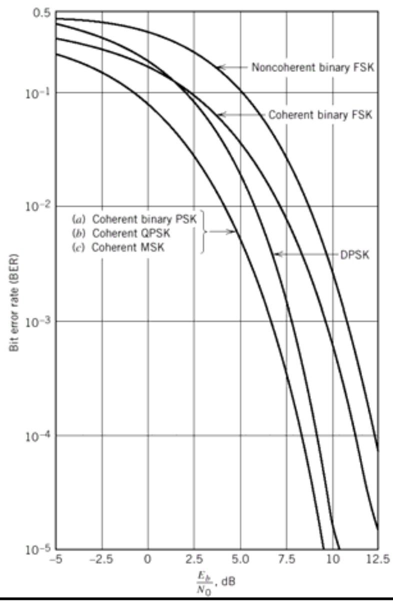
Comparison

TABLE 6.8 ***Summary of formulas for the bit error rate of different digital modulation schemes***

Signaling Scheme	*Bit Error Rate*
(a) Coherent binary PSK Coherent QPSK Coherent MSK	$\frac{1}{2}\operatorname{erfc}(\sqrt{E_b/N_0})$
(b) Coherent binary FSK	$\frac{1}{2}\operatorname{erfc}(\sqrt{E_b/2N_0})$
(c) DPSK	$\frac{1}{2}\exp(-E_b/N_0)$
(d) Noncoherent binary FSK	$\frac{1}{2}\exp(-E_b/2N_0)$



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