Example

8-states rate 1/2 convolutional code.

Analyse the following code with generator polynomials in Octal form:

$$g_0 = (15)_8$$
$$g_1 = (17)_8$$

- Find the state diagram.
- Find the trellis Diagram.
- What is the constraint length.
- what is $d_f \equiv$ minimum Hamming Distance.
- Find the transfer Function using Mason's Rule.
- Expand the transfer function using Long Division.
- Draw the encoder using Shift Registers and adders.

Note on Octal Form Representation:

The Octal representation are read from left to right (dropping leading zeros on the left)

$$g(X) = g_0 + g_1 X + g_2 X^2 + \cdots + g_{\nu-1} X^{\nu-1}.$$

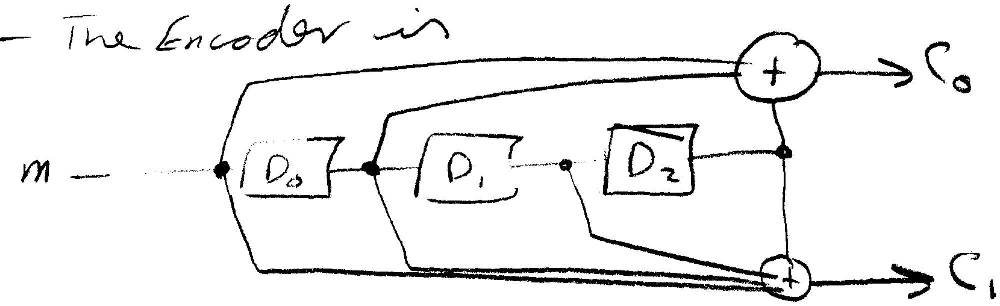
For example

$$(15)_8 \Longrightarrow g(X) = 1 + X + X^3$$

$$(1101)_2$$

- $g_0(x) = 1 + x + x^3$
- $g_1(x) = 1 + x + x^2 + x^3$

— The Encoder is

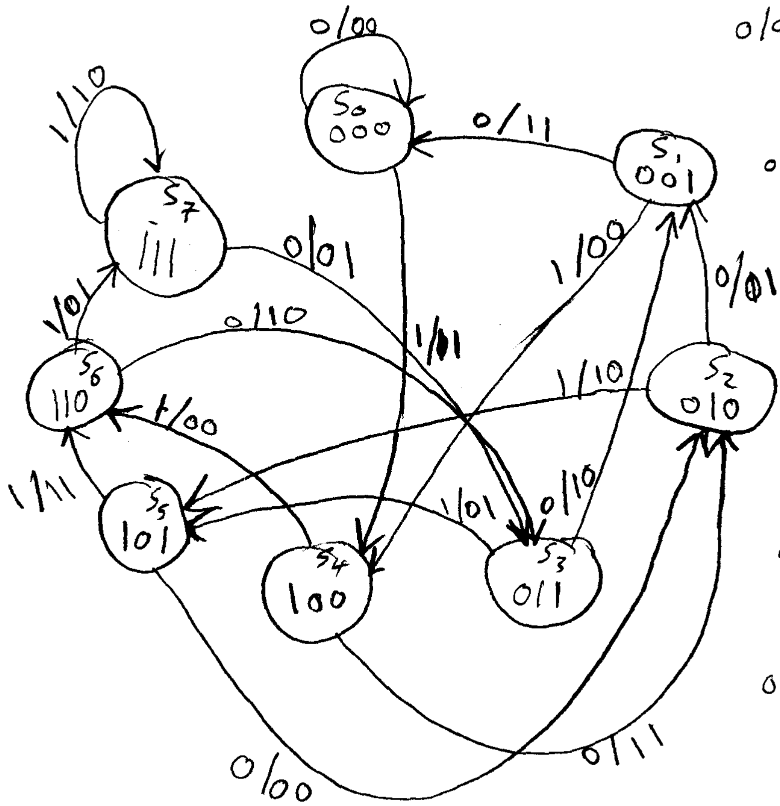


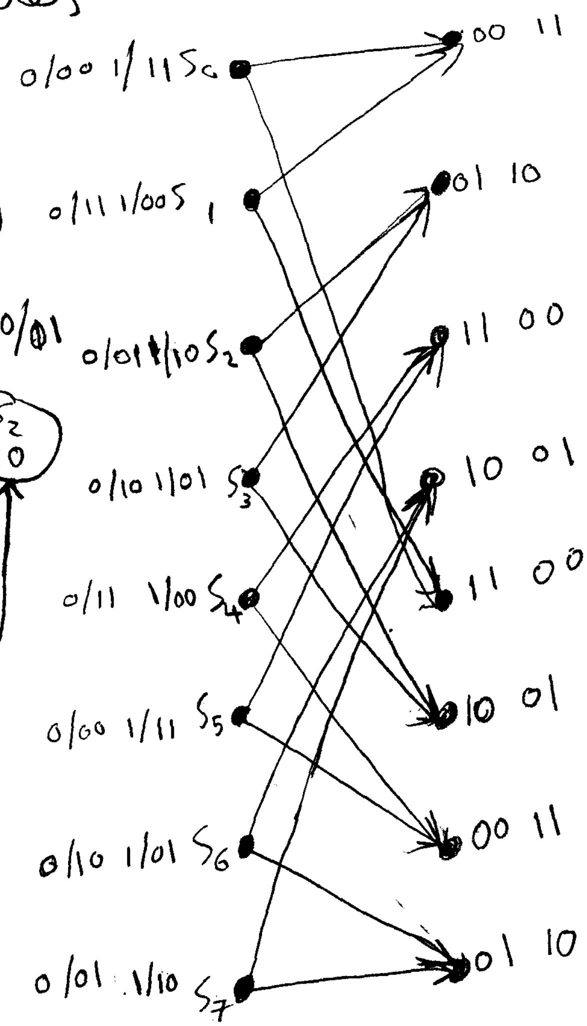
The Memory depth $M = \text{Max deg}(g_0, g_1)$

$= 3$

$\Rightarrow$ constraint length $\nu = 4$

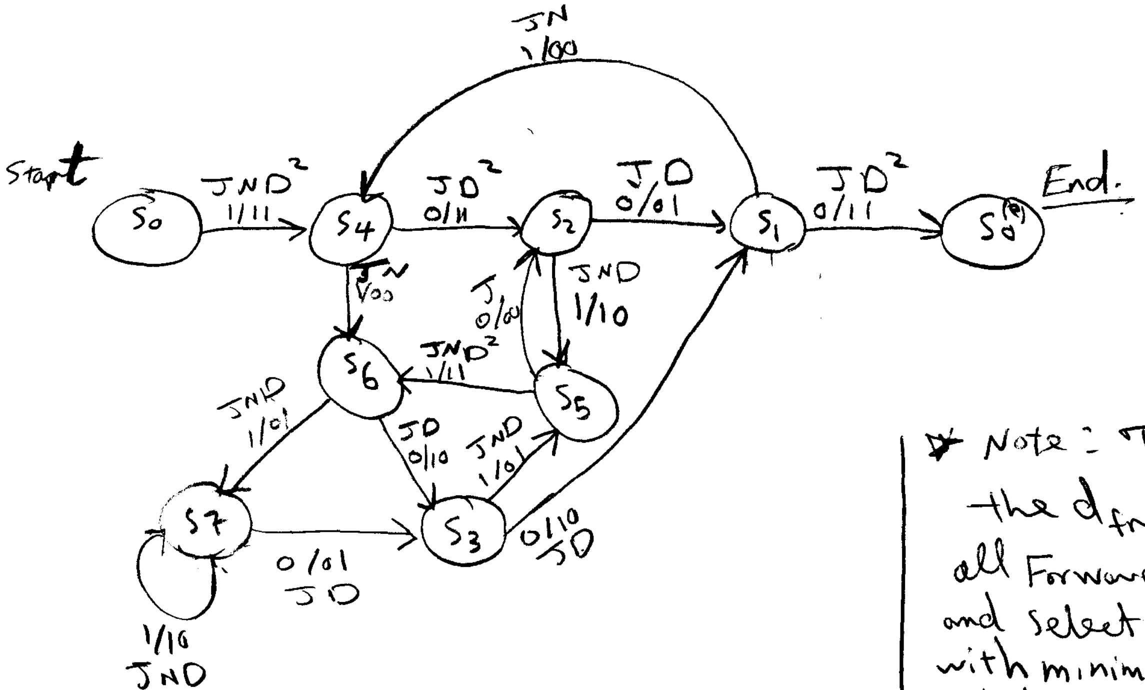
— There are $2^M = 2^3 = 8$ states





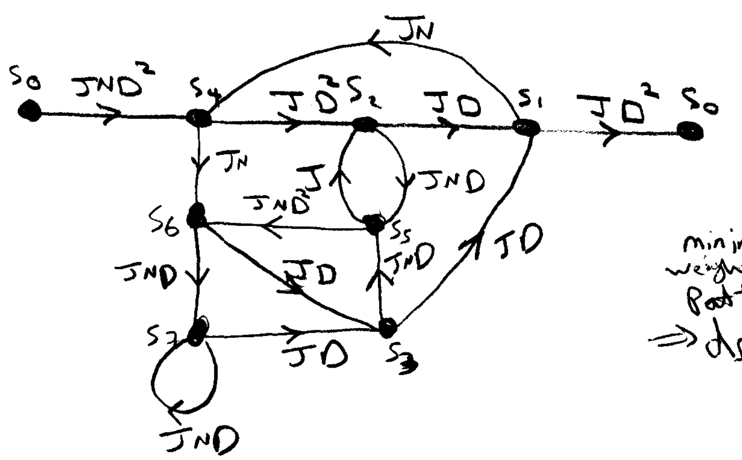
— Tranfer Function

Lable each branch with JND operators.



★ Note: To Find the d_{free}, inspect all Forward paths and select the path with minimum Hamming wieght, (power of D)

— Redraw in Signal Flow Graph. Identify Forward Paths and loops.



Forward Paths

$P_1 = J^4 N D^7$ $[S_0 S_4 S_2 S_1 S_0]$

$P_2 = J^6 N^3 D^7$ $[S_0 S_4 S_6 S_7 S_3 S_1 S_0]$

$P_3 = J^5 N^2 D^6$ $[S_0 S_4 S_6 S_3 S_1 S_0]$ ⟸ minimum weight Path ⟹ $d_f = 6$

$P_4 = J^8 N^4 D^8$ $[S_0 S_4 S_6 S_7 S_3 S_5 S_2 S_1 S_0]$

$P_5 = J^8 N^4 D^{12}$ [042567310]

$P_6 = J^7 N^3 D^{11}$ [04256310]

$P_7 = J^7 N^3 D^7$ [04635210]

~~Two non-touching Paths~~

Loops

$L_1 = J^3 N D^3$ [4214]

~~$L_2 = J^4 N^3 D^5$ [425642]~~

$L_2 = J^2 N D$ [252]

$L_3 = J^3 N^2 D^4$ [5635]

$L_4 = J^4 N^3 D^5$ [56735]

$L_5 = JND$ [77]

$L_6 = J^5 N^3 D^3$ [467314]

$L_7 = J^7 N^3 D^8$ [4256731 4]

$L_8 = J^6 N^3 D^7$ [4256314]

$L_9 = J^4 N^2 D^2$ [46314]

L_1 and L_3 non touching

L_1 and L_4 non touching

L_1 and L_5 non touching

L_2 and L_5 non-touching

L_2 and L_6 non-touching.

L_2 and L_9 non-touching

L_5 and L_8 non-touching

L_5 and L_9 non-touching

Three non-touching Paths

L_1, L_3 and L_5

L_2, L_5 and L_9

Mason's Rule

$$T = \frac{\sum P_i \Delta_i}{\Delta}$$

$$\begin{aligned}
\Delta = 1 - [&J^3 N D^3 + J^2 N D + J^3 N^2 D^4 + J N D + J^5 N^3 D^3 + J^7 N^3 D^8 \\
&+ J^6 N^3 D^7 + J^4 N^2 D^2] \\
+ [&J^6 N^3 D^7 + J^7 N^4 D^8 + J^4 N^2 D^4 + J^3 N^2 D^2 \\
&+ J^6 N^4 D^4 + J^6 N^3 D^3 + J^7 N^4 D^8 \\
&+ J^5 N^3 D^3] \\
- [&J^7 N^4 D^8 +
\end{aligned}$$

incomplete.

Continue following the same procedure.

- Free minimum Hamming Distance (d_f)

I will inspect the trellis Diagram to find d_f: