Numerical Example

Calculations of the orbital coordinates and look angles, using real data for the CTS (Communications Technology Satellite) space craft.

The orbital elements as measured at 00:00:00 UT on 27/12/1978.

a	**Semi-major axis**	**42164.765 km**
e	**Eccentricity**	**0.001181**
i	**Inclination**	**0.802^{o}**
M	**Mean anomaly**	**116.636^{o}**
ω	**Argument of perigee**	**138.167^{o}**
Ω	**RA of ascending node**	**84.178^{o}**

In order to calculate the orbital coordinates and look angles, we proceed as follows:

Mean angular velocity

$$\eta = \frac{1}{a}\sqrt{\frac{\mu}{a}} = 7.29208108 \times 10^{-5} \quad rad/\sec$$

Using the mean anomaly M and the mean angular velocity, the elapsed time since perigee is:

$$(t - t_p) = \frac{M}{\eta} = 2.79163413 \times 10^{4} \ \sec$$

$$= \{7 \text{ h}, \ 45 \text{ min}, 16.3 \text{ s}\}$$

To locate the satellite in its orbit, the eccentric anomaly {E} is obtained from:

M = E – e sin(E) → E = 116.637056^{O}

The orbital plane radial coordinate can then be evaluated:

r_O = a(1 – e cos E) = 4.218709065 X 10^4 km

The angular coordinate $\Phi_O = 116.5461901^{O}$

Transforming the orbital coordinates to the rectangular system yields:

x_O = -1.885421816 X 10^4 km

y_O = - 3.773948960 X 10^4 km

Transforming these to the geocentric equatorial system yields:

$x_i = 3.93522813 \times 10^4$ **km**

$y_i = -1.519286524 \times 10^4$ **km**

$z_i = -5.6960337 \times 10^2$ **km**

The second transformation is then applied to locate the satellite with respect to the rotating coordinate system.

The value of $\Omega_e T_e$ Is calculated from:

$\Omega_e T_e = \alpha_{g,0} + 0.25068447\ t$ **degrees**

where $\alpha_{g,0} = 99.6909833 + 36000.7689\ T_c + 0.00038708\ T_c^2$ degrees

and $T_c = (JD – 2415020) / 36525$ Julian centuries

JD for 00:00:00 h on 27 December, 1978 is 2443869.5

$T_c = (2443869.5 – 2415020) / 36525 = 0.78985626$

Substituting in ag,0 we get $\alpha_{g,0} = 2.8535124X10^{4\ O}$

$\Omega_e T_e = \alpha_{g,0} = 2.8535124X10^{4\ O} = (79.2642333)X360^O$

$= 95.12399880^O$

Now we can find the satellite coordinates in the rotating coordinate system:

X_r **= -1.864676149 x 10^4 km**

Y_r **= - 3.783812210 x 10^4 km**

Z_r **= - 5.696033700 x 10^2 km**

The sub-satellite point lies in the third quadrant

$$\therefore \text{ } Longitude \quad l_S = 90^\circ + \tan^{-1}\left|\frac{x_r}{y_r}\right| = 116.2342^\circ \ west$$

$$\therefore \text{ } Latitude \quad L_S = 90^\circ - \cos^{-1}\sqrt{\frac{z_r^2}{x_r^2 + y_r^2 + z_r^2}} = -0.7736^\circ \ north$$

Earth station data

Longitude and latitude

$$l_e = 80.438^\circ \ west \ \ and \ \ L_e = 37.229^\circ \ north$$

Look angles can then be calculated

Central angle is given by:

$$\gamma = \cos^{-1}[\cos(L_e)\cos(L_s)\cos(l_s - l_e) + \sin(L_e)\sin(L_s)] = 50.387511^\circ$$

Orbital radius → $r_s = \sqrt{x_r^2 + y_r^2 + z_r^2} = 42183.23 \;\; km$

Earth's radius → $r_e = 6370 \;\; km$

Cos (*El*) = 0.8455

Elevation = 32.28^O

To find the azimuth,

$c = \left| L_e - L_s \right| = 35.7962^\circ$

tan [0.5(Y + X)] =9.359795

tan [0.5(Y – X)] = 1.061419

which leads to bearings **X = 37.1950^O**

Y = 130.6081^O

Azimuth = 229.39^O