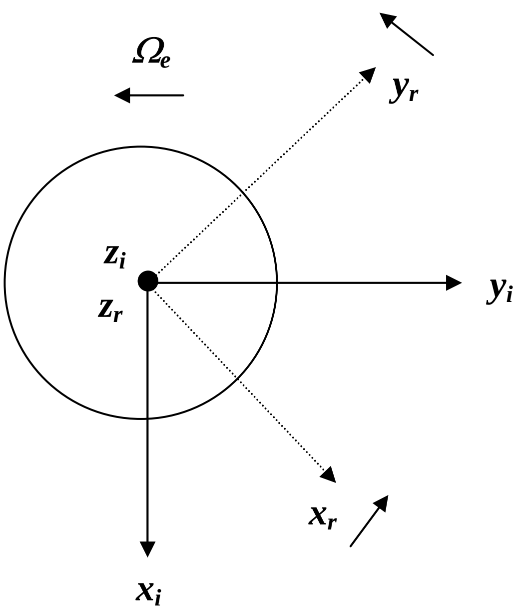
ORBITAL TO GEOCENTRIC EQUATORIAL COORDINATE SYSTEM TRANSFORMATION

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} (\cos\Omega\cos\omega - \sin\Omega\cos i\sin\omega) & (-\cos\Omega\sin\omega - \sin\Omega\cos i\cos\omega) & \sin\Omega\sin i \\ (\sin\Omega\cos\omega + \cos\Omega\cos i\sin\omega) & (-\sin\Omega\sin\omega + \cos\Omega\cos i\cos\omega) & -\cos\Omega\sin i \\ \sin i\sin\omega & \sin i\cos\omega & \cos i \end{bmatrix} \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix}$$

GEOCENTRIC EQUTORIAL TO ROTATING COORDINATE SYSTEM TRANSFORMATION

- The equatorial plane coincides with the plane of the paper.
- The earth rotates anti-clockwise with angular velocity Ω_e.
- x_r and y_r axes are attached to the earth and rotate with it.
- z_i and z_r axes coincide.



$$\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} \cos(\Omega_e T_e) & \sin(\Omega_e T_e) & 0 \\ -\sin(\Omega_e T_e) & \cos(\Omega_e T_e) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

$\boldsymbol{T_e}$ is the time elapsed since the x_r axis coincided with the x_i axis.

The value of $\Omega_e T_e$ at any time t, expressed in minutes after midnight UT is given by:

$$\Omega_e T_e = \alpha_{g,0} + 0.25068447\ t \qquad \text{degrees.}$$

Where $\alpha_{g,0}$ is the right ascension of the Greenwich meridian at 0 h UT at Julian day JD and is given by:

$$\alpha_{g,0} = 99.690983 + 36000.7689 T_c + 0.00038708 T_c^2 \quad \text{degrees}$$

where Tc is the elapsed time in Julian centuries between 0h UT on Julian day JD and noon UT on January 1, 1900.

$$T_c = (JD - 2415020) / 36525$$

Add 0.5 to the JD value used in this equation before substituting in the previous equation (since it is calculated at 0h UT).

JULIAN DAYS AND JULIAN DATES

Standard time is Universal time UT

(mean solar time at Greenwich observatory near London).

- Astronomers use Julian days and Julian dates.
- Julian days start at noon.
- Julian date time reference is 1200 noon UT on January 1, 4713 BC

- Examples:
  - ❖ Noon on December 31, 1899 was the beginning of Julian day 2,415,020
  - ❖ Noon UT on December 31, 1984 was the start of Julian day 2,446,066
  - ❖ 00:00:00 hours UT on January 1, 1985 was Julian date 2,446,066.5

JULIAN DATES AT THE BEGINNING OF EACH YEAR FOR (1986-2000)

Year	Julian date	Year	Julian date
	2400000 +	1993	48 987.5
1986	46 430.5	1994	49 352.5
1987	46 795.5	1995	49 717.5
1988	47 160.5	1996	50 082.5
1989	47 526.5	1997	50 448.5
1990	47 891.5	1998	50 813.5
1991	48 256.5	1999	51 178.5
1992	48 621.5	2000	51 543.5

DAY NUMBER FOR NOON ON THE LAST DAY OF EACH MONTH

Date	Day No.	Leap year	Date	Day No.	Leap year
Jan 31	31.5	31.5	July 31	212.5	213.5
Feb 28/29	59.5	60.5	Aug 31	243.5	244.5
March 31	90.5	91.5	Sept 30	273.5	274.5
Apr 30	120.5	121.5	Oct 31	304.5	305.5
May 31	151.5	152.5	Nov 30	334.5	335.5
June 30	181.5	182.5	Dec 31	365.5	366.5

Example of Julian date calculation:

1. Find the Julian date JD corresponding to 3 h UT on Oct 11,1986.

   Oct 11 is day number 273.5 + 11 = 284.5
   Start of Oct 11 (0h UT) is 284
   At 03:00:00 UT is (3/24) = 0.125 day
   Day and time will be 284.125

   Add this to the Julian date for Jan 1, 1986

   We get: 2,400,000 + 46,430.5 + 284.125 = 2,446,714.625

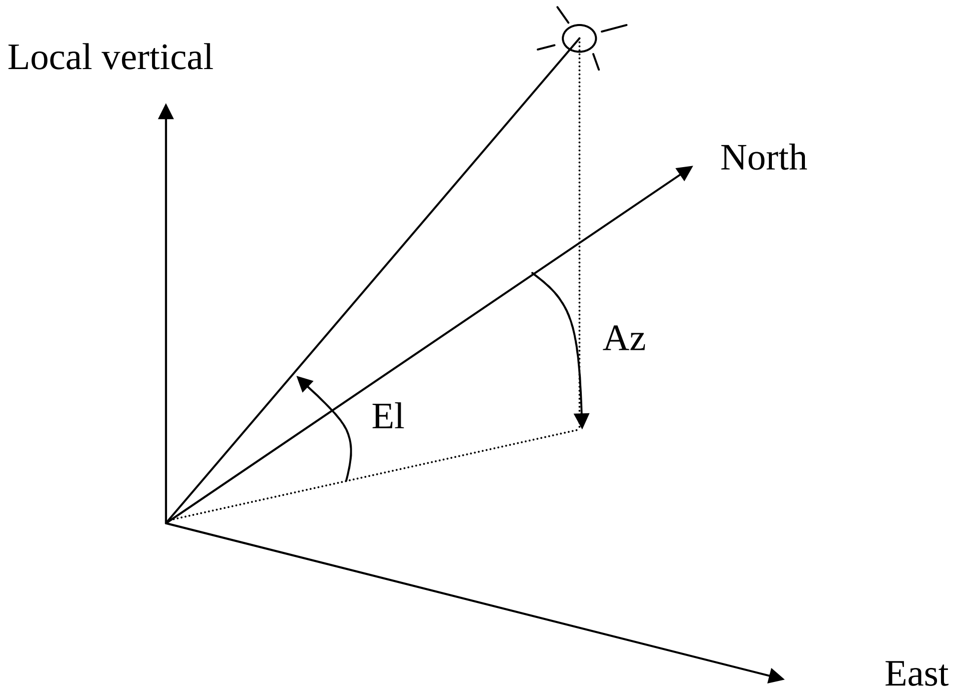
2. Find the Julian date JD corresponding to 15:00:00 h UT on March 10, 1999.

   March 10 is day number 59.5 + 10 = 69.5
   At 15:00:00 UT is 0.125 day after noon
   Day and time will be 69.5 + 0.125 = 69.625

   Add this to the Julian date for Jan 1, 1999
   We get: 2,400,000 + 51,178.5 + 69.625 = 2,451,248.125

LOOK ANGLE DETERMINATION

Definition: Look angles are the coordinates to which an earth station antenna must be pointed to communicate with the satellite.



Azimuth (Az) The angle measured eastward from geographic north to the projection of the satellite path on a locally horizontal plane at the earth station.

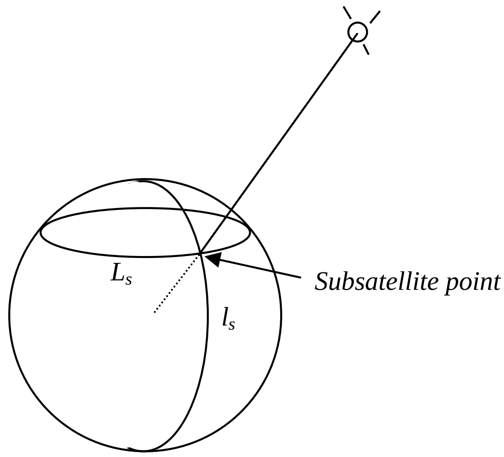
Elevation (El) The angle measured upward from the horizontal plane to the satellite path.

THE SUBSATELLITE POINT

The point where a line drawn from the centre of the earth to the satellite passes through the earth's surface.

L_s → The north latitude of the subsatellite point.

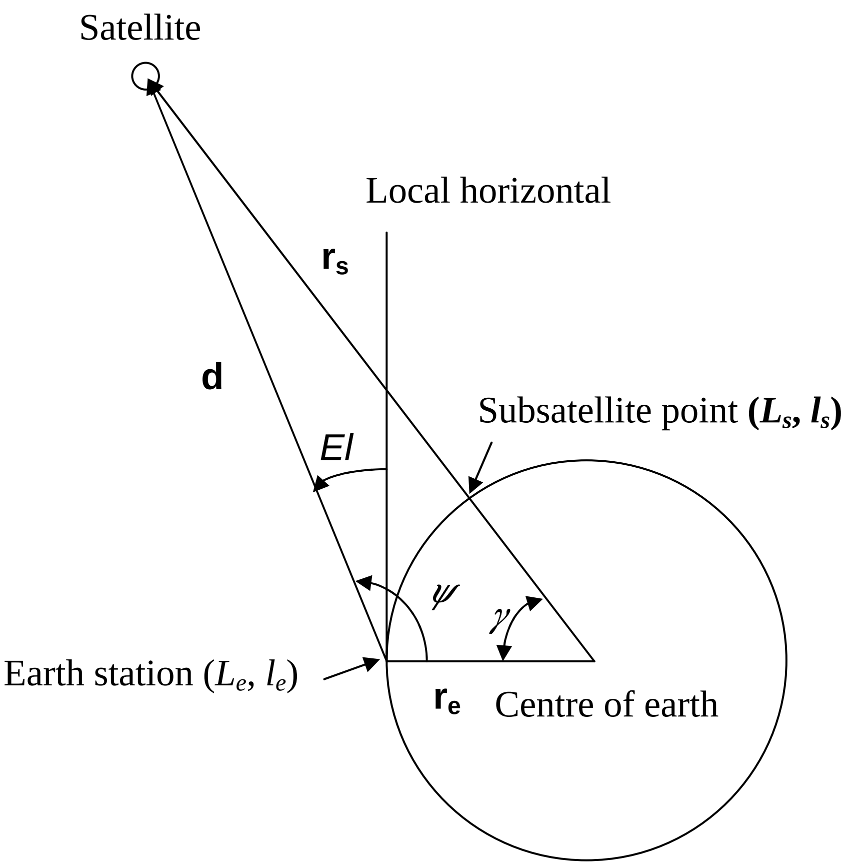
l_s → The west longitude of the subsatellite point.



$$L_s = 90^\circ - \cos^{-1}\left[\frac{z_r}{\sqrt{x_r^2 + y_r^2 + z_r^2}}\right]$$

$$l_s = \begin{cases} -\tan^{-1}\left(\frac{y_r}{x_r}\right) & y_r \geq 0 \quad x_r \geq 0 \quad \textit{first quadrant} \\ 180^\circ + \tan^{-1}\left(\frac{y_r}{|x_r|}\right) & y_r \geq 0 \quad x_r \leq 0 \quad \textit{second quadrant} \\ 90^\circ + \tan^{-1}\left|\frac{x_r}{y_r}\right| & y_r \leq 0 \quad x_r \leq 0 \quad \textit{third quadrant} \\ \tan^{-1}\left(\frac{|y_r|}{x_r}\right) & y_r \leq 0 \quad x_r \geq 0 \quad \textit{fourth quadrant} \end{cases}$$

ELEVATION EVALUATION



$$\cos(\gamma) = \cos(L_e)\cos(L_s)\cos(l_s - l_e) + \sin(L_e)\sin(L_s)$$

$$d = r_s \sqrt{\left[1 + \left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)\cos(\gamma)\right]}$$

$$El = \psi - 90°$$

Using the law of sines : $$\frac{r_s}{\sin(\psi)} = \frac{d}{\sin(\gamma)}$$

$$\therefore \cos(El) = \frac{r_s \sin(\gamma)}{d} = \frac{\sin(\gamma)}{\sqrt{1 + \left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)\cos(\gamma)}}$$

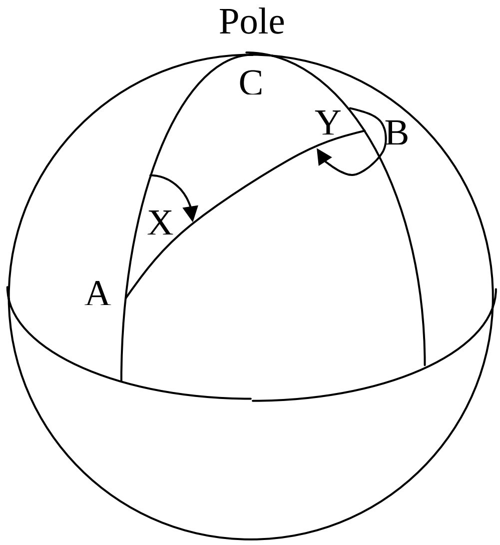
These equations permit the evaluation of the elevation angle from a knowledge of the subsatellite point and earth station coordinates.

AZIMUTH CALCULATION

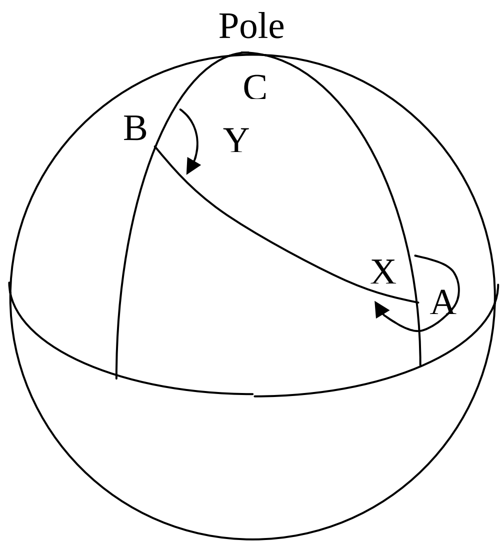
The satellite, sub-satellite point and the earth station lie on the same vertical plane. Therefore the azimuth angle can be measured from the north direction going eastward towards the sub-satellite point.

The geometry used for the calculation depends on whether the sub-satellite point is east or west of the earth station and which hemisphere contains the sub-satellite point and the earth station.

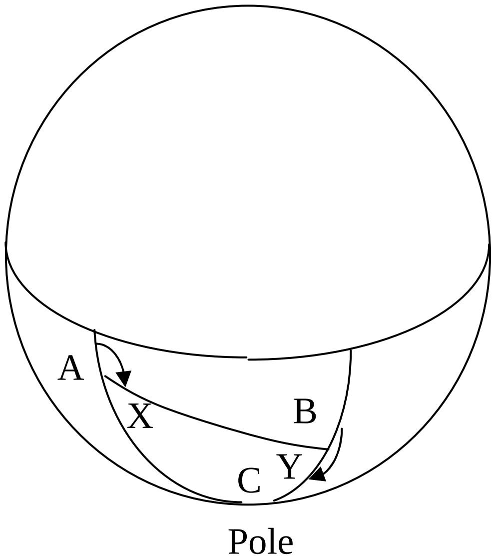
This calculation is simplified for the ideal geo-stationary orbit.



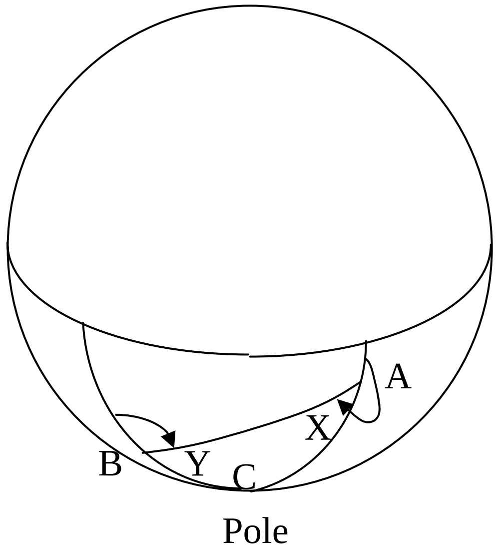
Northern hemosphere, A west of B



Northern hemosphere, B west of A



Southern hemosphere, A west of B



Southern hemosphere, B west of A

- Either point A or point B can be the earth station; the other must be the sub-satellite point.
- B is closer to the pole that is nearer to both points.
- Points A, B, and the pole form a spherical triangle with polar angle C and angles X and Y at the vertices A and B.

$$C = |l_A - l_B| \qquad \text{or} \qquad C = \left| 360 - |l_A - l_B| \right|$$

Whichever makes $C \leq 180 \text{ degrees}$

Case 1: At least one point in the northern hemisphere.

B is chosen to be closer to the north pole. → $L_B > L_A$

The bearings X and Y may be found from:

$$\tan[0.5(Y - X)] = \frac{\cot(0.5C)\sin[0.5(L_B - L_A)]}{\cos[0.5(L_B + L_A)]}$$

$$\tan[0.5(Y + X)] = \frac{\cot(0.5C)\cos[0.5(L_B - L_A)]}{\sin[0.5(L_B + L_A)]}$$

$$X = 0.5(Y + X) + 0.5(Y - X)$$

$$Y = 0.5(Y + X) - 0.5(Y - X)$$

The relationship between X, Y, and the azimuth Az depends on the identity of points A and B and on their geographical relationship. These are given in the following table.

Formulas for calculating the azimuth.

At least one point in the northern hemisphere			
Sub-satellite point	Earth Station	Relation	Azimuth
A	B	A west of B	360 - Y
B	A	A west of B	X
A	B	B west of A	Y
B	A	B west of A	360 - X

Both points in the southern hemisphere			
Sub-satellite point	Earth Station	Relation	Azimuth
A	B	A west of B	180 + Y
B	A	A west of B	180 - X
A	B	B west of A	180 - Y
B	A	B west of A	180 + X

CALCULATION OF LOOK ANGLES
FOR GEO-STATIONARY SATELLITES

Sub-satellite point is at the equator. → therefore $L_s = 0$.

The geo-synchronous radius r_s = 42242 Km

The earth's radius re = 6370 Km

The central angle γ is given by:

$$\cos(\gamma) = \cos(L_e)\cos(l_s - l_e)$$

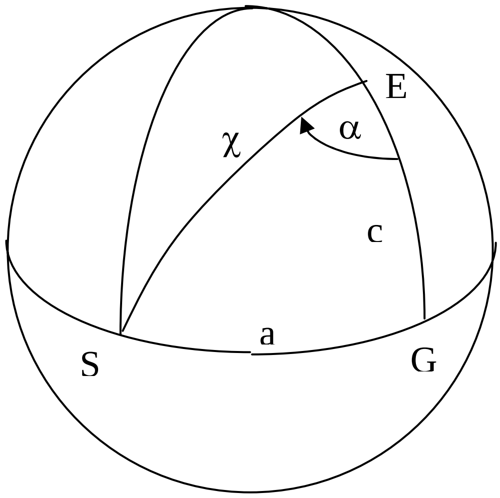
The distance d from the earth station to the satellite is given by:

$$d = 42242[1.02274 - .301596\cos(\gamma)]^{1/2} \quad Km$$

The elevation angle is then given by:

$$\cos(El) = \frac{\sin(\gamma)}{[1.02274 - 0.301596\cos(\gamma)]^{1/2}}$$

zimuth calculation is simpler than the general case, because the sub-satellite point lies on the equator. We refer to the following figure for this calculation.



$$a = |l_s - l_e|$$

$$c = |L_e - L_s|$$

Considering the half perimeter of

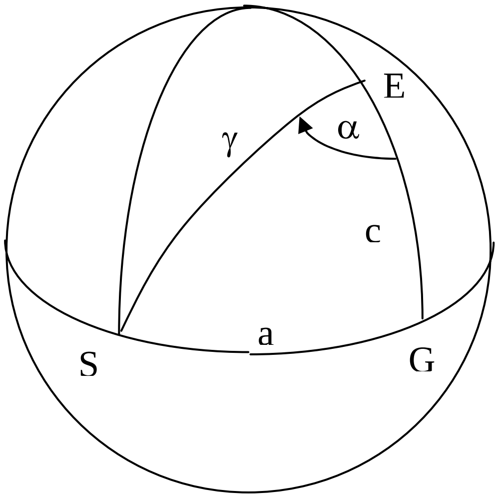
the triangle = s

$$\therefore s = 0.5(a + c + \gamma)$$

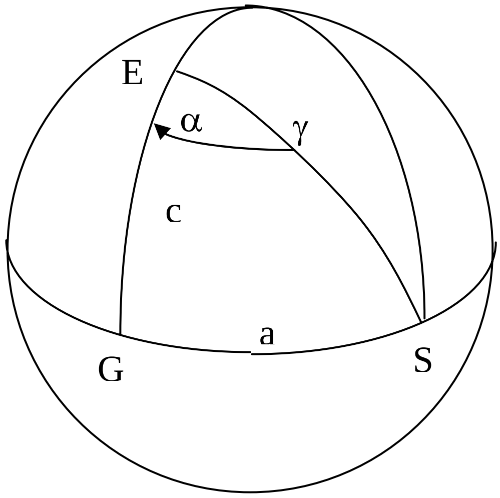
The angle a at the vertex may be obtained from:

$$\tan^2\left(\frac{\alpha}{2}\right) = \frac{\sin(s-\gamma)\sin(s-c)}{\sin(s)\sin(s-a)}$$

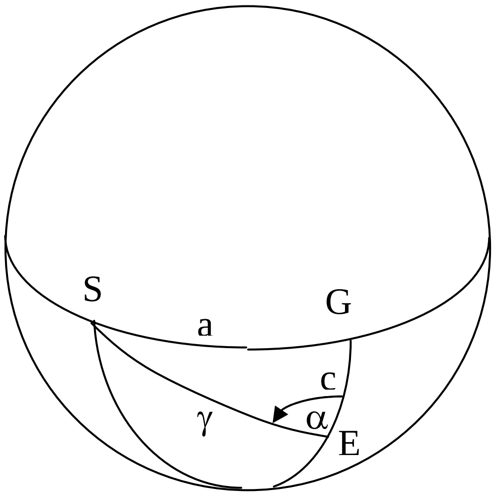
and $\alpha = 2\tan^{-1}\sqrt{\dfrac{\sin(s-\gamma)\sin(s-|L_e|}{\sin(s)\sin(s-|l_e - l_s|}}$



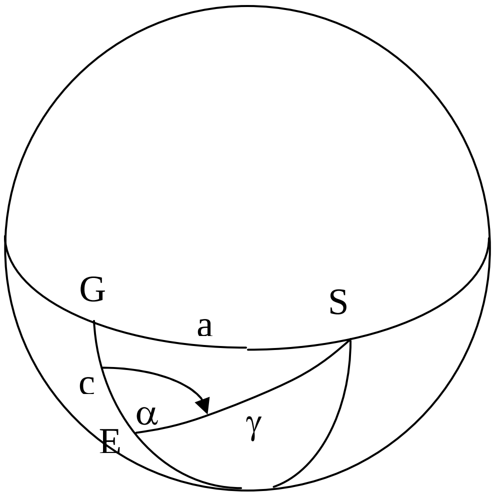
SSP south-west of ES



SSP south-east of ES



SSP north-west of ES



SSP north-east of ES

Equations for calculating azimuth from spherical triangle angle α

SSP → Sub-satellite point

ES → Earth Station

Situation	Equation
1. SSP South-west of ES	$Az = 180^{o} + \alpha$
2. SSP South-east of ES	$Az = 180^{o} - \alpha$
3. SSP North-west of ES	$Az = 360^{o} - \alpha$
4. SSP North-east of ES	$Az = \alpha$