SATELLITE PATH IN SPACE

Assumptions:

1. The satellite and earth are symmetric, spherical and therefore may be treated as point masses.
2. No forces other than their gravitational forces act on the system.
3. The mass of the earth is much greater than that of the satellite.

Equation of motion may be formulated:

$$F = -\frac{GM_E m \hat{r}}{r^2} \quad , \qquad F = m\frac{d^2 r}{dt^2}\hat{r}$$

$$\therefore -\frac{\mu \hat{r}}{r^2} = \frac{d^2 r}{dt^2}\hat{r} \quad \rightarrow \quad \frac{1}{r}\frac{d^2 r}{dt^2} + \frac{\mu r}{r^3} = 0$$

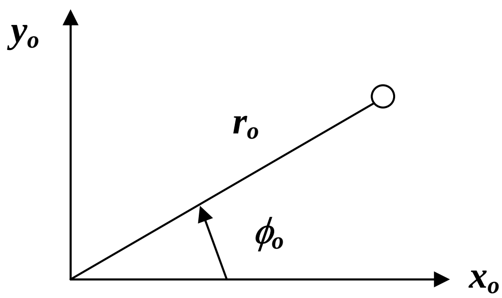
Taking r **X** with each term $\quad \rightarrow \quad r \times \dfrac{d^2 r}{dt^2} = 0$

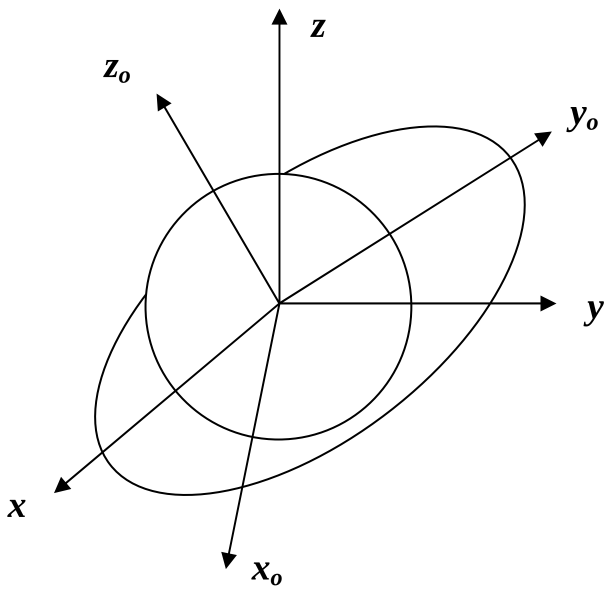
But $$\frac{d}{dt}[r \times \frac{dr}{dt}] = \frac{dr}{dt} \times \frac{dr}{dt} + r \times \frac{d^2 r}{dt^2}$$

$$\therefore \frac{d}{dt}[r \times \frac{dr}{dt}] = 0 \quad \text{or} \quad r \times \frac{dr}{dt} = h = \text{Orbital angular momentum.}$$

Orbital angular momentum can only be constant if the orbit lies in a plane.

To simplify the analysis, we use the orbital plane co-ordinate system:





Using the rectangular to polar transformation, we obtain the equation relating r_o and ϕ_o

$$r_o = \frac{1}{\frac{\mu}{h^2} + C\cos(\phi_o - \theta_o)} = \frac{(\frac{h^2}{\mu})}{1 + (\frac{h^2}{\mu})C\cos(\phi_o - \theta_o)}$$

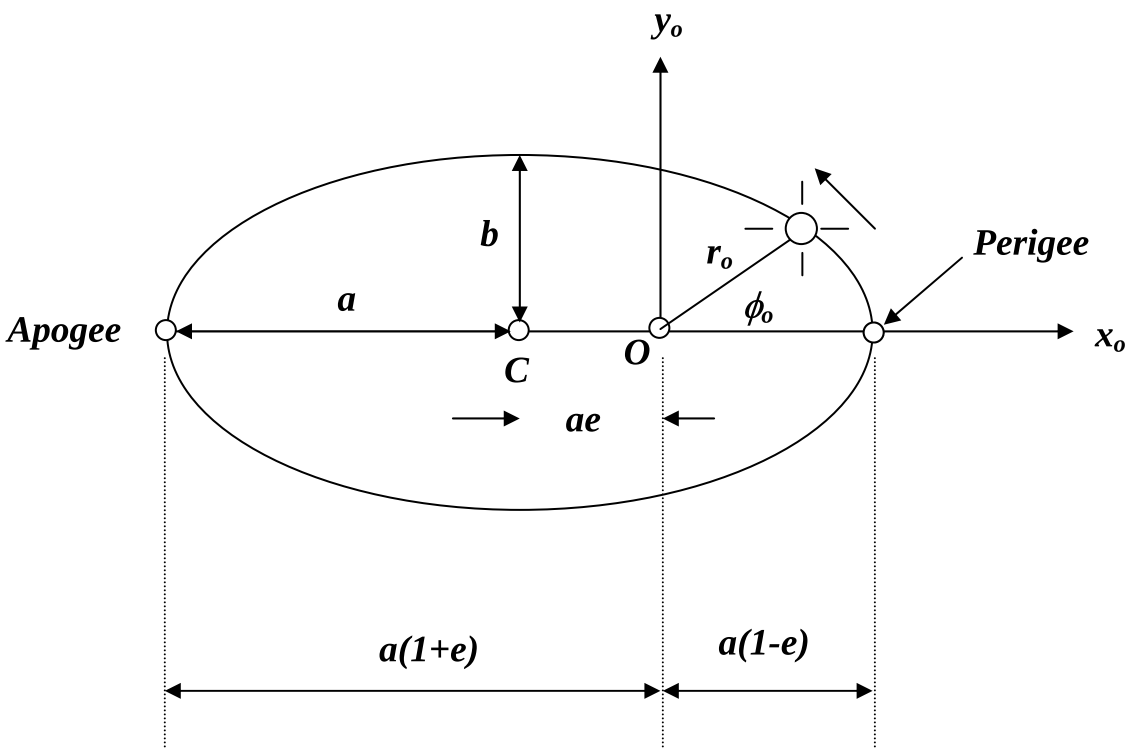
C and θ_o are constants.

$$r_o = \frac{p}{1+e\cos(\phi_o - \theta_o)}$$

Where $0 \le e < 1$ for elliptical path. The path is circular if e=0.

e is the eccentricity and is given by $e = \frac{h^2 C}{\mu}$ and $p = \frac{h^2}{\mu}$.

THE ORBIT DESCRIPTION:



Θo is taken = 0. So that xo coincides with the major axis.

$$r_o = \frac{p}{1+e\cos\phi_o}$$

$$a = \frac{p}{1-e^2} \qquad \text{and} \qquad b = a(1-e^2)^{1/2}$$

eccentricity $e = \sqrt{1-(\frac{b}{a})^2}$

SATELLITE PERIOD:

$$T^2 = \frac{4\pi^2 a^3}{\mu}$$

We may use this expression to calculate the radius of a geo-synchronous circular orbit.

If T = 86,400 Se. → a = 42,241.558 Km.

A geo-synchronous orbit that lies in the earth's equatorial plane (having zero inclination) is geo-stationary.

For a satellite in a circular orbit around the earth, we have:

$$T^2 = \frac{4\pi^2 (R_E + h)^3}{\mu}$$

Where R_E is the earth's radius & *h is the satellite altitude.*

LOCATING THE SATELLITE IN THE ORBIT

$$r_o = \frac{p}{1+e\,\cos\phi_o} = \frac{a(1-e^2)}{1+e\,\cos\phi_o}$$

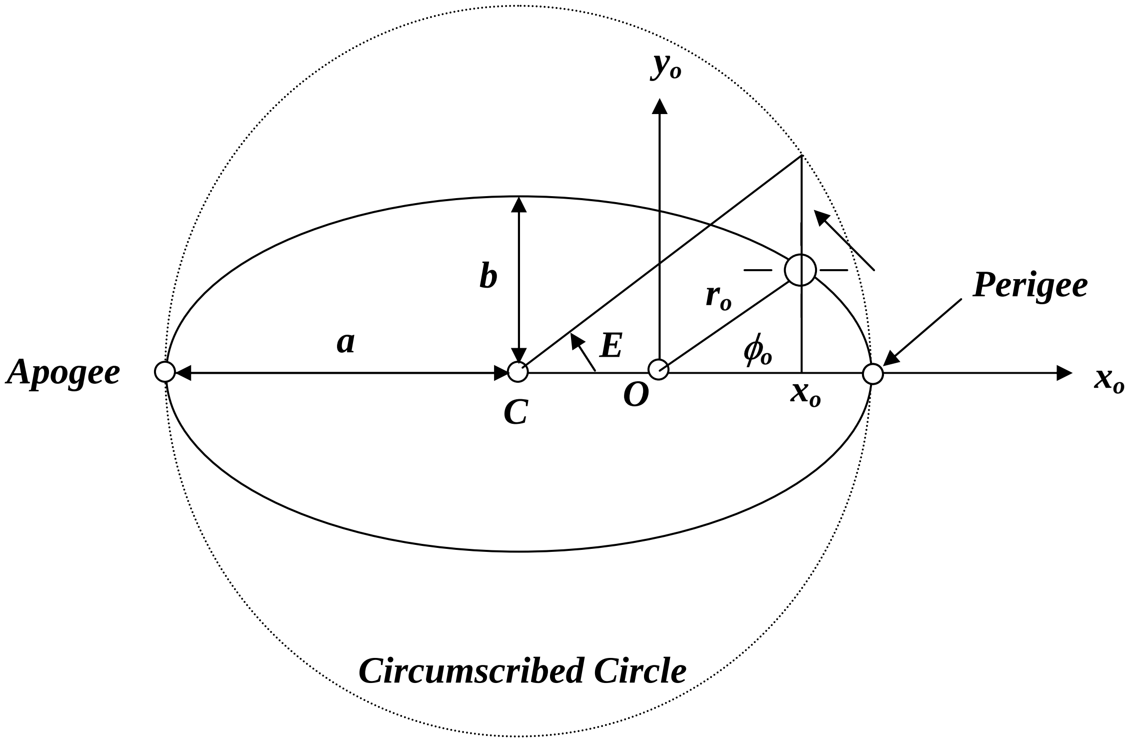
ϕ_o is measured from the x_o axis and is called the *true anomaly*.

The rectangular co-ordinates of the satellite are given by:

$x_o = r_o\ \cos\phi_o$ and $y_o = r_o\ \sin\phi_o$

The satellite average angular velocity is given by:

$\eta = \frac{2\pi}{T} = \sqrt{\frac{\mu}{a^3}}$ The time required for the satellite moving with this angular velocity to go around any circle is T sec.



$$v^2 = (\frac{dx_o}{dt})^2 + (\frac{dy_o}{dt})^2 \qquad = (\frac{dr_o}{dt})^2 + r_o{}^2 (\frac{d\phi_o}{dt})^2$$

It can be shown that $v^2 = (\frac{\mu}{a})(\frac{2a}{r_o} - 1)$

We also had : $r_o{}^2 (\frac{d\phi_o}{dt})^2 = \frac{h^2}{r_o{}^2} = \frac{\mu p}{r_o{}^2} = \frac{\mu a(1-e^2)}{r_o{}^2}$

$$\therefore (\frac{\mu}{a})(\frac{2a}{r_o} - 1) = (\frac{dr_o}{dt})^2 + (\frac{\mu a}{r_o{}^2})(1-e^2)$$

and $\frac{dr_o}{dt} = \{(\frac{\mu}{a r_o{}^2})[a^2 e^2 - (a - r_o)^2]\}^{\frac{1}{2}}$

Solving for *dt* and multiplying by the mean angular velocity we get :

$$\eta\, dt = (\frac{r_o}{a}) \frac{dr_o}{[a^2 e^2 - (a - r_o)^2]^{1/2}}$$

Angle E is called eccentric anomaly and is related to the radius r_o by :

$$r_o = a - ae\cos E \qquad \rightarrow \qquad a - r_o = ae\cos E$$

$$\therefore \eta\, dt = (1 - e\cos E)\, dE$$

If t_p is the time of perigee, then integrating the last equation, we get:

$$\eta(t-t_p)=E-e\sin E$$

$\eta(t-t_p) \quad \Rightarrow \quad M$ called the mean anomaly.

ϕ_o is the true anomaly.

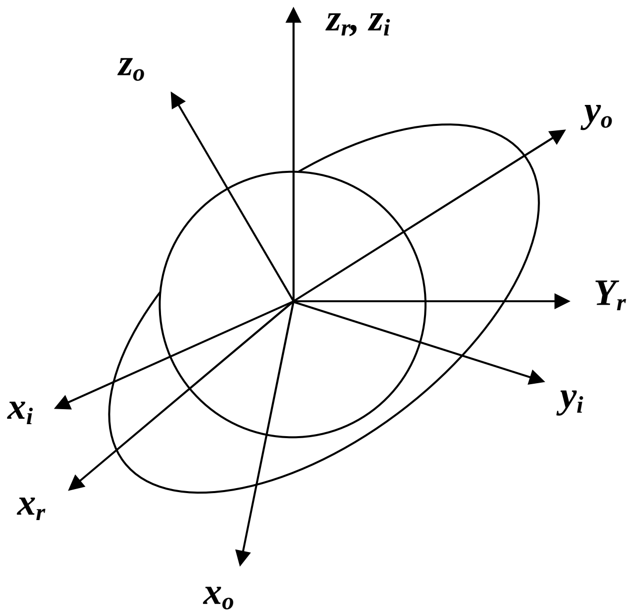
Provided that we know time of perigee (t_p), the eccentricity(e), and semimajor axis(a), then we have all the equations to determine the coordinates of the satellite in the orbital plane:

PROCEDURE:

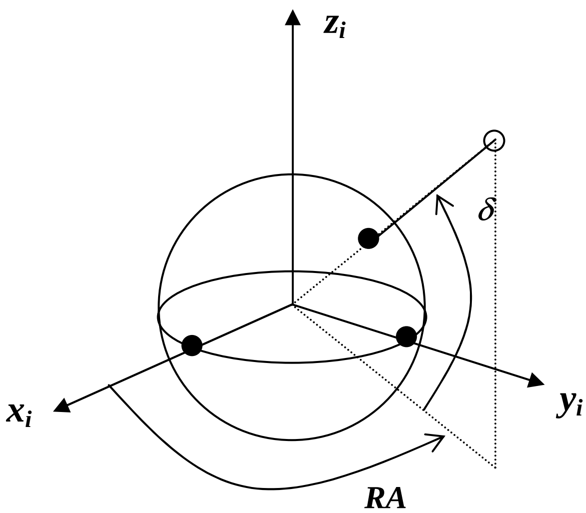
1. Calculate the average angular velocity from
$$\eta=\frac{2\pi}{T}=\sqrt{\frac{\mu}{a^3}}$$
2. Calculate the mean anomaly from $M=\eta(t-t_p)$
3. Find the eccentric anomaly from $M=E-e\sin E$
4. Find r_o from $a-r_o=ae\cos E$
5. Find ϕ_o from $r_o=\dfrac{A(1-e^2)}{1+e\cos\phi_o}$
6. x_o *and* y_o *can be found from*

$$x_o=r_o\cos\phi_o \qquad \text{and} \qquad y_o=r_o\sin\phi_o$$

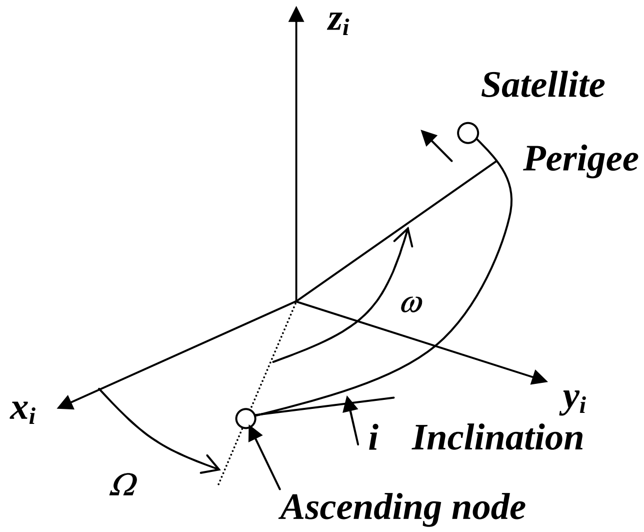
LOCATING THE SATELLITE WITH RESPECT TO THE EARTH



$x_i\ ,\ y_i\ , and\ z_i$	Geocentric equatorial co-ordinate system.
$x_o\ ,\ y_o\ , and\ z_o$	Orbital plane co-ordinate system.
$x_r\ ,\ y_r\ , and\ z_r$	Rotating co-ordinate system.



The geocentric equatorial system



Ω is the right ascension of the ascending node.

ω is the argument of perigee in the orbital plane.