EE 204
Lecture 12
Nodal Analysis with Voltage Sources

Voltage Source Connected to the Reference Node

This case is illustrated by an example.

Example 1:

Calculate the nodal voltages V_1 , V_2 , V_3 .

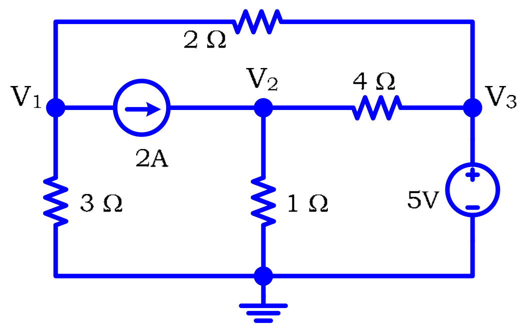

Figure 1

Solution:

Nodes 1 & 2 $\Rightarrow$ No voltage sources connected $\Rightarrow$ No special treatment

Node 3 $\Rightarrow$ Voltage source connected $\Rightarrow$ Needs special treatment

KCL at node 1 $\Rightarrow$ $\frac{V_1 - 0}{3} + 2 + \frac{V_1 - V_3}{2} = 0$ $\Rightarrow$ $5V_1 - 3V_3 = -6$ (1)

KCL at node 2 $\Rightarrow$ $-2 + \frac{V_2 - 0}{1} + \frac{V_2 - V_3}{4} = 0$ $\Rightarrow$ $5V_2 - V_3 = 8$ (2)

KCL at node 3 $\Rightarrow$ $\frac{V_3 - V_2}{4} + \frac{V_3 - V_1}{2} + i_x = 0$ (problem!)

i_x *cannot directly* be replaced with nodal voltages, because *Ohm's Law does not apply to voltage sources*.

We have three unknowns (V_1 & V_2 & V_3) $\Rightarrow$ we need 3 equations $\Rightarrow$ 1 equation is missing!

For node 3, the basic Nodal Analysis procedure *must be* revised.

The $5V$ source is connected to the reference node

$\Downarrow$

$$KVL \quad \Rightarrow \quad V_3 - 0 = 5 \quad \Rightarrow \quad V_3 = 5 \qquad (3)$$

The missing 3rd equation is *easy* to obtain, using **KVL** between node 3 and the reference node.

$5V_1 - 3V_3 = -6$ (1) Using the regular Nodal Analysis procedure

$5V_2 - V_3 = 8$ (2) Using the regular Nodal Analysis procedure

$V_3 = 5$ (3) Using KVL

Solving the above set $\Rightarrow$ $V_1 = 1.8V$ & $V_2 = 2.6V$ & $V_3 = 5V$

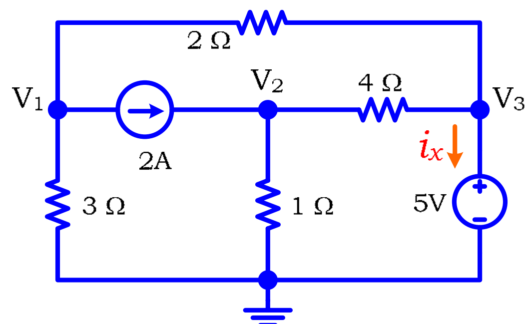

Figure 2

Voltage source connected to reference $\Rightarrow$ Use KVL only (do not use KCL)

Voltage Source *not* Connected to the Reference Node

Example 2:

Calculate the nodal voltages V_1 , V_2 , V_3 .

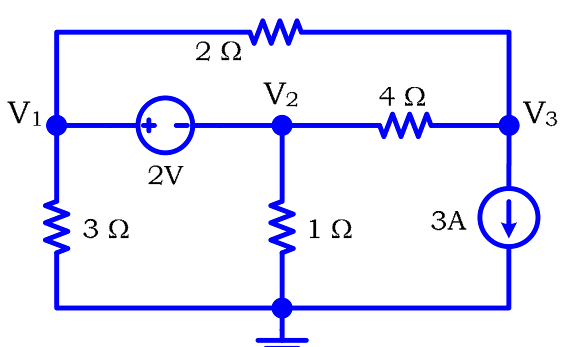

Figure 3

Solution:

KCL at node 1 $\Rightarrow$ $\frac{V_1 - 0}{3} + i_x + \frac{V_1 - V_3}{2} = 0$ $\Rightarrow$ (problem!)

KCL at node 2 $\Rightarrow$ $-i_x + \frac{V_2}{1} + \frac{V_2 - V_3}{4} = 0$ $\Rightarrow$ (another problem!)

The 2V sources is connected to nodes 1 & 2 $\Rightarrow$ KCL at nodes 1& 2 contains i_x

We need two equations, one equation for each node.

How should we proceed now?

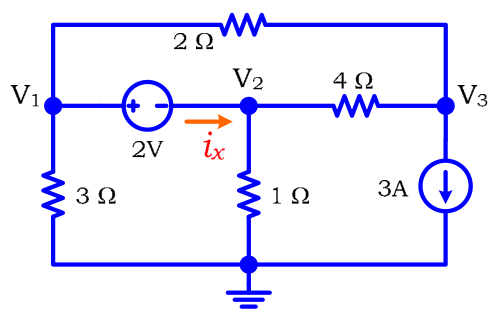

Figure 4

1) Draw a super node around nodes 1 & 2

2) KCL at the super node $\Rightarrow$ $\frac{V_1 - 0}{3} + \frac{V_2 - 0}{1} + \frac{V_2 - V_3}{4} + \frac{V_1 - V_3}{2} = 0$ (Avoids i_x)

$$\Downarrow$$

$$10V_1 + 9V_2 - 3V_3 = 0 \qquad (1)$$

3) KVL $\Rightarrow$

$$V_1 - V_2 = 2 \qquad (2)$$

We obtained the required number of equations.

KCL at node 3 $\Rightarrow$ $\frac{V_3 - V_2}{4} + 3 + \frac{V_3 - V_1}{2} = 0$ $\Rightarrow$ $-2V_1 - V_2 + 3V_3 = -24$ (3)

Solving (1), (2) & (3) $\Rightarrow$ $V_1 = -0.5V$ & $V_2 = -2.5V$ & $V_3 = -9.17V$

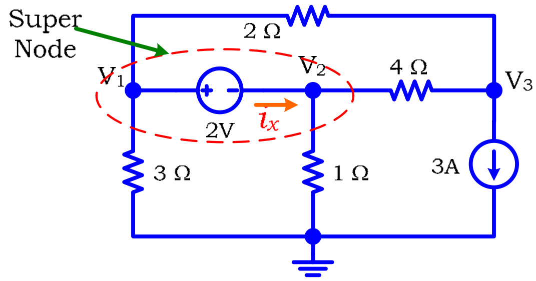

Figure 5

Voltage source *not* connected to reference $\Rightarrow$ KCL at super node & KVL

Intersecting Super Nodes

Example 3:

Calculate the nodal voltages V_1 , V_2 , V_3 .

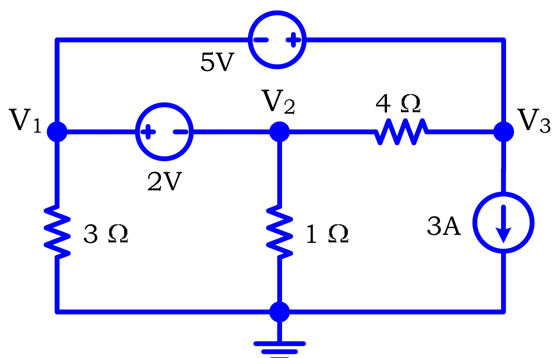

Figure 6

Solution:

Nodes 1 & 2 are connected by the 2V source $\Rightarrow$ draw super node 1

Nodes 1 & 3 are connected by the 5V source $\Rightarrow$ draw super node 2

If we apply KCL at super node 1 $\Rightarrow$ KCL contains current through the 5V source!

If we apply KCL at super node 2 $\Rightarrow$ KCL contains current through the 2V source!

Do not apply KCL at nodes 1 or 2.

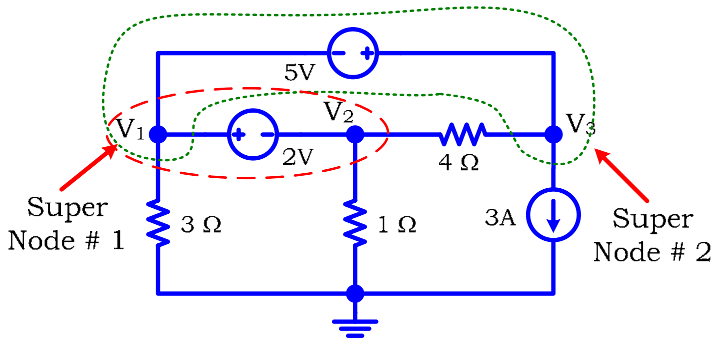

Figure 7

Combine the two super nodes into a single super node.

KCL at the new super node

$$\Downarrow$$

$$\frac{V_1 - 0}{3} + \frac{V_2 - 0}{1} + \frac{V_2 - V_3}{4} + \frac{V_3 - V_2}{4} + 3 = 0$$

$$\Downarrow$$

$$\frac{V_1 - 0}{3} + \frac{V_2 - 0}{1} + 3 = 0$$ (*actual* current through 4Ω leaves and enters the super node)

$$\Downarrow$$

$$V_1 + 3V_2 = -9 \qquad (1)$$

To obtain the remaining two equations $\Rightarrow$ Apply KVL

KVL $\Rightarrow$ $V_1 - V_2 = 2$ (2)

KVL $\Rightarrow$ $V_1 - V_3 = 5$ (3)

Solving (1) & (2) & (3) $\Rightarrow$ $V_1 = -0.75V$ & $V_2 = -2.75V$ & $V_3 = -5.75V$

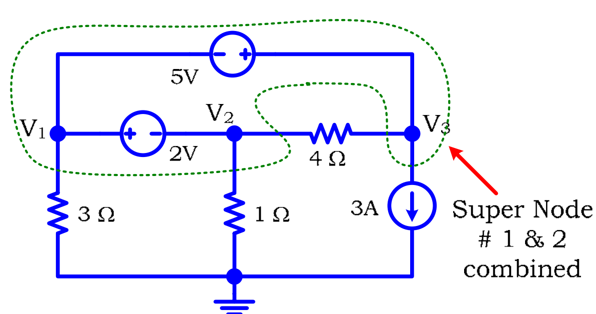

Figure 8

Every voltage source provides *one nodal equation* using KVL

Strategy A

$\Downarrow$

1) Apply KVL to *every* voltage source

2) If two super nodes intersect $\Rightarrow$ *join them*

3) Apply KCL at a node, super node, or combined super node (*only as necessary*)

Let us apply strategy A to the next example.

Example 4:

Write down the nodal equations (do not simplify and do not solve).

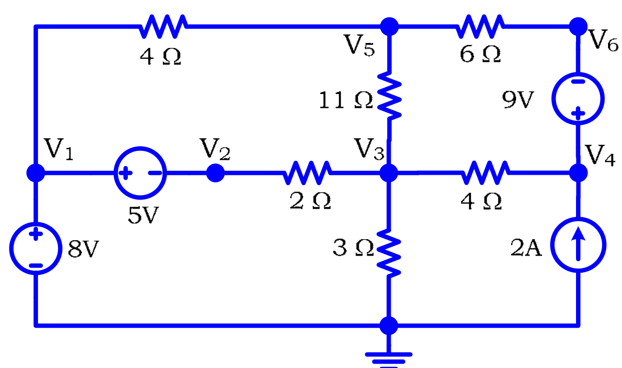

Figure 9

Solution:

KVL for every voltage source $\Rightarrow$

$V_1 = 8$ (1)

$V_1 - V_2 = 5$ (2)

$V_4 - V_6 = 9$ (3)

Three more equation required $\Rightarrow$ Apply KCL

(We *already* obtained *two* equations from nodes 1 & 2 $\Rightarrow$ No KCL at nodes 1 & 2)

KCL at node 3 $\Rightarrow$ $$\frac{V_3 - V_2}{2} + \frac{V_3}{3} + \frac{V_3 - V_4}{4} + \frac{V_3 - V_5}{11} = 0 \quad \Rightarrow \quad (4)$$

KCL at node 5 $\Rightarrow$ $$\frac{V_5 - V_1}{4} + \frac{V_5 - V_3}{11} + \frac{V_5 - V_6}{6} = 0 \quad \Rightarrow \quad (5)$$

One more equation is required!

KCL at super node $\Rightarrow$ $$\frac{V_4 - V_3}{4} - 2 + \frac{V_6 - V_5}{6} = 0 \quad \Rightarrow \quad (6)$$

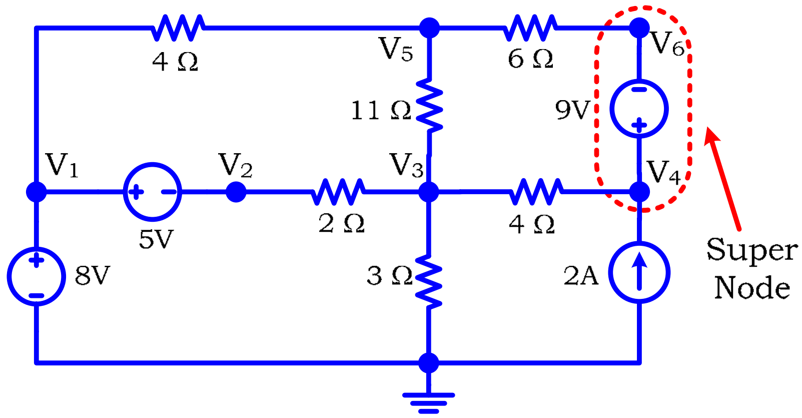

Figure 10

In the previous example, we *did not* apply KCL at the S.N. 1, or S.N. 2 because we *do not need* KCL there (since we have enough equations from nodes 1 & 2).

Another way to think about this point:

S. N. 1 contains current through the 8V source $\Rightarrow$ Do not apply KCL at S.N. 1

S. N. 2 contains current through the 5V & 8V sources $\Rightarrow$ Do not apply KCL at S.N. 2

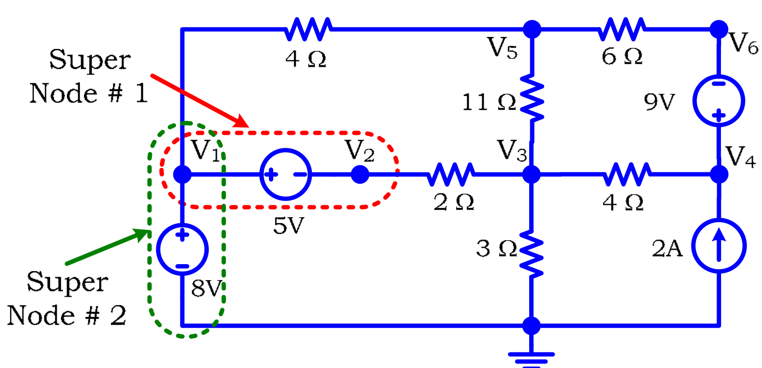

Figure 11

Combine S.N. 1 & 2 into a new S.N.

The new S.N. *still* contains current through the 8V source $\Rightarrow$ Do not apply KCL at the new S.N.!!

The *basic* reason for this is that the new S.N. contains the *reference* node.

IMPORTANT

$\Downarrow$

Super node contains the *reference* node $\Rightarrow$ Do not apply KCL

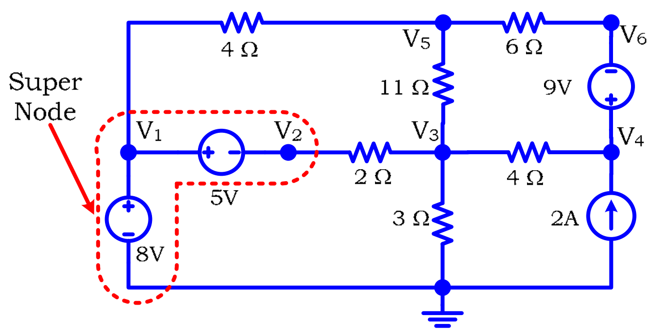

Figure 12

Strategy B

$\Downarrow$

1) Apply KVL to *every* voltage source

2) Draw a super node around *every* voltage source

3) If two super nodes intersect $\Rightarrow$ *join them*

4) Do *not* apply KCL to *any* super node that contains the *reference* node

5) Apply KCL to the remaining nodes and super nodes

Strategies A and B are *basically* the same, they produce *exactly* the same set of equations.