Image Enhancement in the Frequency Domain
Part III

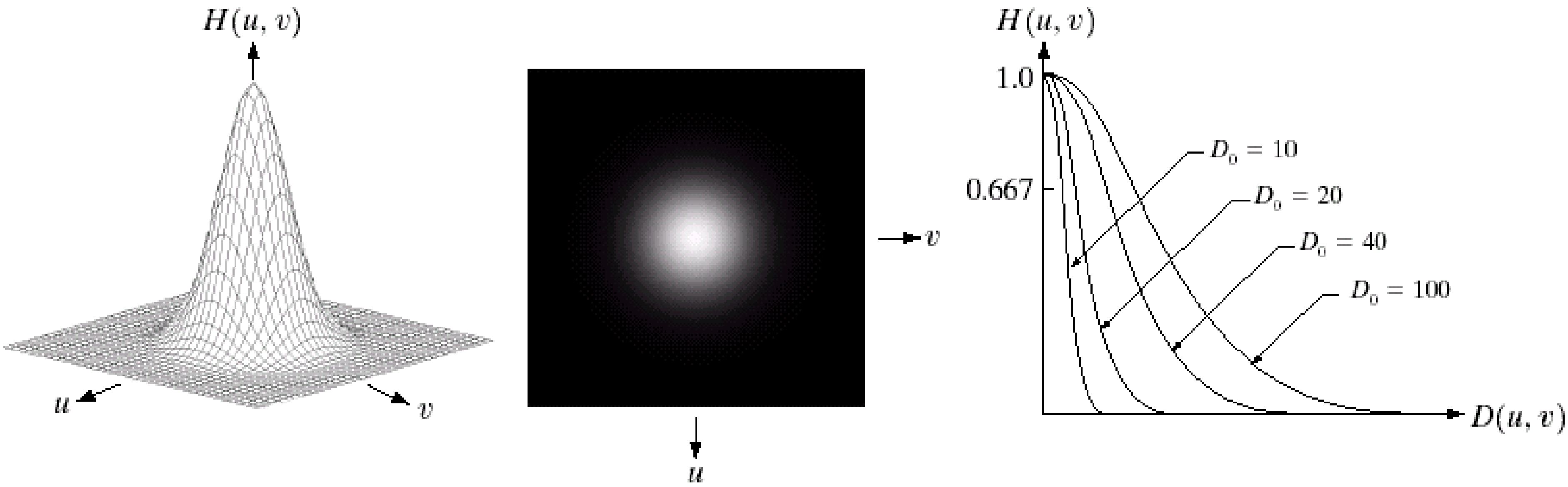
Dr. Samir H. Abdul-Jauwad
Electrical Engineering Department
College of Engineering Sciences
King Fahd University of Petroleum & Minerals
Dhahran – Saudi Arabia

Gaussian Lowpass Filter

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

- D(u,v): distance from the origin of FT
- Parameter: $\sigma = D_0$ (cutoff frequency)
- The inverse FT of the Gaussian filter is also a Gaussian

Image Enhancement in the Frequency Domain



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

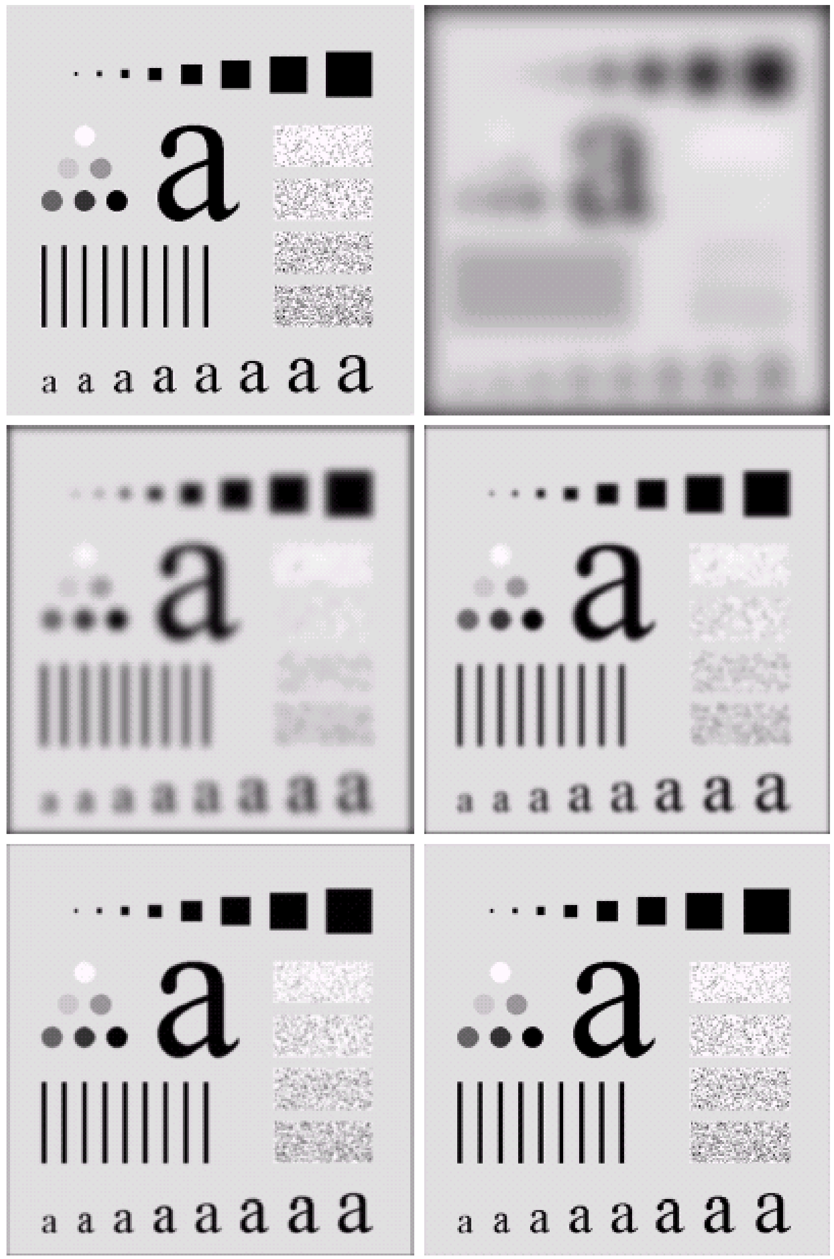


FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

a	b
c	d
e	f

Image Enhancement in the Frequency Domain

a b

FIGURE 4.19
(a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

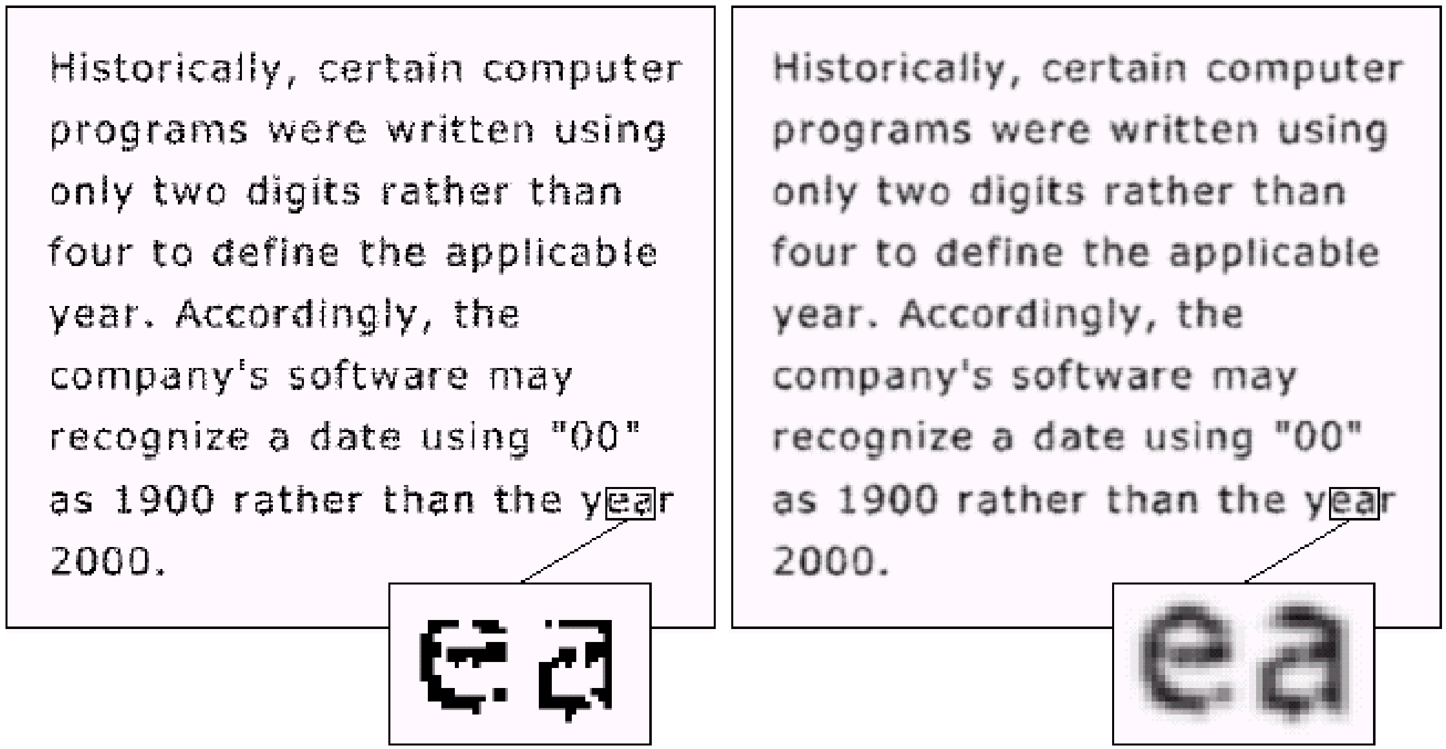


Image Enhancement in the Frequency Domain

FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

Image Enhancement in the Frequency Domain

a b c

FIGURE 4.21 (a) Image showing prominent scan lines. (b) Result of using a GLPF with $D_0 = 30$. (c) Result of using a GLPF with $D_0 = 10$. (Original image courtesy of NOAA.)

Sharpening (Highpass) Filtering

- Image sharpening can be achieved by a highpass filtering process, which attenuates the low-frequency components without disturbing high-frequency information.
- Zero-phase-shift filters: radially symmetric and completely specified by a cross section.

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

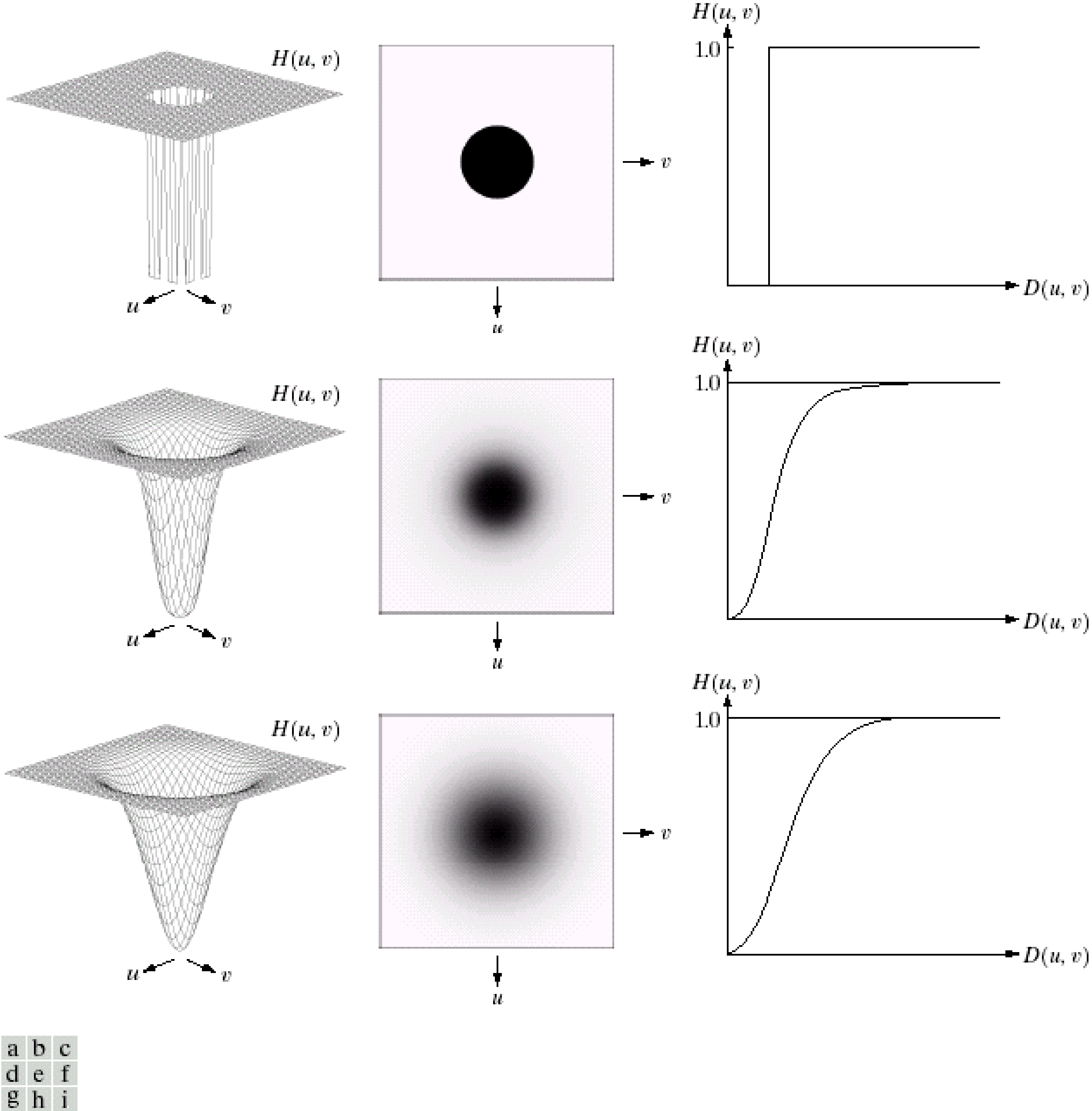


FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Ideal Filter (Highpass)

$$H(u,v) = \begin{cases} 0 & \text{if} \quad D(u,v) \le D_0 \\ 1 & \text{if} \quad D(u,v) > D_0 \end{cases}$$

- This filter is the opposite of the ideal lowpass filter.

Butterworth Filter (Highpass)

$$H(u,v) = \frac{1}{1 + [D_0 / D(u,v)]^{2n}}$$

- High-frequency emphasis: Adding a constant to a highpass filter to preserve the low-frequency components.

Gaussian Highpass Filter

$$H(u,v) = 1 - e^{-D^2(u,v)/2\sigma^2}$$

- D(u,v): distance from the origin of FT
- Parameter: $\sigma = D_0$ (cutoff frequency)

Laplacian (recap)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial^2 x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial^2 y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)$$

Laplacian in the FD

- It can be shown that:

$$\Im\left[\nabla^2 f(x,y)\right] = -(u^2 + v^2)F(u,v)$$

- The Laplacian can be implemented in the FD by using the filter

$$H(u,v) = -(u^2 + v^2)$$

- FT pair:

$$\nabla^2 f(x,y) \Leftrightarrow -[(u - M/2)^2 + (v - N/2)^2]F(u,v)$$

Laplacian in the Frequency Domain

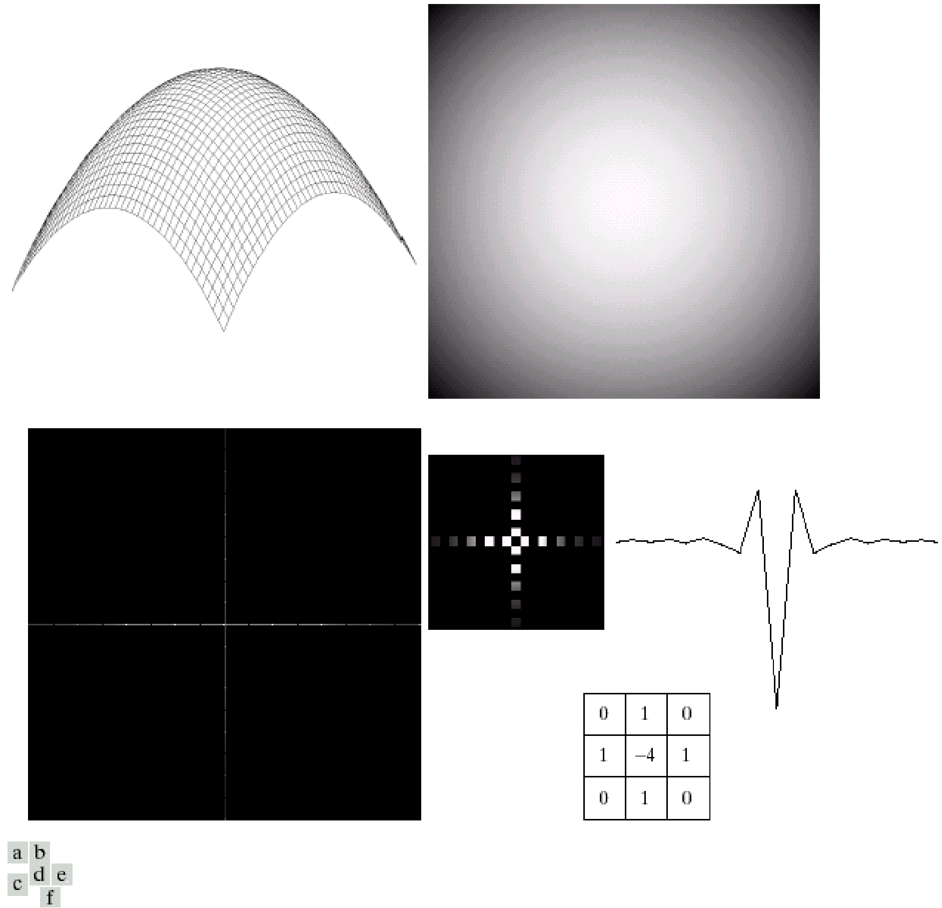


FIGURE 4.27 (a) 3-D plot of Laplacian in the frequency domain. (b) Image representation of (a). (c) Laplacian in the spatial domain obtained from the inverse DFT of (b). (d) Zoomed section of the origin of (c). (e) Gray-level profile through the center of (d). (f) Laplacian mask used in Section 3.7.

Questions?