Image Enhancement in the Frequency Domain Part II

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Spatial & Frequency Domain

$$f(x,y) * h(x,y) \Leftrightarrow F(u,v)\, H(u,v)$$

$$\delta(x,y) * h(x,y) \Leftrightarrow \Im[\delta(x,y)]\, H(u,v)$$

$$h(x,y) \Leftrightarrow H(u,v)$$

Filters in the spatial and frequency domain form a FT pair, i.e. given a filter in the frequency domain we can get the corresponding one in the spatial domain by taking its inverse FT

Image Enhancement in the Frequency Domain

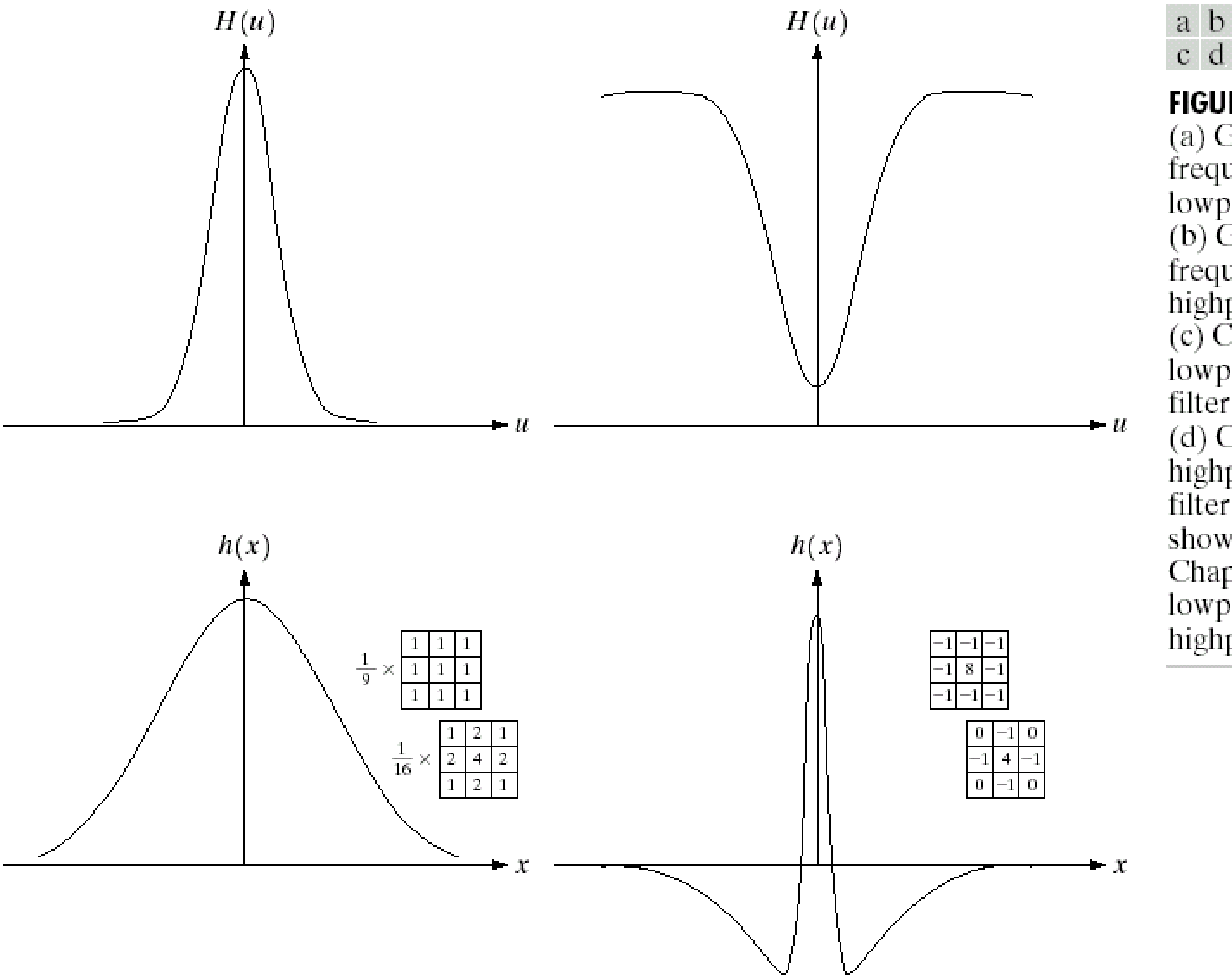


FIGURE 4.9 (a) Gaussian frequency domain lowpass filter. (b) Gaussian frequency domain highpass filter. (c) Corresponding lowpass spatial filter. (d) Corresponding highpass spatial filter. The masks shown are used in Chapter 3 for lowpass and highpass filtering.

Enhancement in the Frequency Domain

- Types of enhancement that can be done:
  - Lowpass filtering: reduce the high-frequency content -- blurring or smoothing
  - Highpass filtering: increase the magnitude of high-frequency components relative to low-frequency components -- sharpening.

Image Enhancement in the Frequency Domain

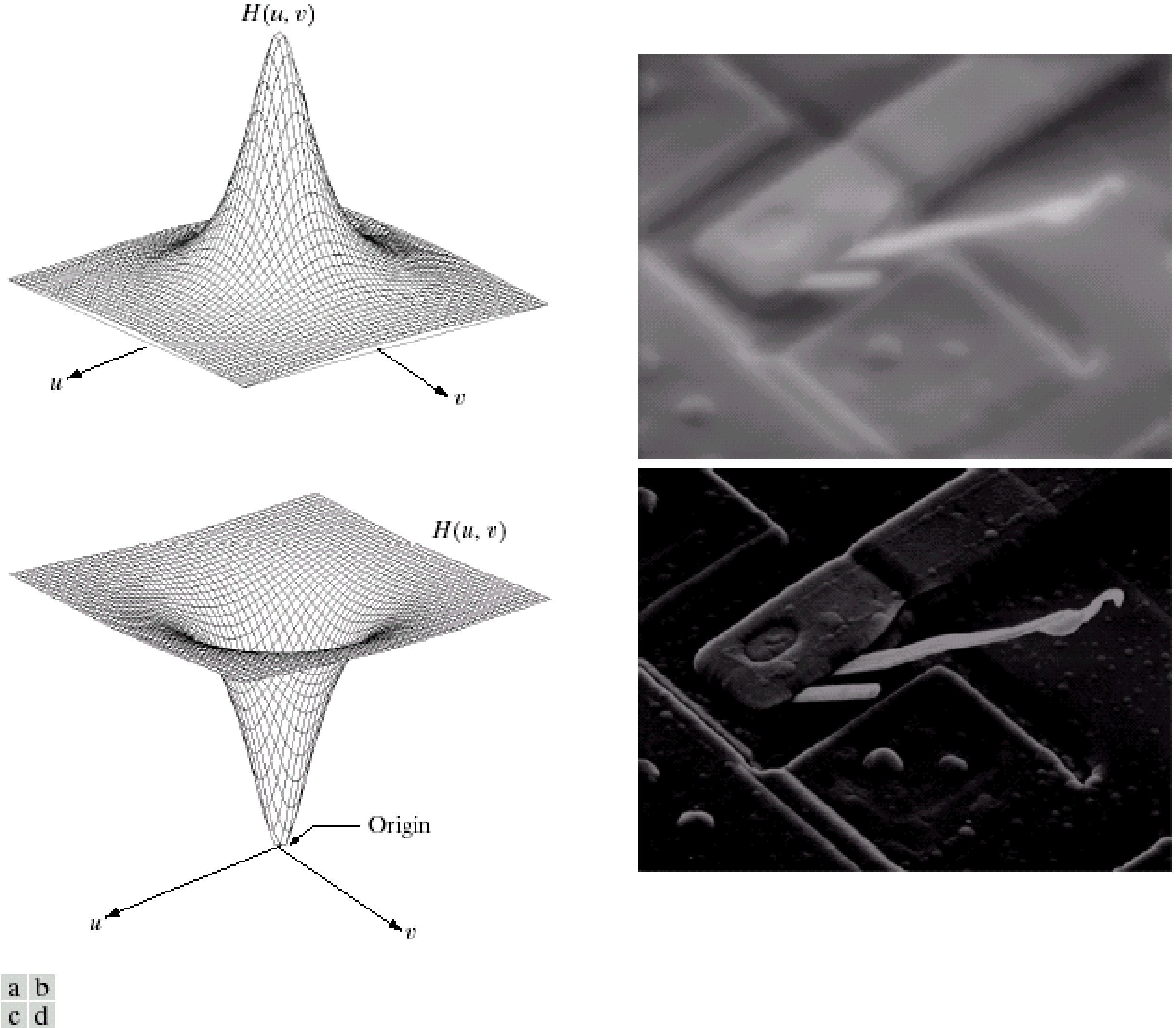


FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

Lowpass Filtering in the Frequency Domain

- Edges, noise contribute significantly to the high-frequency content of the FT of an image.
- Blurring/smoothing is achieved by reducing a specified range of high-frequency components:

$$G(u,v) = H(u,v)F(u,v)$$

Smoothing in the Frequency Domain

$$G(u,v) = H(u,v)\,F(u,v)$$

- Ideal
- Butterworth (parameter: *filter order*)
- Gaussian

Ideal Filter (Lowpass)

- A 2-D ideal low-pass filter:

$$H(u,v) = \begin{cases} 1 \text{ if } & D(u,v) \le D_0 \\ 0 \text{ if } & D(u,v) > D_0 \end{cases}$$

where D_0 is a specified nonnegative quantity and D(u,v) is the distance from point (u,v) to the center of the frequency rectangle.

- Center of frequency rectangle: (u,v)=(M/2,N/2)
- Distance from any point to the center (origin) of the FT:

$$D(u,v) = (u^2 + v^2)^{1/2}$$

Image Enhancement in the Frequency Domain

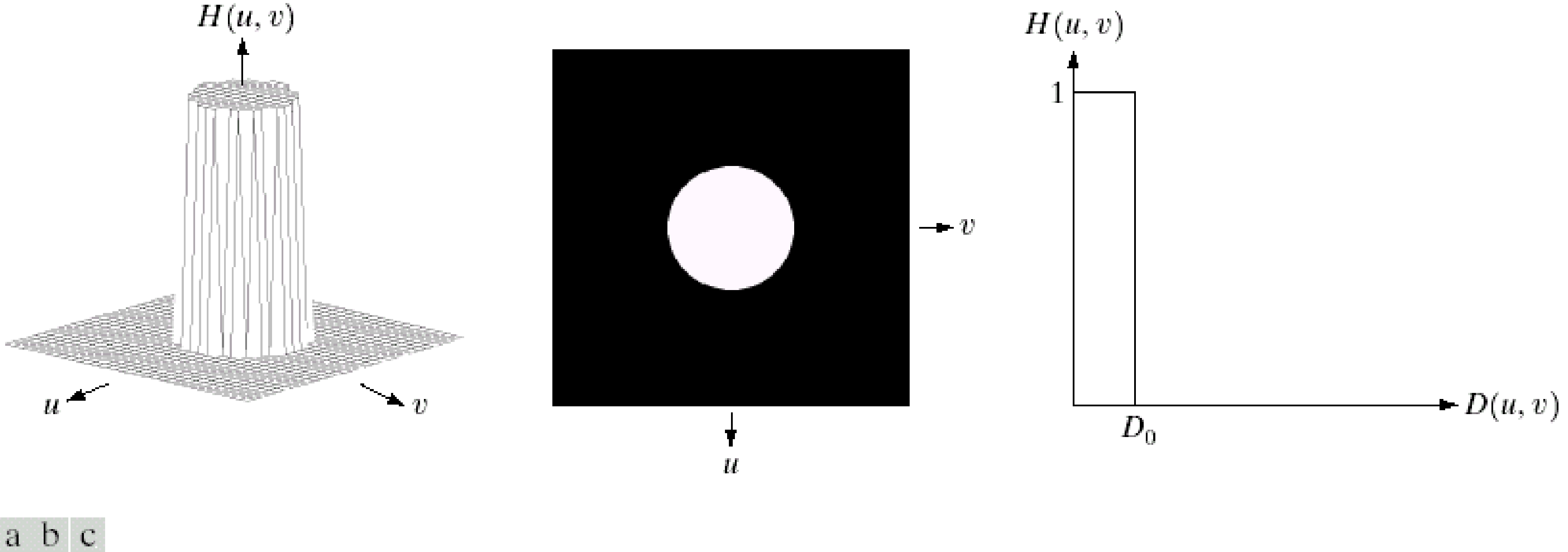


FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Ideal Filter (Lowpass)

- Ideal:
  - all frequencies inside a circle of radius D_0 are passed with no attenuation
  - all frequencies outside this circle are completely attenuated.

Ideal Filter (Lowpass)

- Cutoff-frequency: the point of transition between $H(u,v)=1$ and $H(u,v)=0$ (D_0)
- To establish cutoff frequency loci, we typically compute circles that enclose specified amounts of total image power P_T.

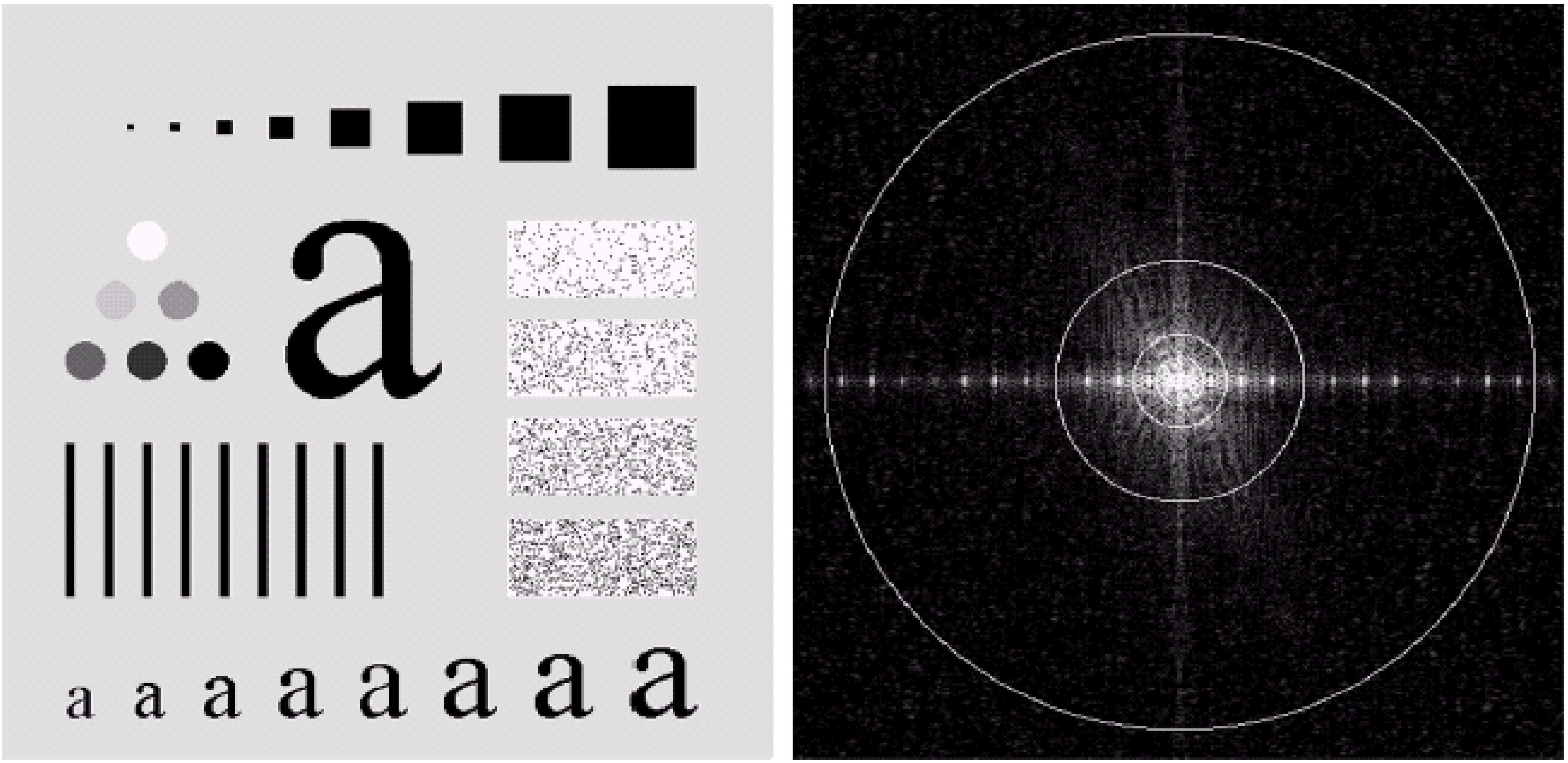
Ideal Filter *(cont.)*

- P_T is obtain by summing the components of power spectrum P(u,v) at each point for u up to M-1 and v up to N-1.
- A circle with radius *r*, origin at the center of the frequency rectangle encloses a percentage of the power which is given by the expression

- The summation is taken within the circle *r*

$$100[\sum_u \sum_v P(u,v)/P_T$$

Image Enhancement in the Frequency Domain



a b

FIGURE 4.11 (a) An image of size 500×500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

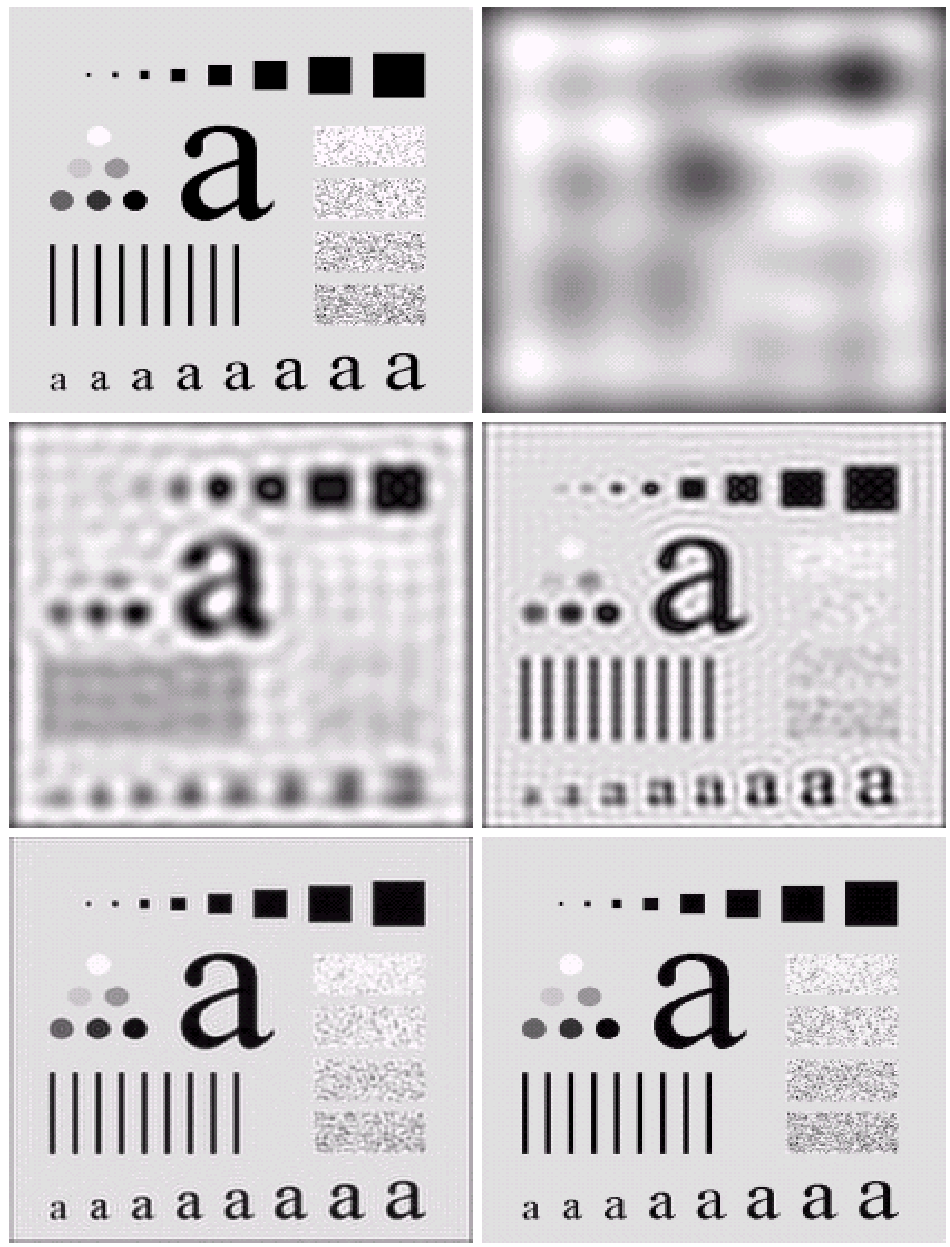


FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

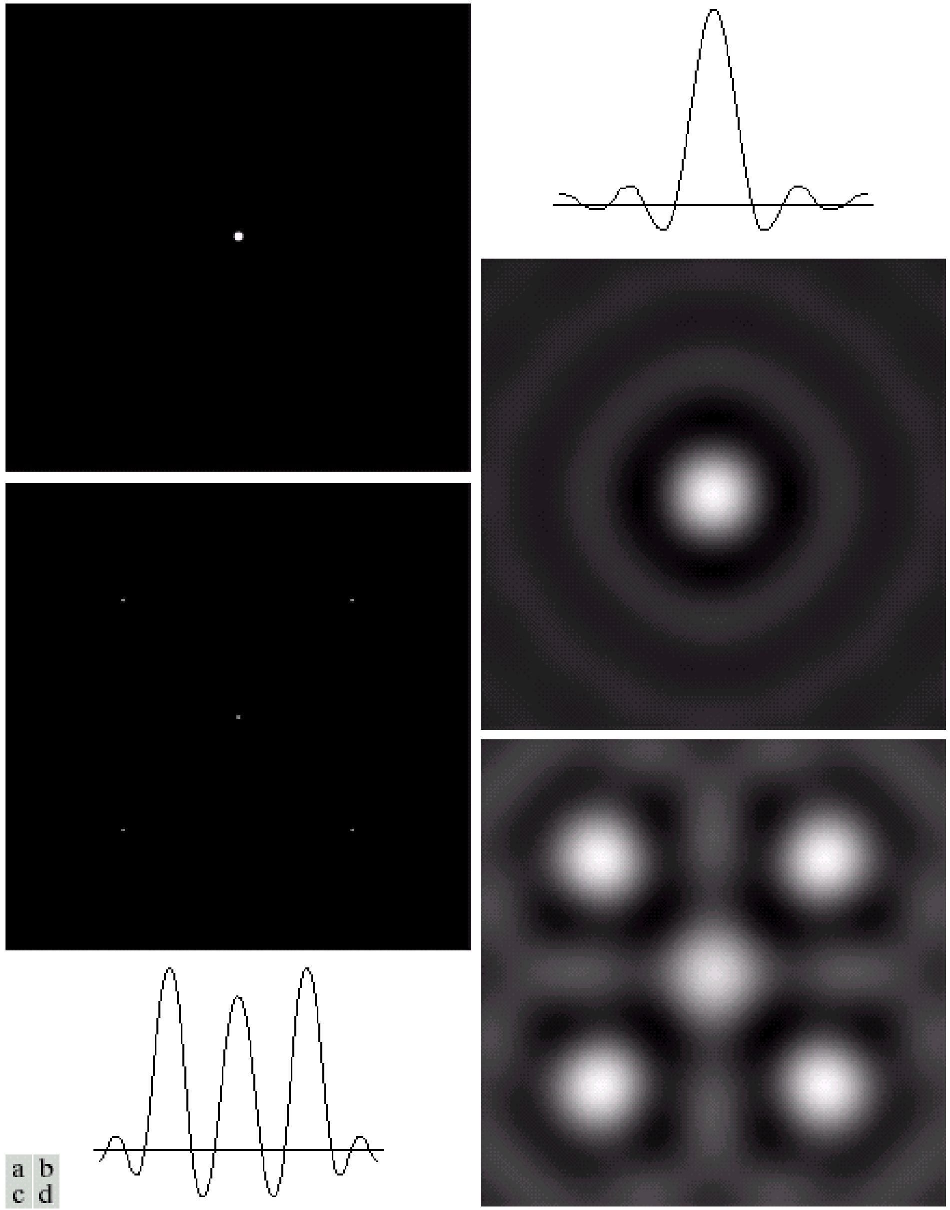


FIGURE 4.13 (a) A frequency-domain ILPF of radius 5. (b) Corresponding spatial filter (note the ringing). (c) Five impulses in the spatial domain, simulating the values of five pixels. (d) Convolution of (b) and (c) in the spatial domain.

Butterworth Filter (Lowpass)

- This filter does not have a sharp discontinuity establishing a clear cutoff between passed and filtered frequencies.

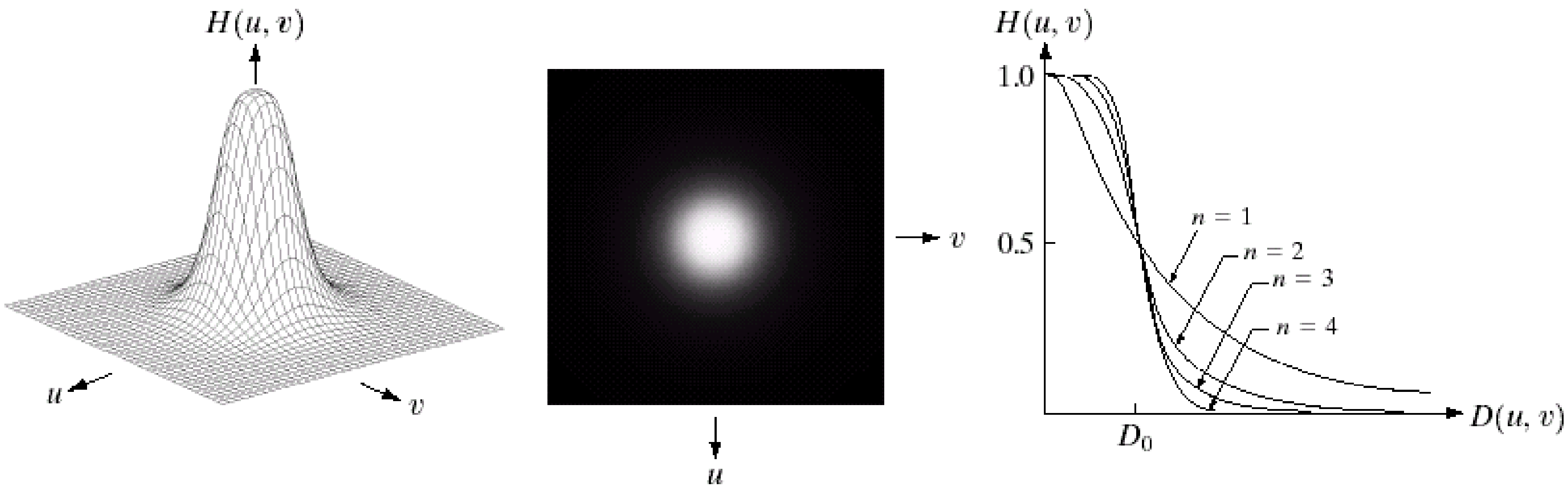
$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

Butterworth Filter (Lowpass)

- To define a cutoff frequency locus: at points for which H(u,v) is down to a certain fraction of its maximum value.

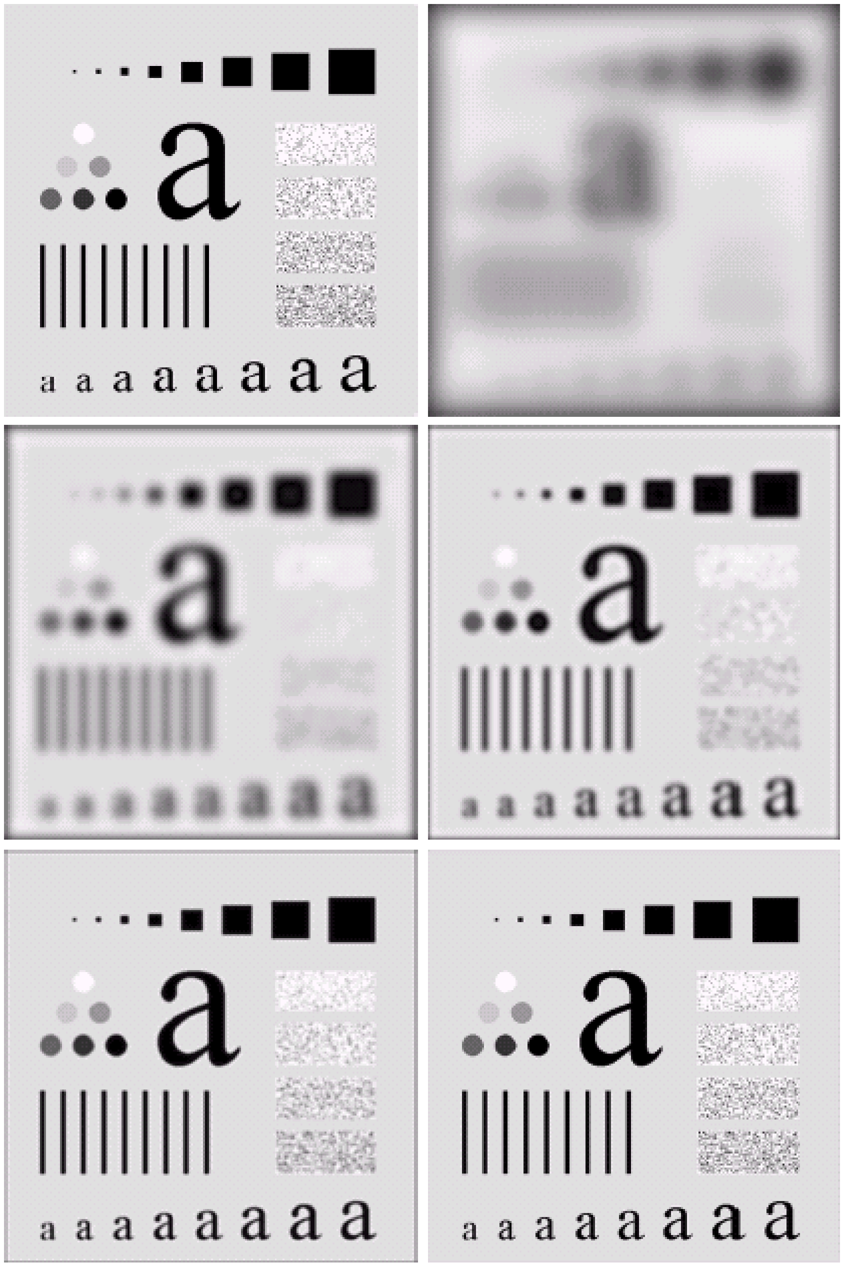
- When $D(u,v) = D_0$, $H(u,v) = 0.5$
  - i.e. down 50% from its maximum value of 1.

Image Enhancement in the Frequency Domain



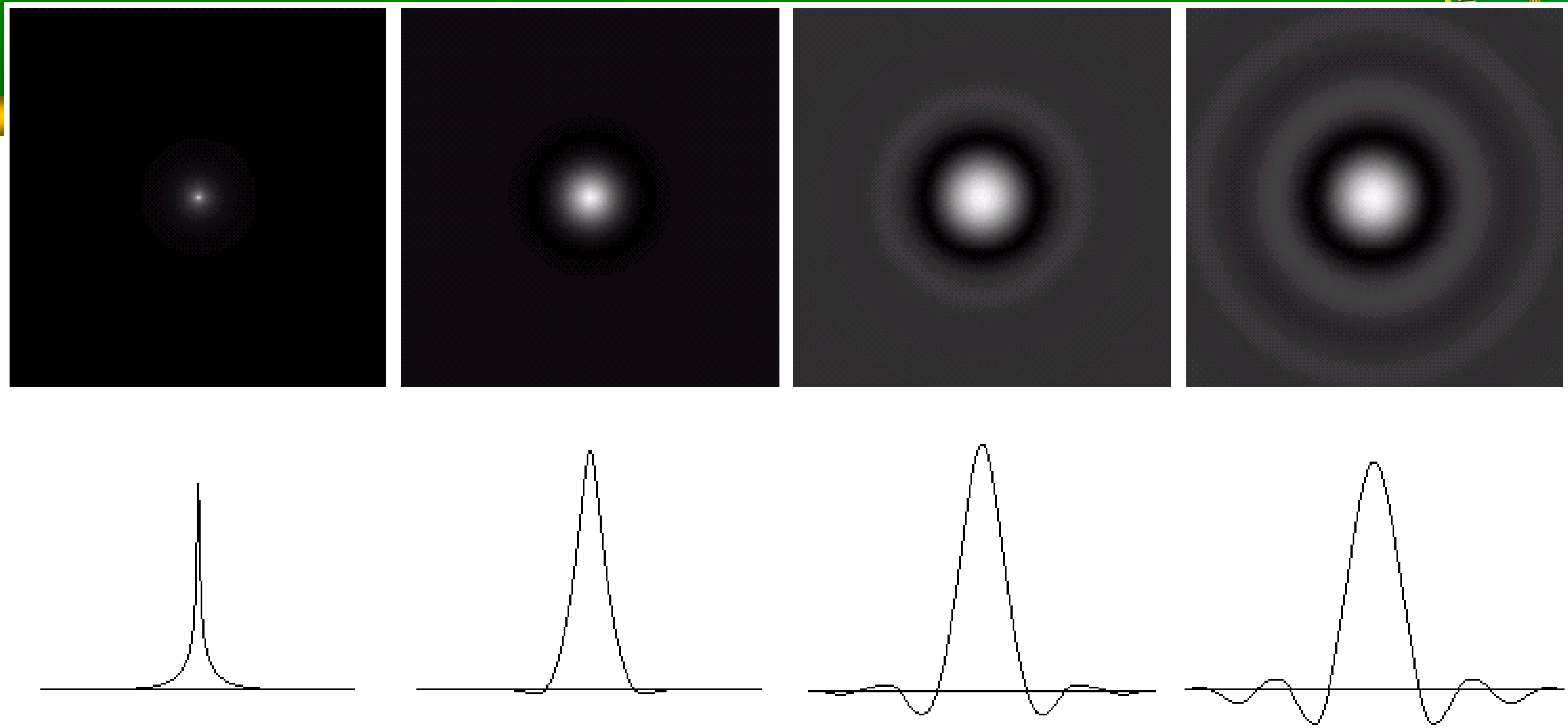
a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.



a b c d

FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

Questions?