Digital Image Processing

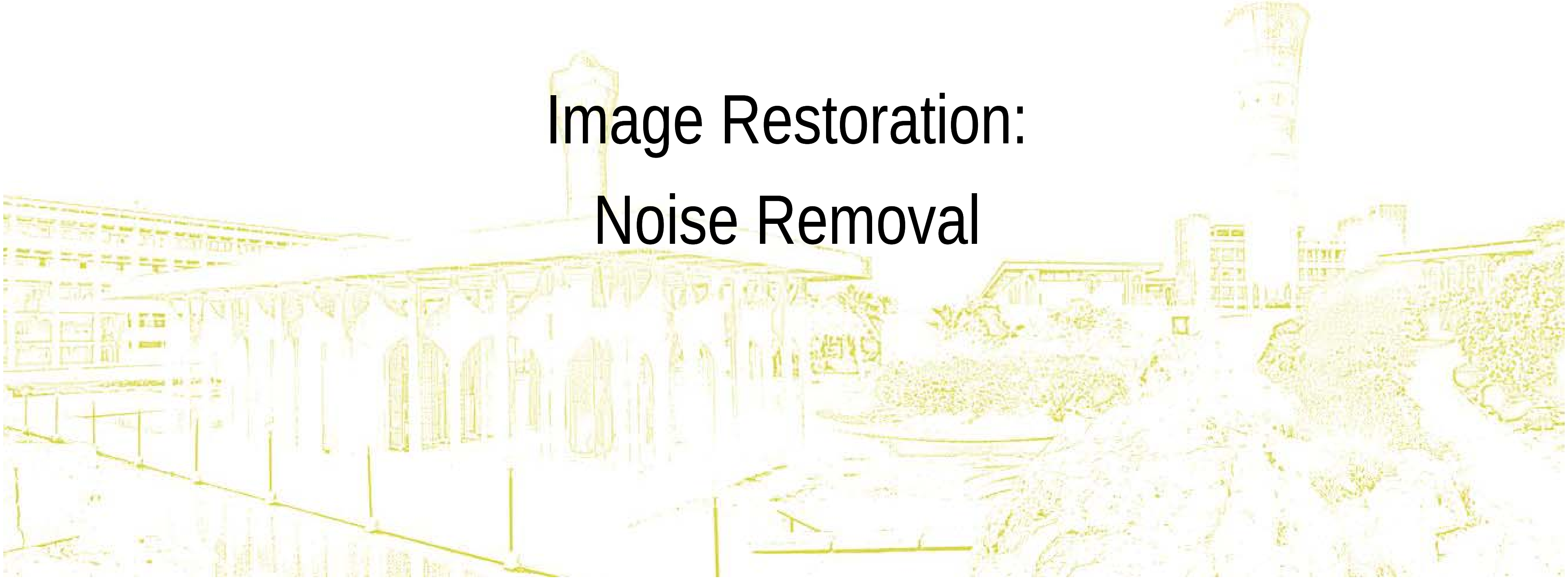


Image Restoration:

Noise Removal

Contents

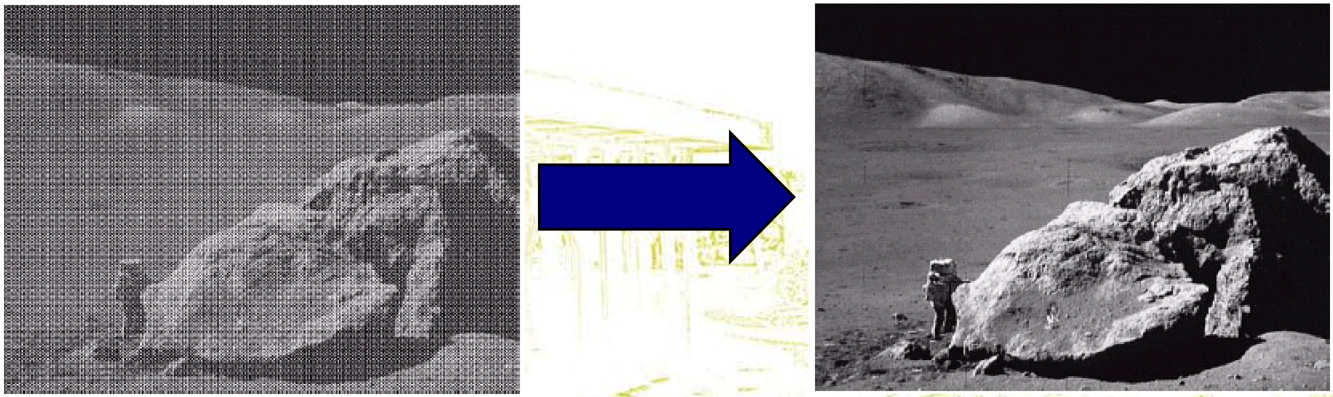
In this lecture we will look at image restoration techniques used for noise removal

- What is image restoration?
- Noise and images
- Noise models
- Noise removal using spatial domain filtering
- Periodic noise
- Noise removal using frequency domain filtering

What is Image Restoration?

Image restoration attempts to restore images that have been degraded

- Identify the degradation process and attempt to reverse it
- Similar to image enhancement, but more objective



Noise and Images

The sources of noise in digital images arise during image acquisition (digitization) and transmission

- Imaging sensors can be affected by ambient conditions
- Interference can be added to an image during transmission

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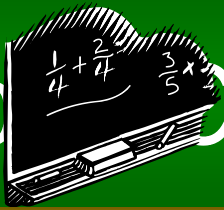
Noise Model

We can consider a noisy image to be modelled as follows:

$$g(x, y) = f(x, y) + \eta(x, y)$$

where $f(x, y)$ is the original image pixel, $\eta(x, y)$ is the noise term and $g(x, y)$ is the resulting noisy pixel

If we can estimate the model the noise in an image is based on this will help us to figure out how to restore the image

No Corruption Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	51	52	52	56	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
148	154	157	160	163	167	170
151	155	159	162	165	169	172

y

Image f (x, y)

Noisy Image

x

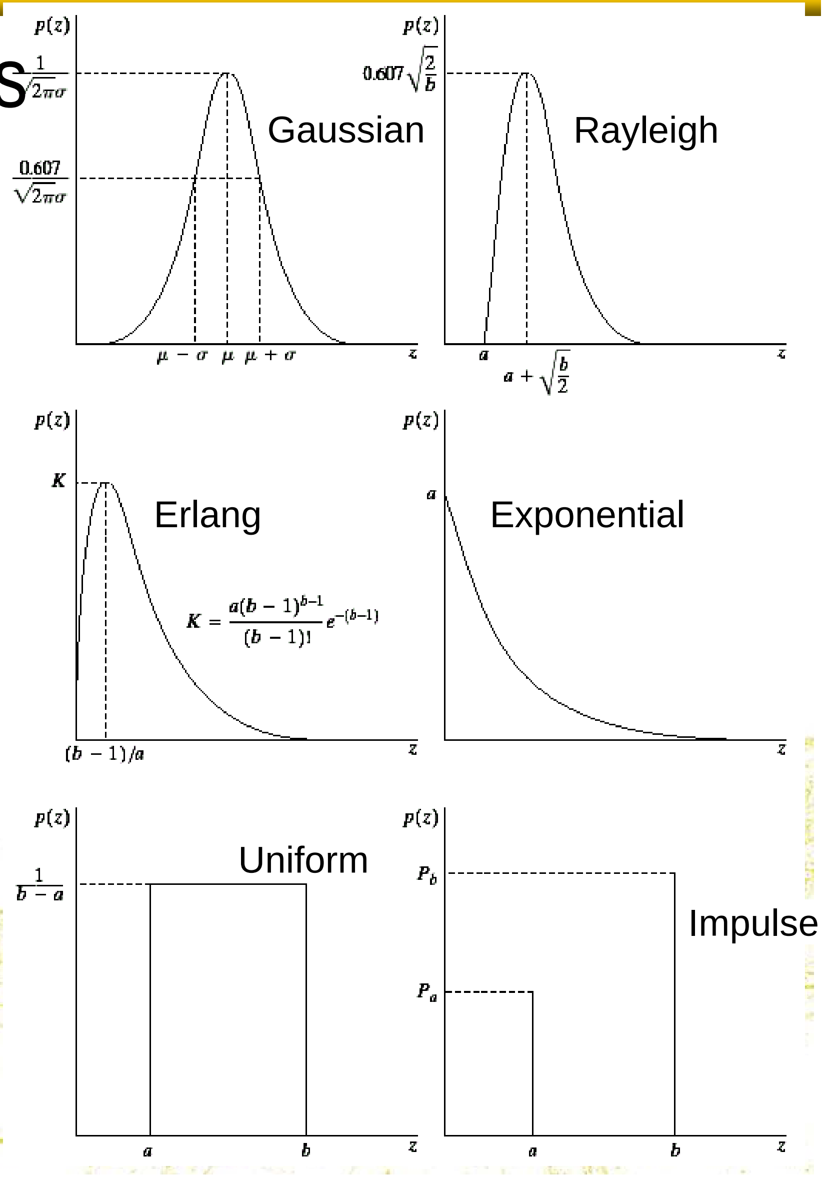
y

Image f (x, y)

Noise Models

There are many different models for the image noise term $\eta(x, y)$:

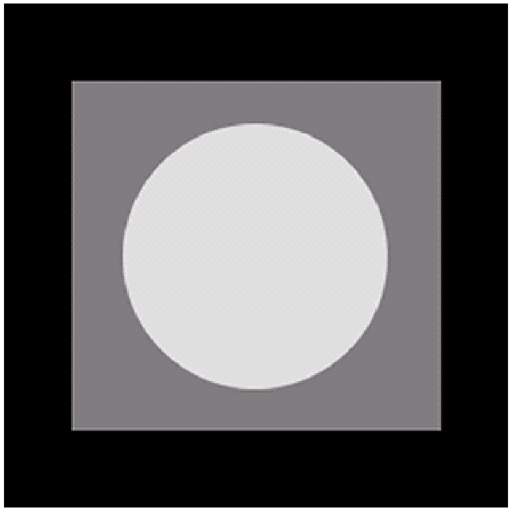
- Gaussian
  - Most common model
- Rayleigh
- Erlang
- Exponential
- Uniform
- Impulse
  - *Salt and pepper* noise



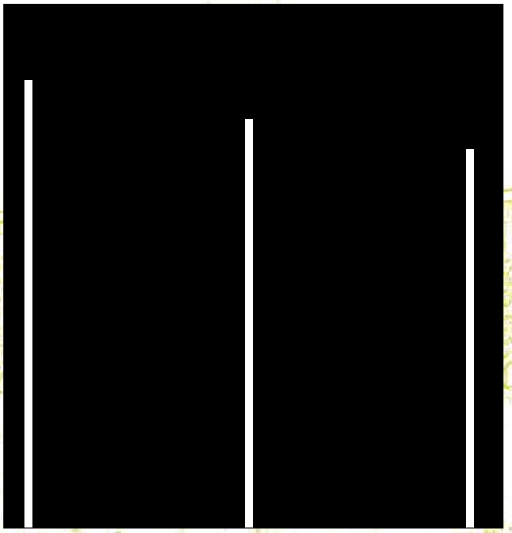
Noise Example

The test pattern to the right is ideal for demonstrating the addition of noise

The following slides will show the result of adding noise based on various models to this image

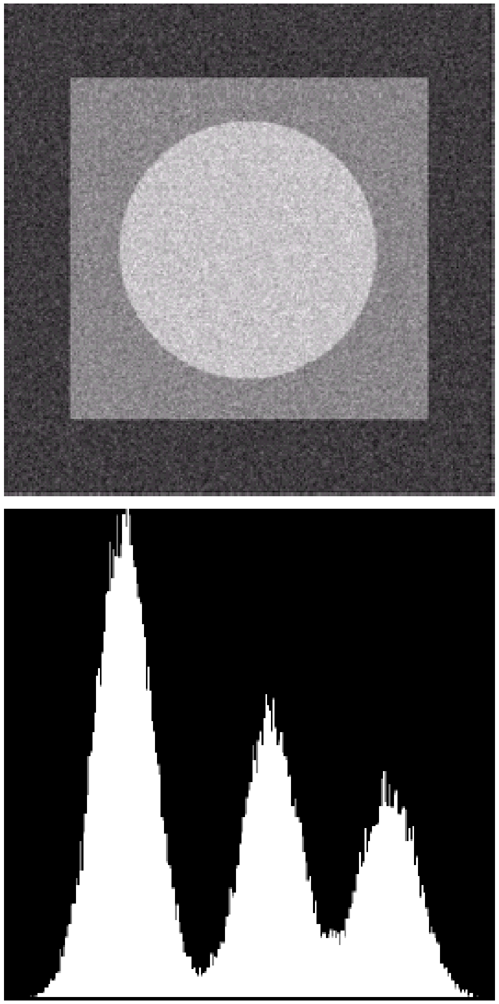


Image

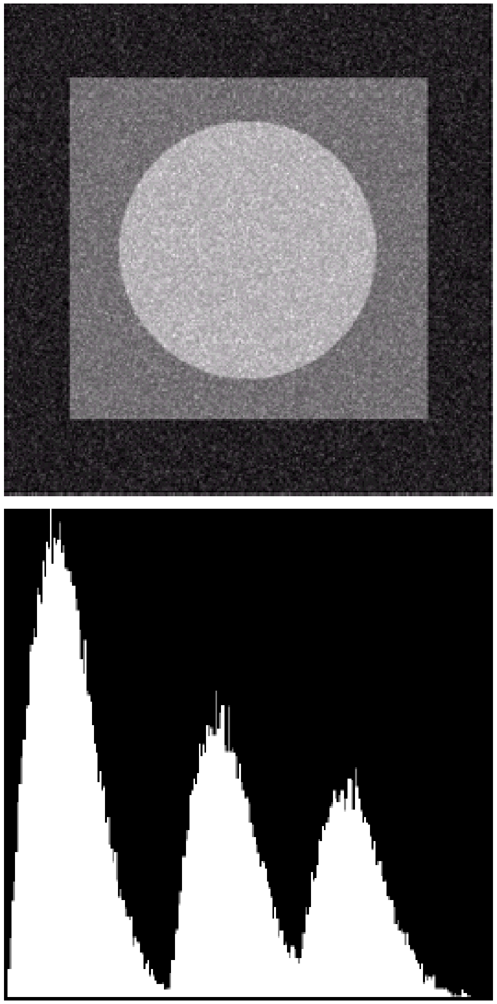


Histogram

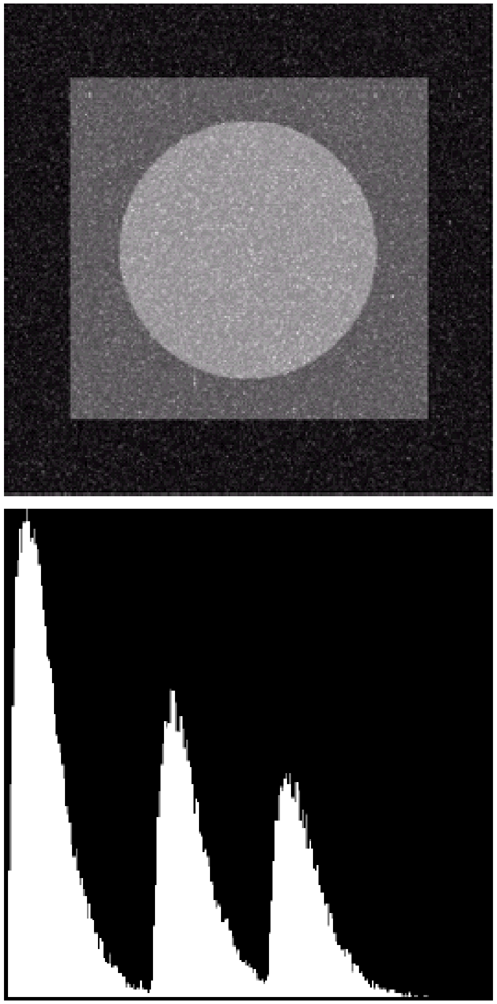
Noise Example (cont…)



Gaussian

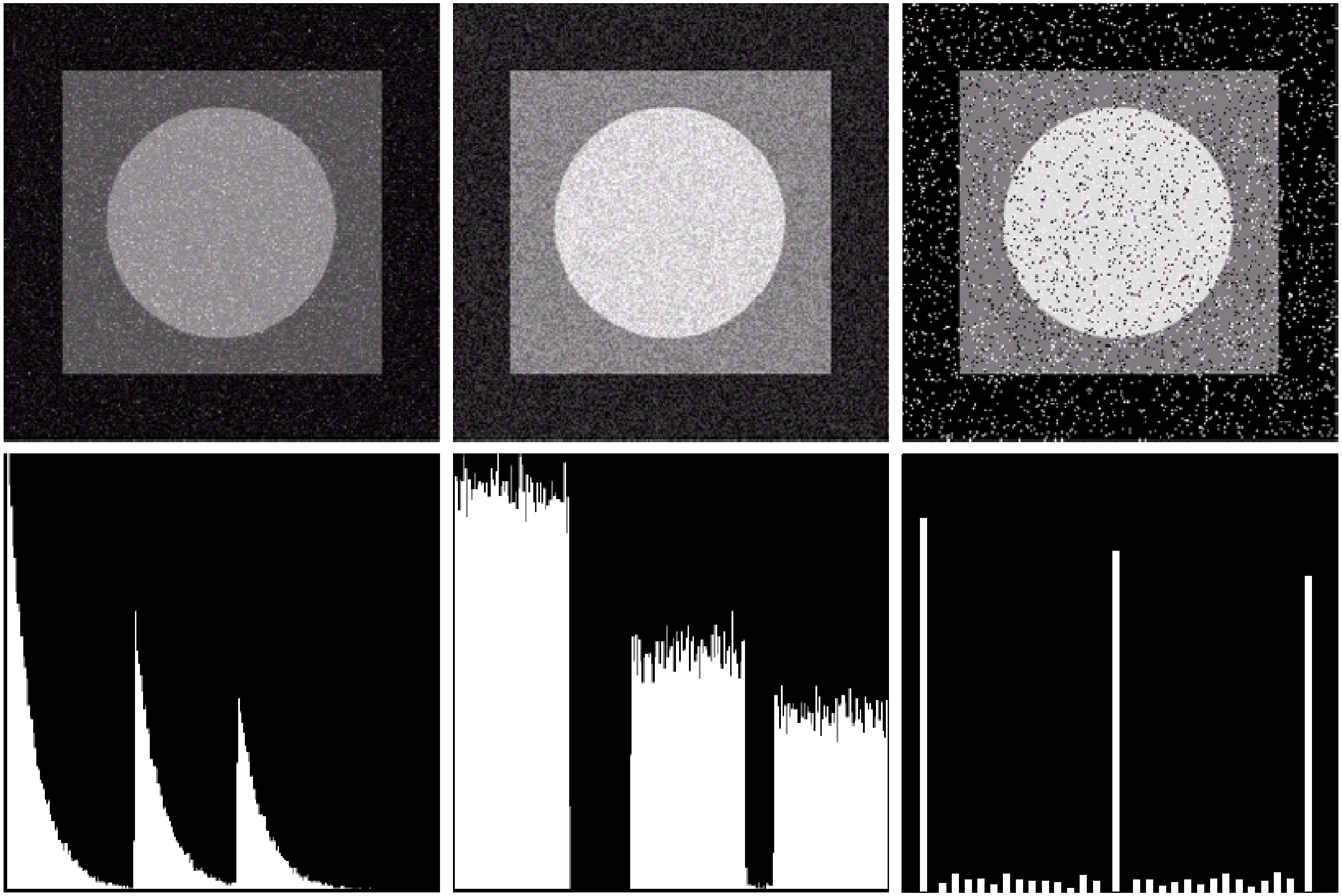


Rayleigh



Erlang

Noise Example (cont…)



Exponential Uniform Impulse

Filtering to Remove Noise

We can use spatial filters of different kinds to remove different kinds of noise

The *arithmetic mean* filter is a very simple one and is calculated as follows:

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

$^{1}/_{9}$	$^{1}/_{9}$	$^{1}/_{9}$
$^{1}/_{9}$	$^{1}/_{9}$	$^{1}/_{9}$
$^{1}/_{9}$	$^{1}/_{9}$	$^{1}/_{9}$

This is implemented as the simple smoothing filter

Blurs the image to remove noise

Noise Removal Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
148	154	157	160	163	167	170
151	155	159	162	165	169	172

y

Image f (x, y)

Filtered Image

x

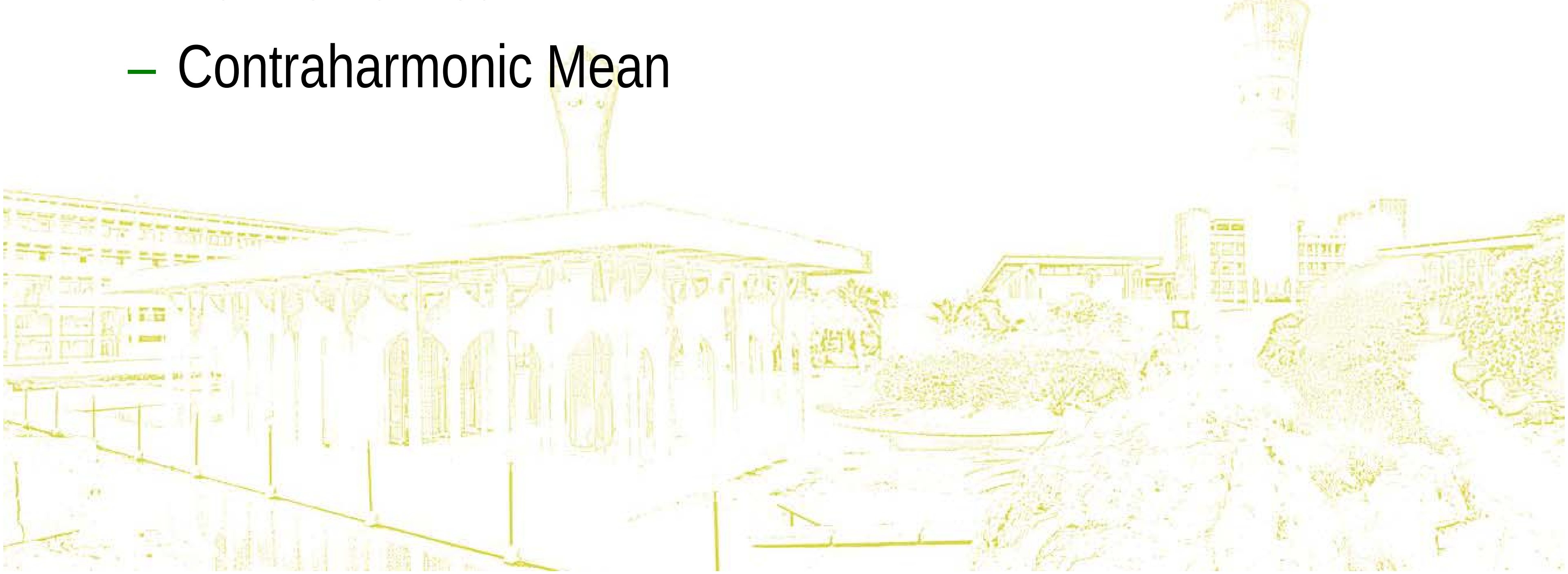
y

Image f (x, y)

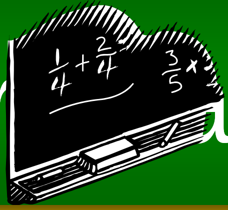
Other Means

There are different kinds of mean filters all of which exhibit slightly different behaviour:

- Geometric Mean
- Harmonic Mean
- Contraharmonic Mean



Other ans (cont…)

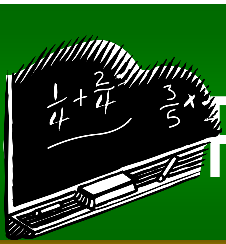


There are other variants on the mean which can give different performance

Geometric Mean:

$$\hat{f}(x,y)=\left[\prod_{(s,t)\in S_{xy}} g(s,t)\right]^{\frac{1}{mn}}$$

Achieves similar smoothing to the arithmetic mean, but tends to lose less image detail

N Removal Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
148	154	157	160	163	167	170
151	155	159	162	165	169	172

Image f (x, y)

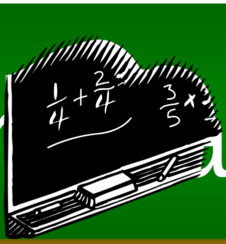
y

Filtered Image

x

Image f (x, y)

y



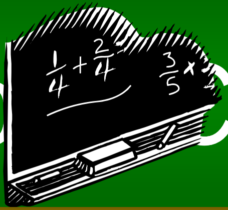
Other ans (cont…)

Harmonic Mean:

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise, but fails for pepper noise

Also does well for other kinds of noise such as Gaussian noise



No Corruption Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
50	54	57	60	63	67	70
51	55	59	62	65	69	72

y

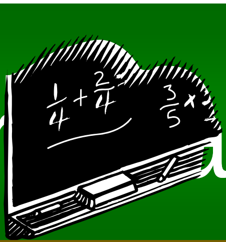
Image f (x, y)

Filtered Image

x

y

Image f (x, y)



Other ans (cont…)

Contraharmonic Mean:

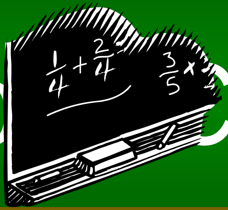
$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q is the *order* of the filter and adjusting its value changes the filter's behaviour

Positive values of Q eliminate pepper noise

Negative values of Q eliminate salt noise

No Corruption Example



Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
50	54	57	60	63	67	70
51	55	59	62	65	69	72

Image f (x, y)

y

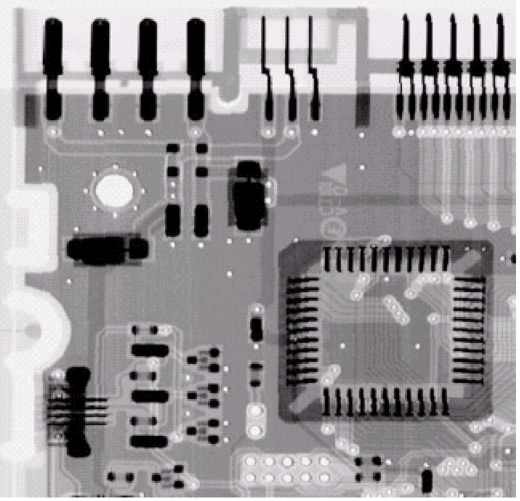
Filtered Image

x

Image f (x, y)

y

Noise Removal Examples



Original Image

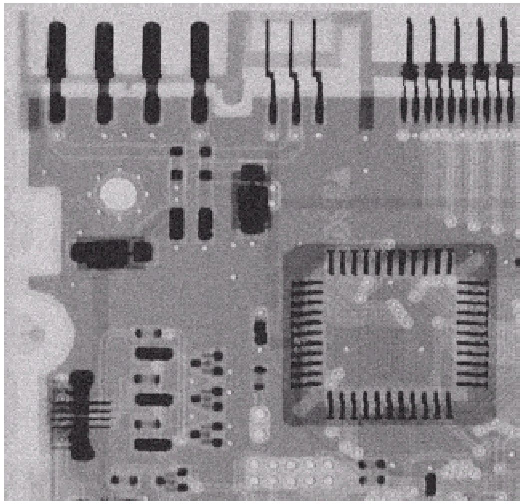
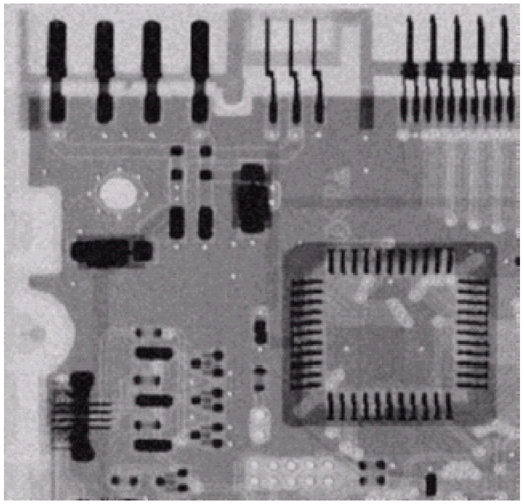
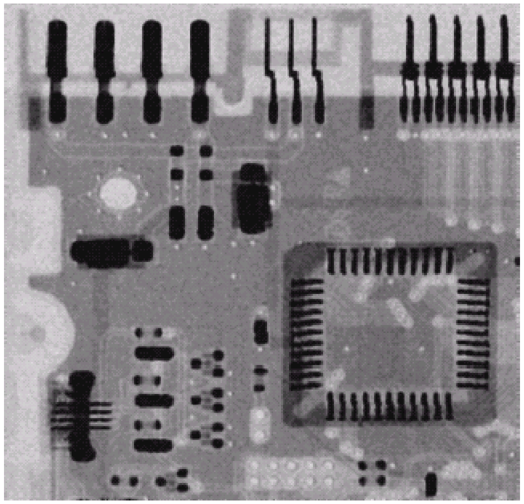


Image Corrupted By Gaussian Noise



After A 3*3 Arithmetic Mean Filter



After A 3*3 Geometric Mean Filter

Noise Removal Examples (cont…)

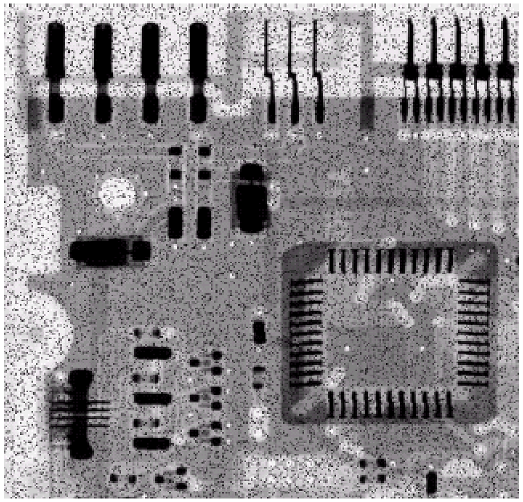
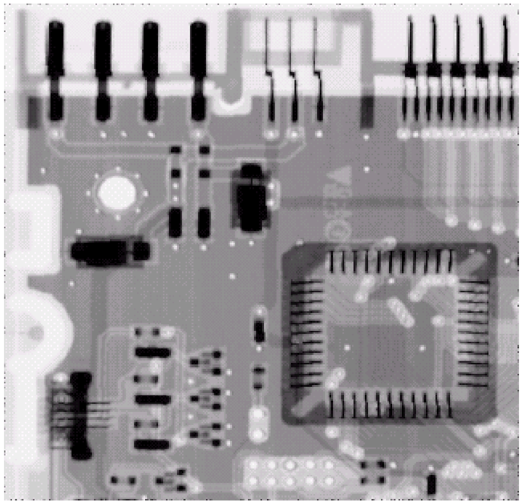


Image Corrupted By Pepper Noise



Result of Filtering Above With 3*3 Contraharmonic Q=1.5

Noise Removal Examples (cont…)

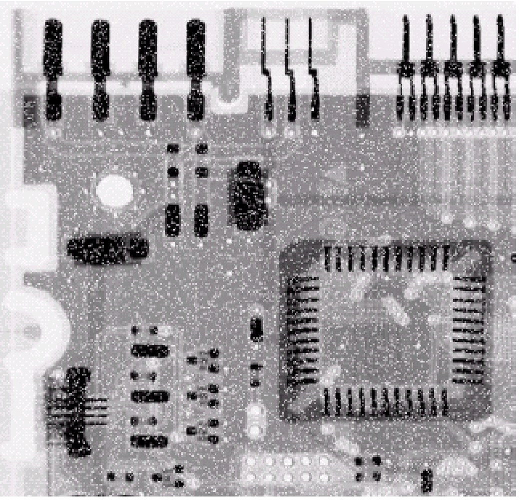
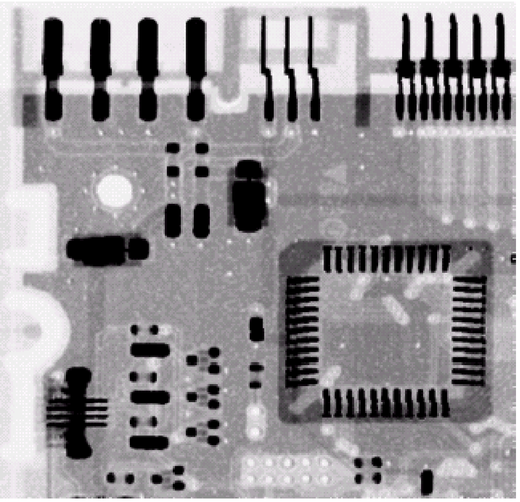


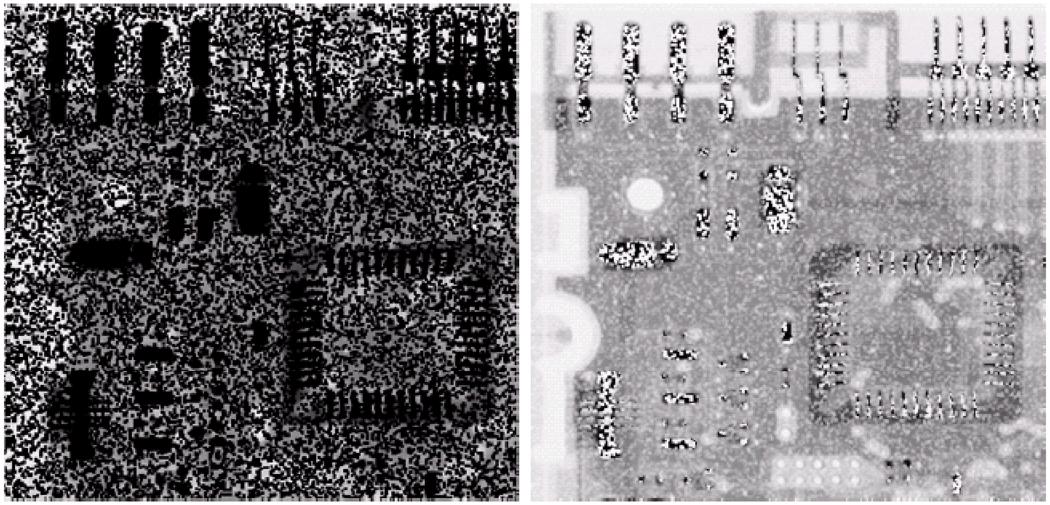
Image
Corrupted
By Salt
Noise



Result of
Filtering Above
With 3*3
Contraharmonic
Q=-1.5

Contraharmonic Filter: Here Be Dragons

Choosing the wrong value for Q when using the contraharmonic filter can have drastic results



Order Statistics Filters

Spatial filters that are based on ordering the pixel values that make up the neighbourhood operated on by the filter

Useful spatial filters include

- Median filter
- Max and min filter
- Midpoint filter
- Alpha trimmed mean filter

Median Filter

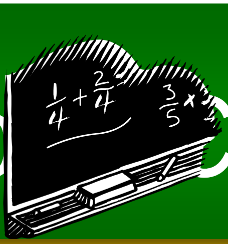
Median Filter:

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

Excellent at noise removal, without the smoothing effects that can occur with other smoothing filters

Particularly good when salt and pepper noise is present

No Corruption Example



Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
50	54	57	60	63	67	70
51	55	59	62	65	69	72

y

Image f (x, y)

Filtered Image

x

y

Image f (x, y)

Max and Min Filter

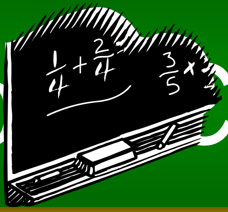
Max Filter:

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Min Filter:

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Max filter is good for pepper noise and min is good for salt noise

No  Corruption Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
50	54	57	60	63	67	70
51	55	59	62	65	69	72

y

Image f (x, y)

Filtered Image

x

y

Image f (x, y)

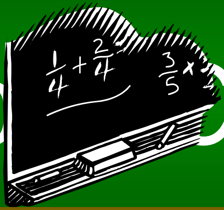
Midpoint Filter

Midpoint Filter:

$$\hat{f}(x,y)=\frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\}+\min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

Good for random Gaussian and uniform noise

No Corruption Example



Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
50	54	57	60	63	67	70
51	55	59	62	65	69	72

Image f (x, y)

y

Filtered Image

x

Image f (x, y)

y

Alpha-Trimmed Mean Filter

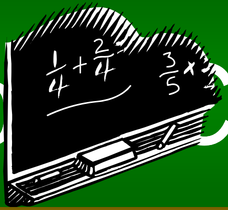
Alpha-Trimmed Mean Filter:

$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

We can delete the $d/2$ lowest and $d/2$ highest grey levels

So $g_r(s, t)$ represents the remaining $mn - d$ pixels

No Corruption Example



Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
50	54	57	60	63	67	70
51	55	59	62	65	69	72

y

Image f (x, y)

Filtered Image

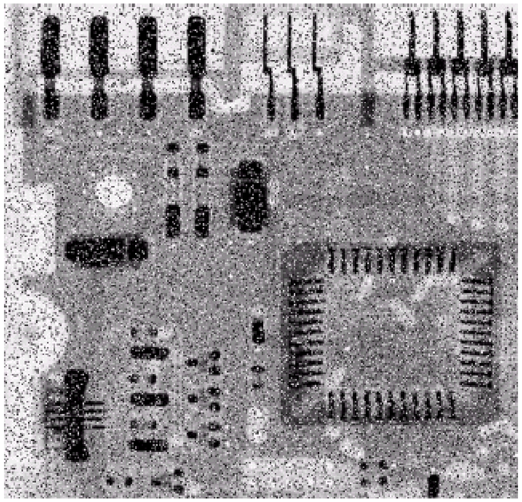
x

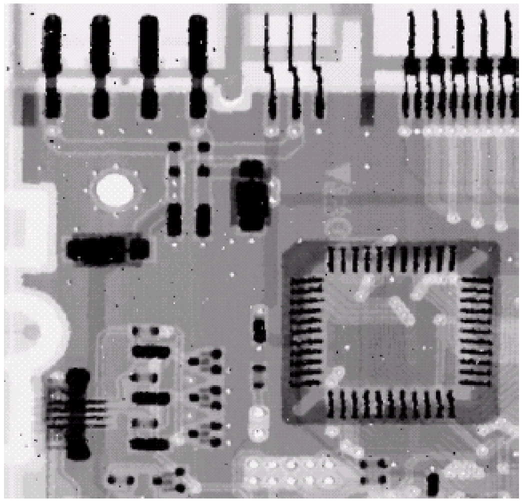
y

Image f (x, y)

Noise Removal Examples

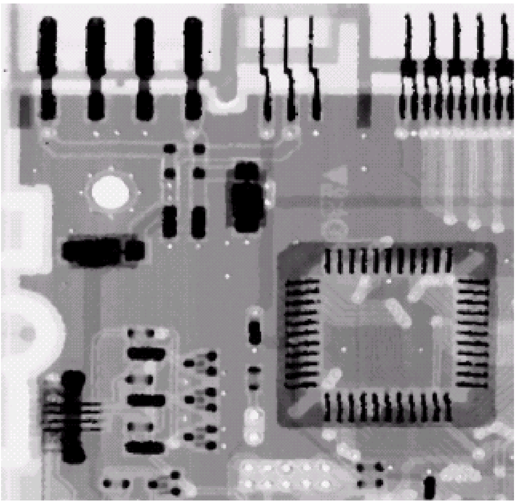
Image Corrupted By Salt And Pepper Noise

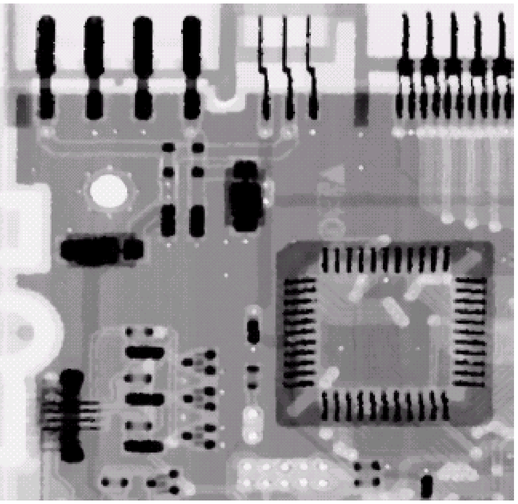




Result of 1 Pass With A 3*3 Median Filter

Result of 2 Passes With A 3*3 Median Filter





Result of 3 Passes With A 3*3 Median Filter

Noise Removal Examples (cont…)

Image Corrupted By Pepper Noise

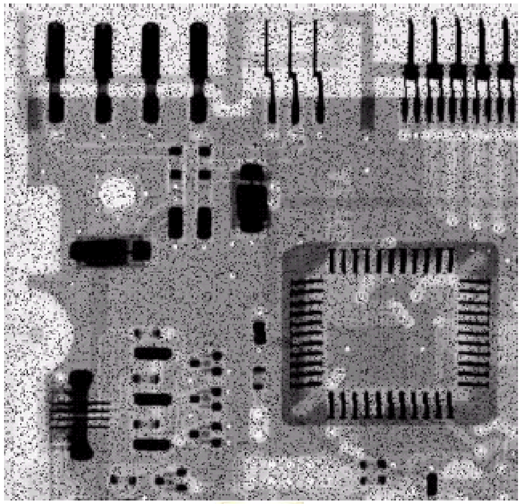
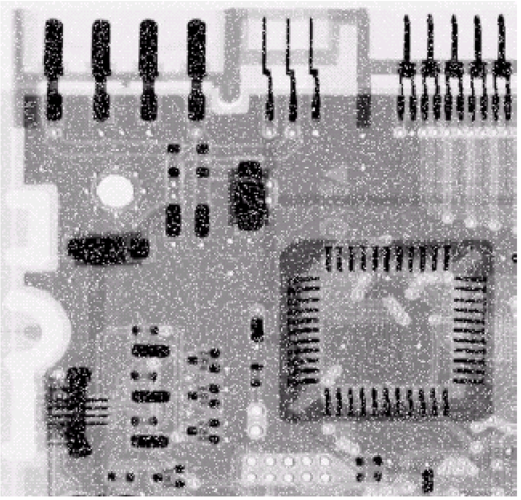
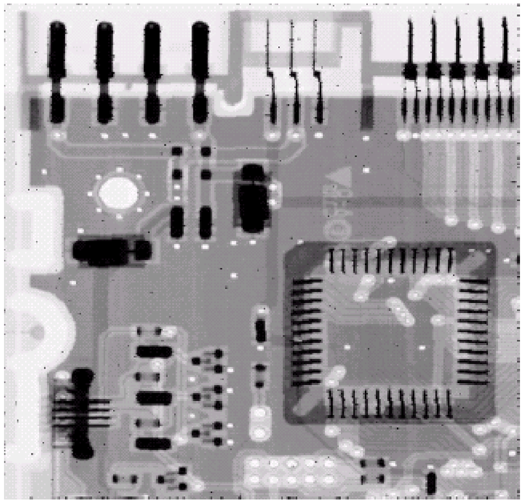


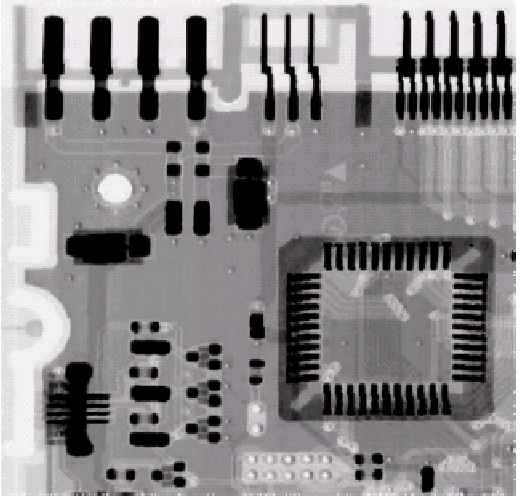
Image Corrupted By Salt Noise



Result Of Filtering Above With A 3*3 Max Filter

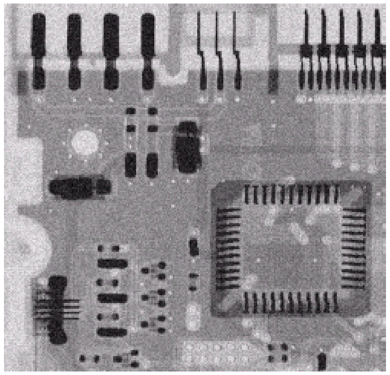


Result Of Filtering Above With A 3*3 Min Filter



Noise Removal Examples (cont…)

Image Corrupted By Uniform Noise



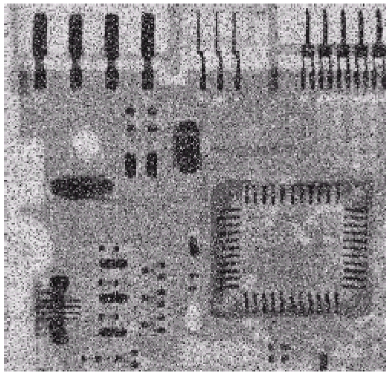
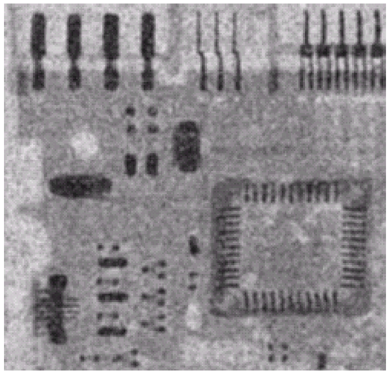
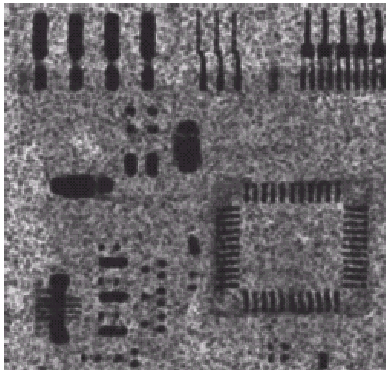


Image Further Corrupted By Salt and Pepper Noise

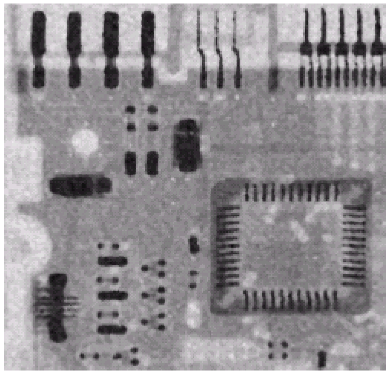
Filtered By 5*5 Arithmetic Mean Filter

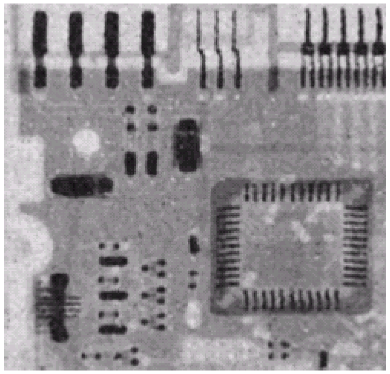




Filtered By 5*5 Geometric Mean Filter

Filtered By 5*5 Median Filter





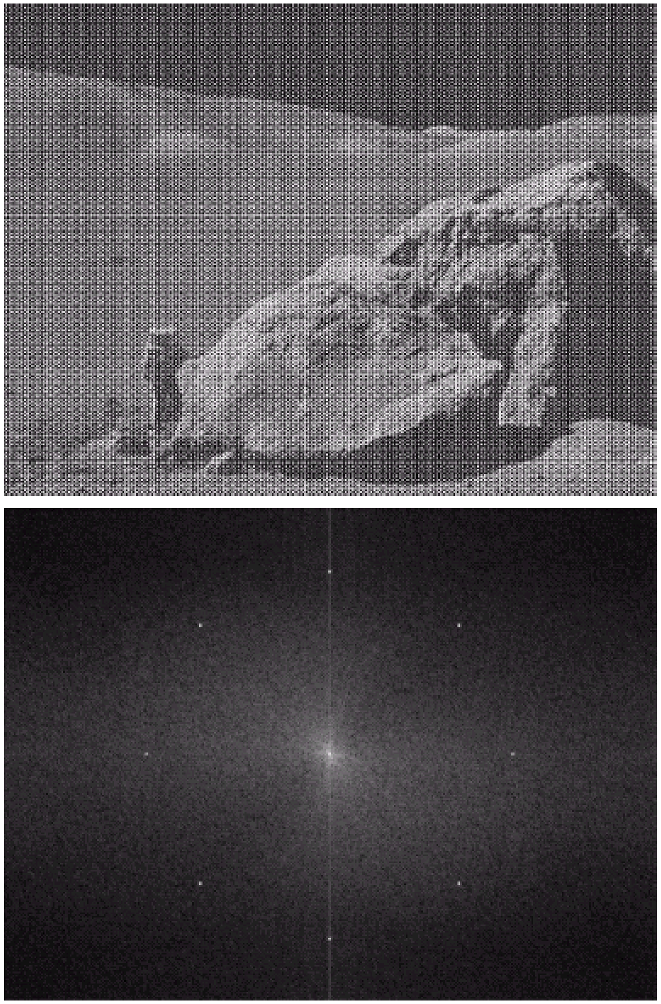
Filtered By 5*5 Alpha-Trimmed Mean Filter

Periodic Noise

Typically arises due to electrical or electromagnetic interference

Gives rise to regular noise patterns in an image

Frequency domain techniques in the Fourier domain are most effective at removing periodic noise



Band Reject Filters

Removing periodic noise form an image involves removing a particular range of frequencies from that image

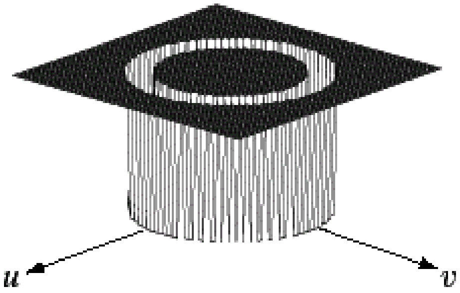
Band reject filters can be used for this purpose

An ideal band reject filter is given as follows:

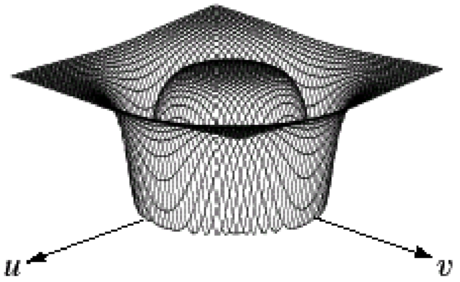
$$H(u,v) = \begin{cases} 1 & if \;\; D(u,v) < D_0 - \frac{W}{2} \\ 0 & if \;\; D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2} \\ 1 & if \;\; D(u,v) > D_0 + \frac{W}{2} \end{cases}$$

Band Reject Filters (cont…)

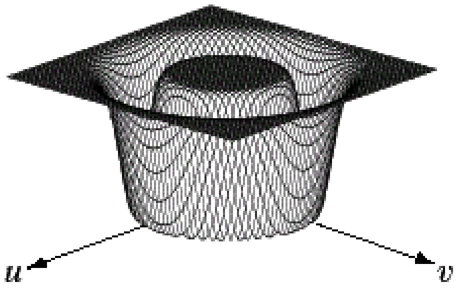
The ideal band reject filter is shown below, along with Butterworth and Gaussian versions of the filter



Ideal Band Reject Filter



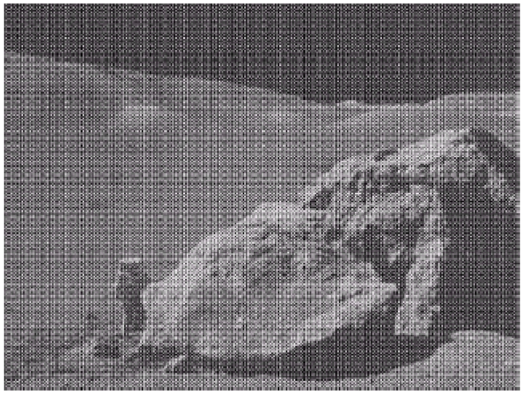
Butterworth Band Reject Filter (of order 1)



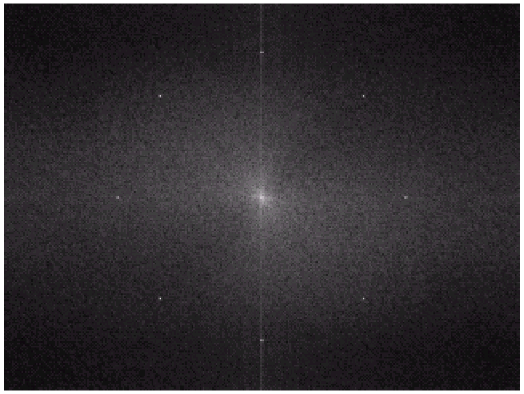
Gaussian Band Reject Filter

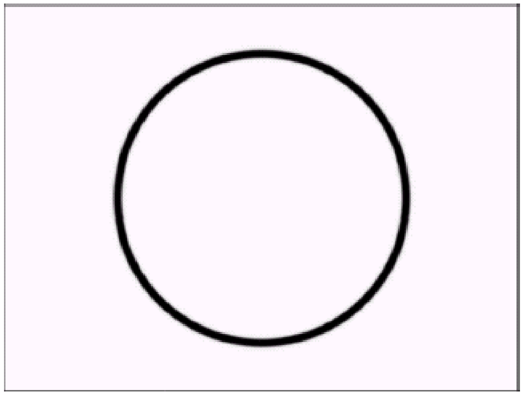
Band Reject Filter Example

Image corrupted by sinusoidal noise

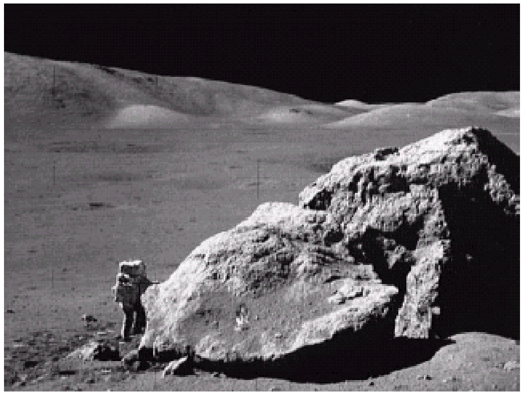


Fourier spectrum of corrupted image





Butterworth band reject filter



Filtered image

Summary

In this lecture we looked at image restoration for noise removal

Restoration is slightly more objective than enhancement

Spatial domain techniques are particularly useful for removing random noise

Frequency domain techniques are particularly useful for removing periodic noise

Adaptive Filtering Example

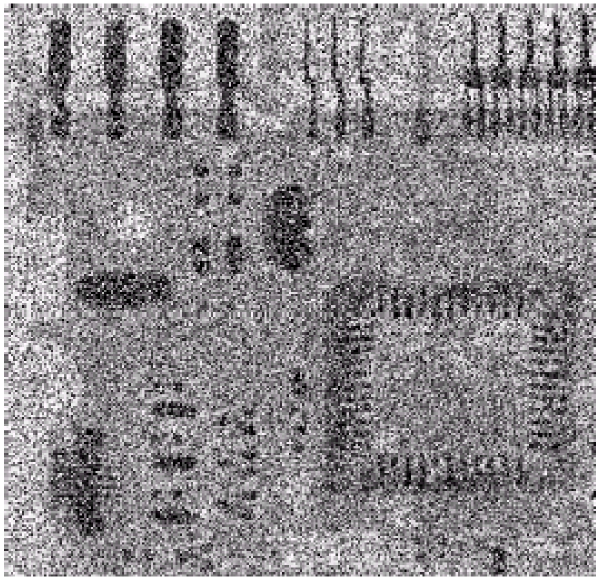
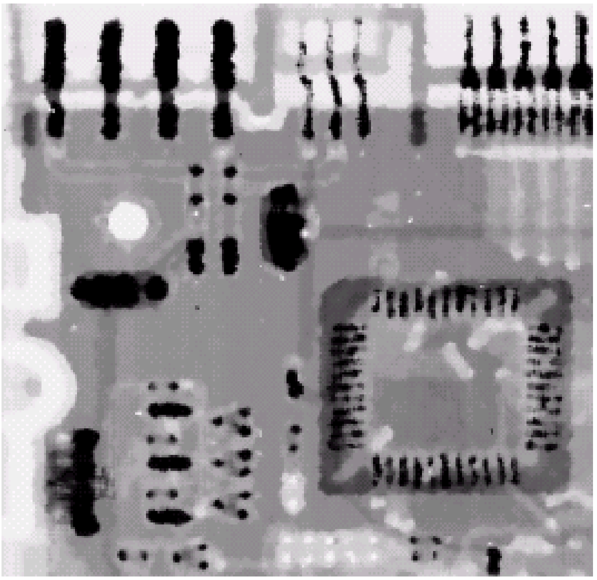
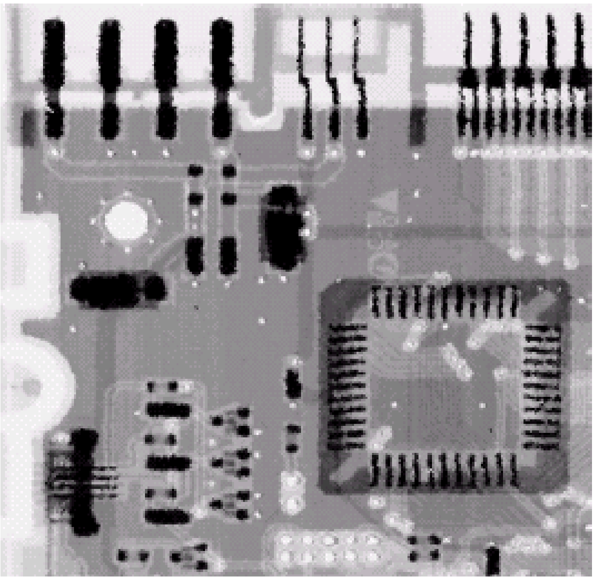


Image corrupted by salt and pepper noise with probabilities $P_a = P_b$=0.25



Result of filtering with a 7 * 7 median filter



Result of adaptive median filtering with i = 7

Questions?