EE 204
Lecture 10
Norton Equivalent Circuits and Maxm. Power

Calculation of TEC (Method 3):

Recall method 1: 1) Find $V_{th} = V_{oc}$ 2) Find $R_{th} = \dfrac{V_{oc}}{i_{sc}}$

The *first* step in method 3 is the *same as in method 1*, we first find $V_{th} = V_{oc}$.

To find R_{th} method 3 uses a *different* technique, as explained next:

1) Set *all independent* sources in circuit A to *zero* [leave dependent sources as they are]

2) Calculate $R_{eq} = R_{th}$

[the equivalent resistance *after all independent* sources have been set to *zero* is equal to the Thevenin's resistance]

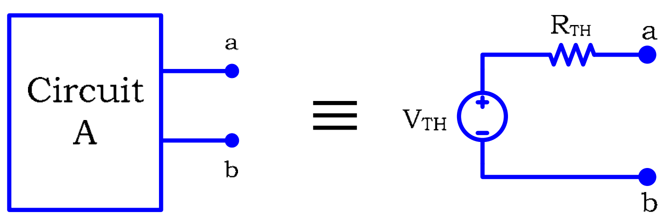


Figure 1

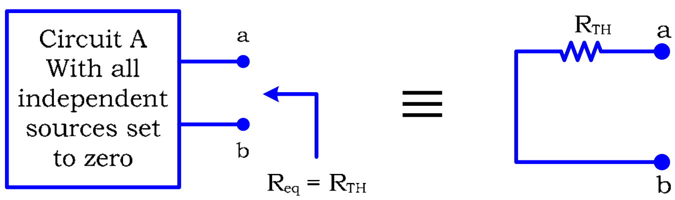


Figure 2

To set an *independent voltage* source to zero $\Rightarrow$ replace it with a *short* circuit

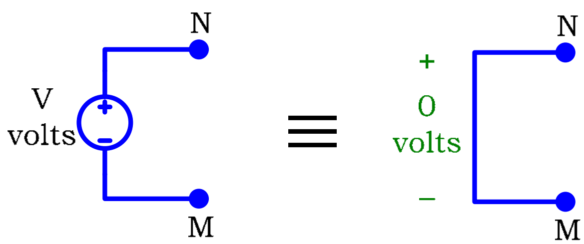


Figure 3

To set an *independent current* source to zero $\Rightarrow$ replace it with an *open* circuit

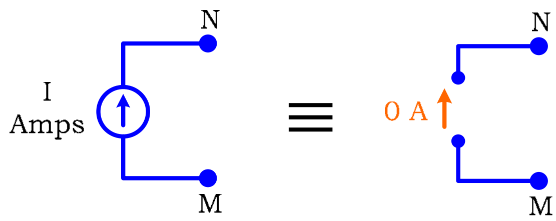


Figure 4

Example 1:

Calculate the TEC.

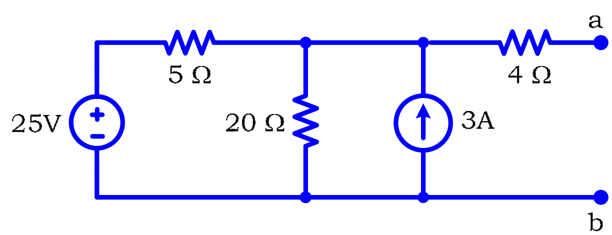


Figure 5

Solution:

Calculate $V_{th} = V_{oc}$ [Let us use the mesh analysis, since there is only *one actual* unknown]

$i_3 = 0$ [because of the open circuit]

$\therefore i_2 = -3$

KVL around mesh 1 $\Rightarrow$ $-25 + 5i_1 + 20(i_1 - (-3)) = 0$ $\Rightarrow$ $i_1 = -1.4A$

KVL around the outer circuit $\Rightarrow$ $-25 + 5(-1.4) + 4(0) + V_{oc} = 0$ $\Rightarrow$ $V_{oc} = 32V$

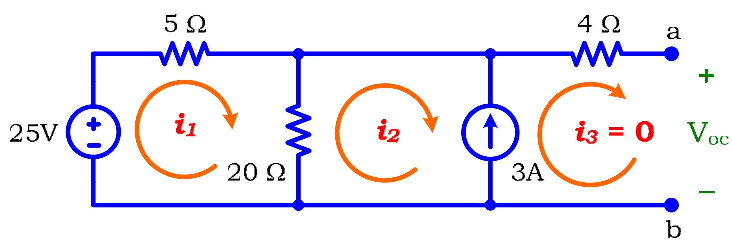


Figure 6

To find R_{th} :

Set *all independent* sources to zero

$3A$ $\Rightarrow$ replace with an *open* circuit

$25V \Rightarrow$ replace with a *short* circuit

Calculate $R_{eq} = R_{th}$

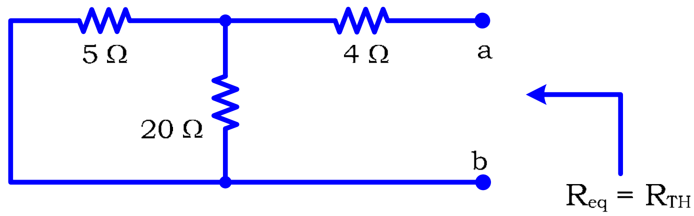


Figure 7

$5\Omega \| 20\Omega \quad \Rightarrow \quad 4\Omega$

$\therefore R_{eq} = R_{th} = 4 + 4 = 8\Omega$

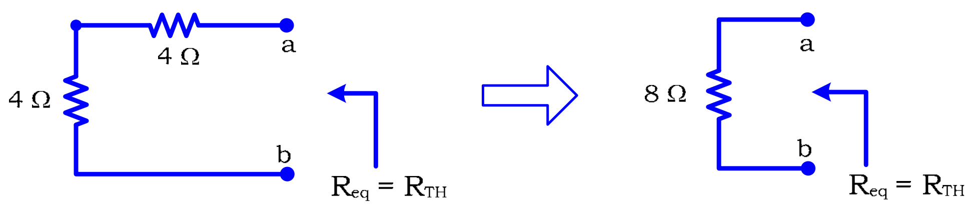


Figure 8

The resulting TEC is:

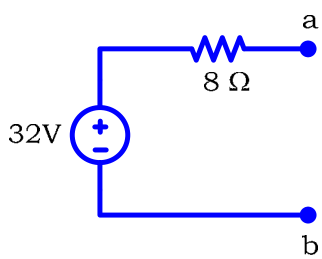


Figure 9

The Norton Equivalent Circuit:

Instead of representing circuit A with a voltage source in series with a resistor, we can represent it with a current source in parallel with a resistor.

The is called Norton Equivalent Circuit (NEC)

In general, the TEC & NEC circuits are related by source transformation. Then:

1) The values of the resistances in the TEC & NEC are the same.

2) Using source transformation, the current source I in the NEC is given by:

$$I = \frac{V_{th}}{R_{th}}$$ [the same as the short circuit current]

$$\therefore I = \frac{V_{th}}{R_{th}} = I_{sc}$$

In general:

We know the TEC ⇔ We Know the NEC

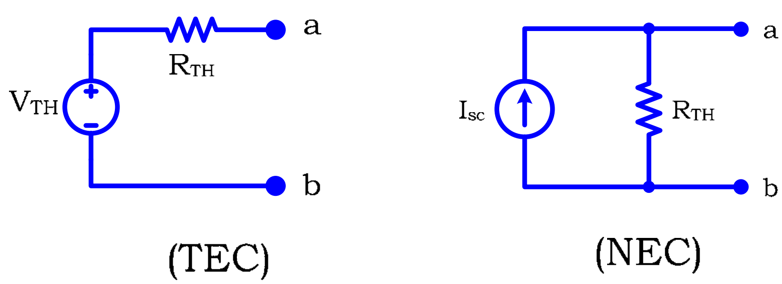
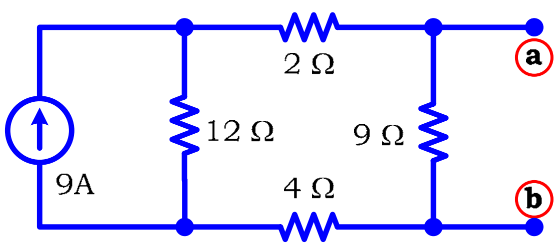


Figure 16

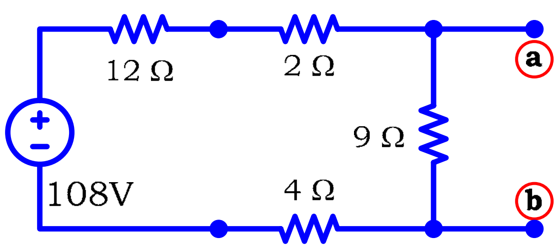
Example1:

Find the Norton Equivalent Circuit with respect to the terminals a, b for the circuit



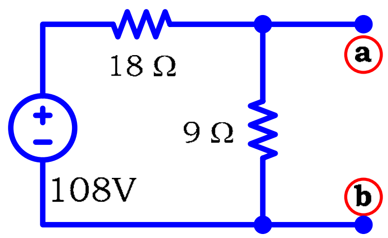
Solution:

Using source transformation: V=9x12=108 V



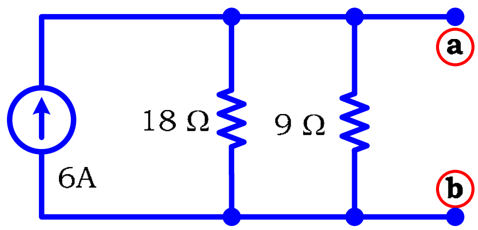
Combine series resistors 12 +2 +4 =18 Ohms

Draw circuit again V=108 V in series with 18 Ohms



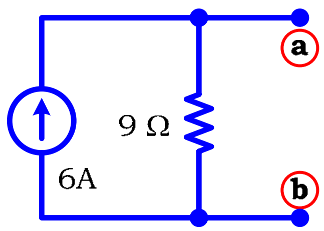
ST again: I=V/R   I=108/18=6A

Draw Circuit 6A(up) in // to 18 //9



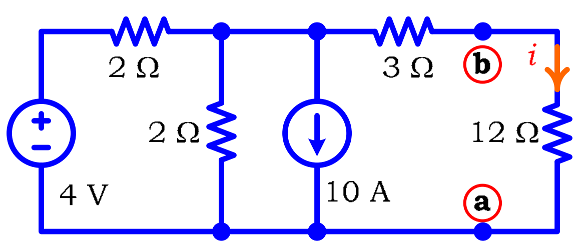
Combine series resistors 18 //9 =6 Ohms

Draw Circuit 6A(up) in // to 6 Ohms



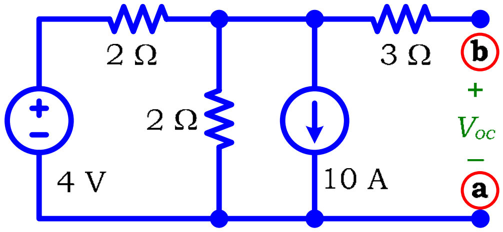
Therefore NEC:
IN=6A
RN=6Ohms

Example2:



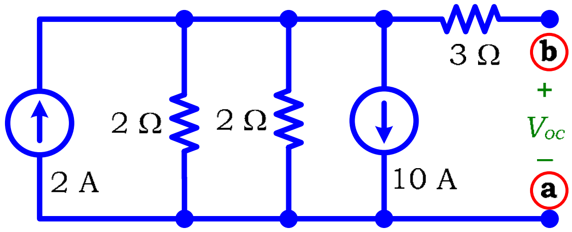
Determine the current I in the circuit by reducing the circuit attached to the $12-\Omega$ **resistor to a TE. Find the NEC.**

Solution:
Remove $12-\Omega$ **resistor mark** V_{OC}

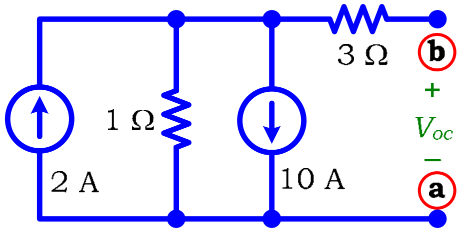


To find Vth let us determine V_{OC}

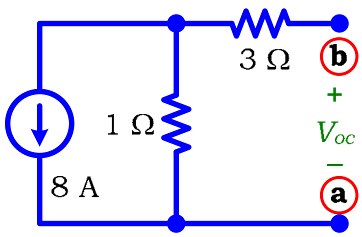
Using ST for the source of 4v in series with $2-\Omega$ **resistor we get: I =2A (up) //** $2-\Omega$



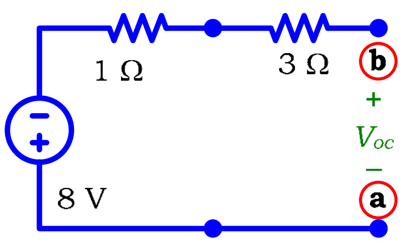
Combine the // resistors: $2//2=1\Omega$



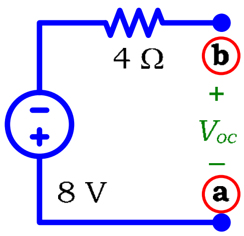
Combine the 2 sources we get I=8A (down)



ST again gives : $V_{OC}=-8V$



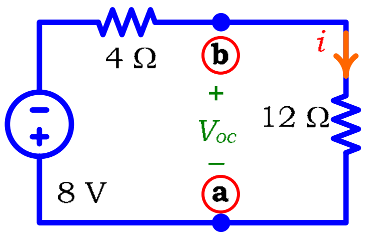
Draw circuit V in series with $3+1=4\Omega$



Find Rth

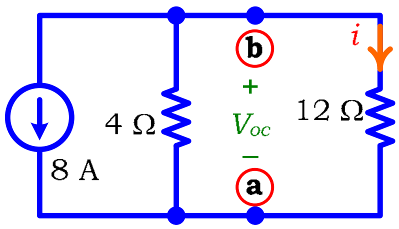
$$R_{TH} = \frac{v_x}{i_x} = 3\Omega + (2\Omega // 2\Omega) = 4\Omega$$

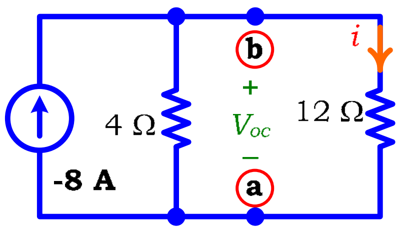
Therefore TEC : Draw circuit



$$\therefore i = \frac{-8}{4+12} = 0.5A$$

To get NEC use ST:

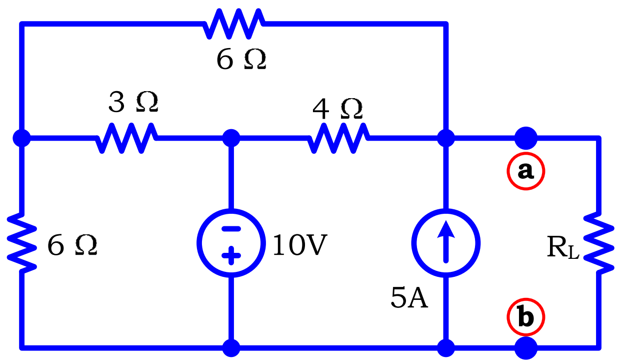


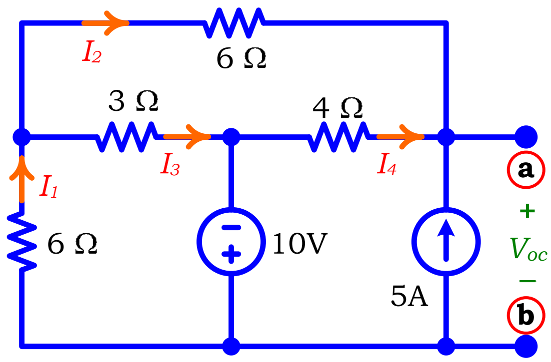


$I_N = -8A$ **(up)**

$R_N = 4\Omega$

Draw Circuit (add the $12-\Omega$ **resistor)**





KVL (Highlight loop)

$V_{OC} + 10 + 4I_4 = 0$ **(1)**

$V_{OC} + 6I_1 + 6I_2 = 0$ **(2)**

1 and 2 give: $6I_1 + 6I_2 = 10 + 4I_4$ **(3)**

KCL at node a:

$I_4 + I_2 + 5 = 0$

or $I_4 = -I_2 - 5$ **(4)**

Substitute (4) into (3) :

$10 - 20 - 4I_2 = 6I_1 + 6I_2 \quad \Rightarrow \quad 10 + 10I_2 + 6I_1 = 0$ **(5)**

KVL $-10 + 6I_1 + 3I_3 = 0$ **(6)**

KCL at node c

$I_3 = I_1 - I_2$ **(7)**

6 and 7 give: $-10 + 6I_1 + 3(I_1 - I_2) = 0$

or $9I_1 - 3I_2 = 10$ **(8)**

Solve equations (5) and (8) [Subtract 3x(5) from 2x(6)] to get: $36I_2 = -50$ **or**

$I_2 = -\frac{25}{18}$ **(9)**

Substitute in (8) to get I_1 $9I_1 = 10 - 3\frac{25}{18}$

$\therefore I_1 = \frac{35}{54} = 0.65A$ **(10)**

Therefore $V_{OC} = -6(I_1 + I_2)$

$\therefore V_{OC} = \frac{40}{9} = 4.44V$