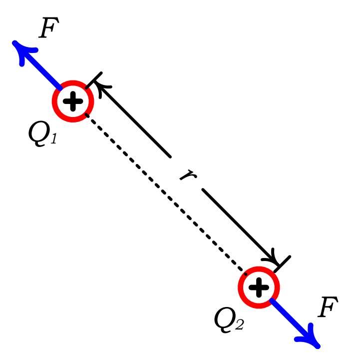
EE 204
Lecture 01
Basic Definitions

1. **Introduction:**
This course is about the fundamentals of Electric Circuit Analysis. No other skill for an Electrical Engineer is more basic than Circuit Analysis. Almost all other courses rely on the mastery of this course.

2. **Basic Definitions:**
   1. **Charge:** An electron carries a negative charge of 1.602 x 10^{-19} C. where the unit of charge is the Coulomb (C). Charges of the same sign repel each other, while charges of opposite sign attract.
   For example, consider two point charges, Q1 and Q2, separated a distance r as shown in Figure. The force exerted by one charge on the other varies directly as the product of the charges (the sign of the charge is included in its value) and inversely as the square of the distance between them according to Coulomb's law:

$$F = k\frac{Q_1 Q_2}{r^2} \qquad \textbf{(1)}$$

   The constant of proportionality, k, is approximately $9\text{x}10^9$ when the other parameters are given in SI units and the surrounding medium is free space.
   Note that a positive charge moving in one direction is equivalent to a positive charge moving in the opposite direction.

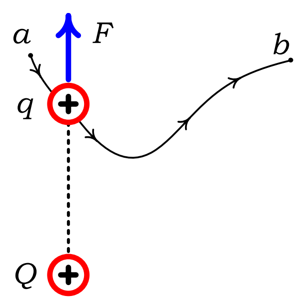


   2. **Voltage:** Since charges exert forces on other charges, energy must be expended in moving a charge in the vicinity of other charges. Thus charges produce a type of force field. The unit of energy is the joule (J), defined as the energy expended in the exertion of ta force of one Newton in moving an object through a distance of one meter (1J = N-m) For example, consider moving a charge q from point a to point b along a chosen path in the presence of another charge Q as illustrated in Fig.

The movement of q from a to b as in Fig. 1.2 requires work w_{ba} on our part then we say that the voltage of point b with respect to point a is the work per unit charge required to move the charge from point a to point b:

$$v_{ba} = \frac{w_{ba}}{q} \quad \textbf{(2)}$$

Alternatively we say that there is a potential difference between points a and b with point b at an assumed higher potential. If this is negative number it means a is at higher potential.



Determining the voltage between two points that is established by point charge Q is a simple matter as shown by the following example.

Example 1
A charge Q establishes a voltage between points a and b as shown in Fig. The two points are at radial distances R_a and R_b from the point charge. Determine the voltage V_{bc} between the two points (point b at the assumed higher potential).

Solution:

Energy required to move a test charge q from a to b is : (we can take any path, but for simplicity we assume that the path is along an arc from a to c and from c to b is perpendicular so there is no work for that portion of the path.)

$$w_{ba} = -\int_{R_a}^{R_b} 9\times10^9 \frac{qQ}{r^2} dR = 9\times10^9 Qq\left(\frac{1}{R_b} - \frac{1}{R_a}\right) \quad \textbf{(3)}$$

Hence, the voltage of point b with respect to point a is

$$v_{ba} = \frac{w_{ba}}{q} = 9\times10^9 Q\left(\frac{1}{R_b} - \frac{1}{R_a}\right) V \quad \textbf{(4)}$$

$$v_{ba}=\frac{w_{ba}}{q}=9\times10^{9}Q\left(\frac{1}{d^{+}}-\frac{1}{d^{-}}\right)V \qquad \textbf{(5)}$$

Note that the sign is as important for a voltage as its magnitude. Here R_b is at higher potential denoted d^+.

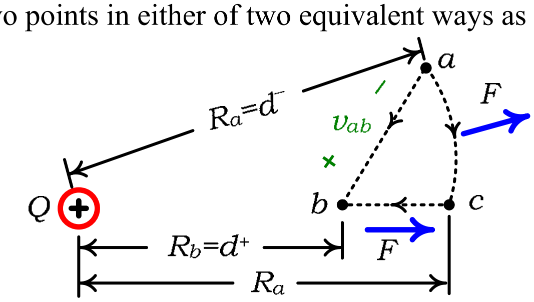
As an example, suppose that Q is a negative charge,

$Q=-1\times10^{-9}$ C, and $R_a=5$ m and $R_b=2$ m.

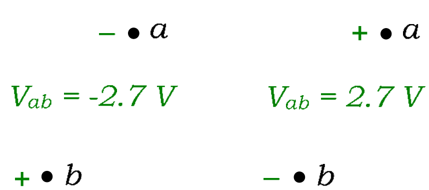
The resulting voltage of point b with respect to point a is

$v_{ba}=-2.7$ V.

This indicates that point a is at the higher potential than point b by 2.7 V. This is a sensible result in that Q being negative actually provides an attractive force for the movement of q from a to b . Hence we could denote the voltage between the two points in either of two equivalent ways as shown in Fig.



(a)



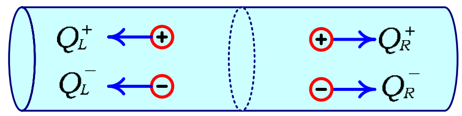
(b)

3. **Current:** Current is the rate of movement of electric charge.
If we observe the charge crossing an area of the cylinder in a certain time interval, Δt, then the current directed to the right is defined as the net positive charge transferred to the right per unit of time:

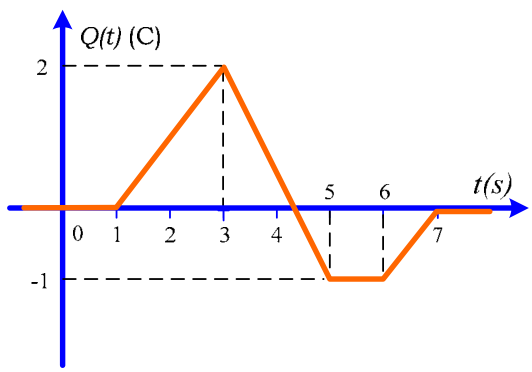
$$i=\frac{\Delta Q}{\Delta t} \qquad \textbf{(6)}$$

The unit of current is the Ampere (A), defined as the movement of one coulomb of net positive charge past a point in one second (1 A=1 C/s)
This rate of movement may not be constant but may vary with time. Hence the general definition of current is:

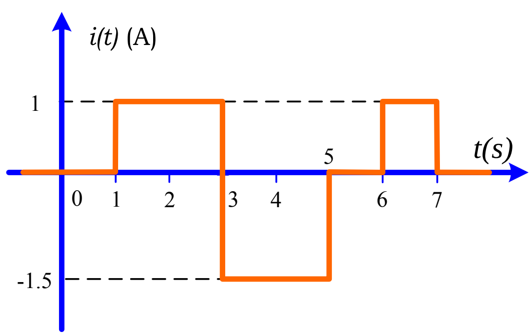
$$i(t) = \frac{dQ(t)}{dt} \quad \textbf{(7)}$$



For example:
Consider the plot of net positive charge moving past a point shown in Fig. Over the time interval 1 s ≤ t ≤ 3 s the net positive charge passing to the right is increasing at a rate of 1C/s, and 1 A of current results. Over 3 s ≤ t ≤ 5 s the net positive charge passing to the right is decreasing at a rate of -1.5C/s and hence the current at any point in this time interval is -1.5A.



(a)



(b)

Conversely, the net positive charge passing a point is:

$$Q(t) = \int_{-\infty}^{t} i(\tau)d\tau \qquad \textbf{(8)}$$

so that the net positive charge passing the point between two times is:

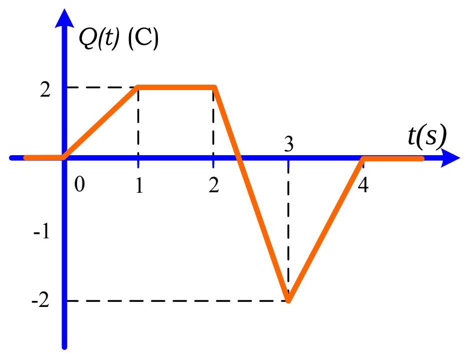
$$Q(t_2) - Q(t_1) = \int_{t_1}^{t_2} i(\tau)d\tau \qquad \textbf{(9)}$$

Equation (9) can be interpreted as the total area under the current -versus-time curve from the beginning of time to the present time.
For example, in Fig the net positive charge transferred to the right at t=6s is the sum of the areas under the i(t) curve up to that time. These areas are 1 Ax2s =2C. over 1s ≤ t ≤ 3s and -1.5 A x 2s = -3 C over 3 s ≤ t ≤ 5 s, giving a

total of -1 C at $t = 6$ s. Over the time interval $6 \text{ s} \leq t \leq 7$ s, 1 C=1 A x 1 s is transferred, yielding a net total charge transfer of 0 for $t > 7$ s.

Self Test:

The positive charge passing a point to the right is sketched in Fig. Determine the current directed to the right over the various time intervals.



Answers:

1A for 0<t<1s, 0A for 1<t<2s, -3A for 2<t<3s, 2A for 3<t<4s, and 0A for t>4s.