Assignment 1

Due Sunday, November 9, 2008

Chapter 4 (Textbook 8th Edition)

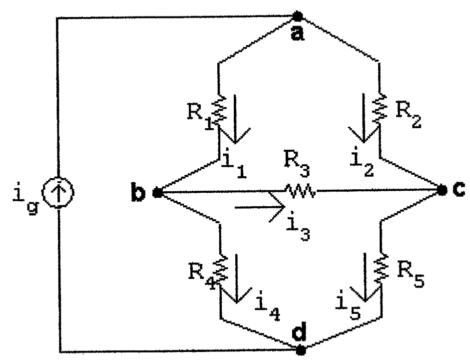
Problems 1,6,8,11,18,31,32,38

Problems

P 4.1 [a] There are six circuit components, five resistors and the current source. Since the current is known only in the current source, it is unknown in the five resistors. Therefore there are **five** unknown currents.

[b] There are four essential nodes in this circuit, identified by the dark black dots in Fig. P4.4. At three of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the fourth node would be dependent on the first three. Therefore there are **three** independent KCL equations.

[c]



Sum the currents at any three of the four essential nodes a, b, c, and d. Using nodes a, b, and c we get

$$-i_g + i_1 + i_2 = 0$$

$$-i_1 + i_4 + i_3 = 0$$

$$i_5 - i_2 - i_3 = 0$$

[d] There are three meshes in this circuit: one on the left with the components i_g, R_1, and R_4; one on the top right with components R_1, R_2, and R_3; and one on the bottom right with components R_3, R_4, and R_5. We cannot write a KVL equation for the left mesh because we don't know the voltage drop across the current source. Therefore, we can write KVL equations for the two meshes on the right, giving a total of **two** independent KVL equations.

[e] Sum the voltages around two independent closed paths, avoiding a path that contains the independent current source since the voltage across the current source is not known. Using the upper and lower meshes formed by the five resistors gives

$$R_1i_1 + R_3i_3 - R_2i_2 = 0$$

$$R_3i_3 + R_5i_5 - R_4i_4 = 0$$

[d] When a conductor joins the lower nodes of the two separate parts, there is now only a single part in the circuit. There would now be 4 nodes, because the two lower nodes are now joined as a single node. The number of branches remains at 7, where each branch contains one of the seven individual circuit components.

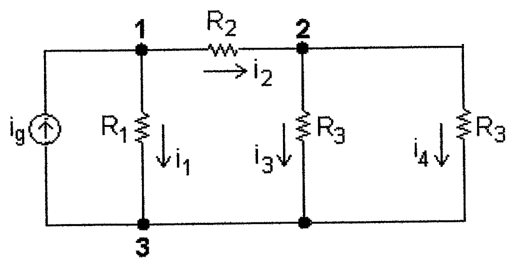
P 4.4 [a] From Problem 4.2(d) there are 10 essential branches were the current is unknown, so we need 10 simultaneous equations to describe the circuit.

[b] From Problem 4.2(f), there are 5 essential nodes, so we can apply KCL at $(5-1) = 4$ of these essential nodes. There would also be two dependent source constraint equations.

[c] The remaining 4 equations needed to describe the circuit will be derived from KVL equations.

[d] We must avoid using the meshes containing current sources, as we have no way of determining the voltage drop across a current source.

P 4.5



[a] At node 1: $\quad -i_g + i_1 + i_2 = 0$

At node 2: $\quad -i_2 + i_3 + i_4 = 0$

At node 3: $\quad i_g - i_1 - i_3 - i_4 = 0$

[b] There are many possible solutions. For example, solve the equation at node 1 for i_g:

$$i_g = i_1 + i_2$$

Substitute this expression for i_g into the equation at node 3:

$$(i_1 + i_2) - i_1 - i_3 - i_4 = 0 \qquad \text{so} \qquad i_2 - i_3 - i_4 = 0$$

Multiply this last equation by -1 to get the equation at node 2:

$$-(i_2 - i_3 - i_4) = -0 \qquad \text{so} \qquad -i_2 + i_3 + i_4 = 0$$

P 4.6 Use the lower terminal of the 5 Ω resistor as the reference node.

$$\frac{v_o - 60}{10} + \frac{v_o}{5} + 3 = 0$$

Solving, $\quad v_o = 10$ V

P 4.7 [a] From the solution to Problem 4.5 we know $v_o = 10$ V, therefore

$$p_{3\text{A}} = 3v_o = 30 \text{ W}$$

$$\therefore \; p_{3\text{A}} \text{ (developed) } = -30 \text{ W}$$

[b] The current into the negative terminal of the 60 V source is

$$i_g = \frac{60 - 10}{10} = 5 \text{ A}$$

$$p_{60\text{V}} = -60(5) = -300 \text{ W}$$

$$\therefore \; p_{60\text{V}} \text{ (developed) } = 300 \text{ W}$$

[c] $p_{10\Omega} = (5)^2(10) = 250$ W

$$p_{5\Omega} = (10)^2/5 = 20 \text{ W}$$

$$\sum p_{\text{dev}} = 300 \text{ W}$$

$$\sum p_{\text{dis}} = 250 + 20 + 30 = 300 \text{ W}$$

P 4.8 [a] $\dfrac{v_0 - 60}{10} + \dfrac{v_o}{5} + 3 = 0; \qquad v_o = 10$ V

[b] Let v_x = voltage drop across 3 A source

$$v_x = v_o - (10)(3) = -20 \text{ V}$$

$$p_{3\text{A}} \text{ (developed) } = (3)(20) = 60 \text{ W}$$

[c] Let i_g = be the current into the positive terminal of the 60 V source

$$i_g = (10 - 60)/10 = -5 \text{ A}$$

$$p_{60\text{V}} \text{ (developed) } = (5)(60) = 300 \text{ W}$$

[d] $\sum p_{\text{dis}} = (5)^2(10) + (3)^2(10) + (10)^2/5 = 360$ W

$$\sum p_{\text{dis}} = 300 + 60 = 360 \text{ W}$$

[e] v_o is independent of any finite resistance connected in series with the 3 A current source

P 4.9 $2.4 + \dfrac{v_1}{125} + \dfrac{v_1 - v_2}{25} = 0$

$$\frac{v_2 - v_1}{25} + \frac{v_2}{250} + \frac{v_2}{375} - 3.2 = 0$$

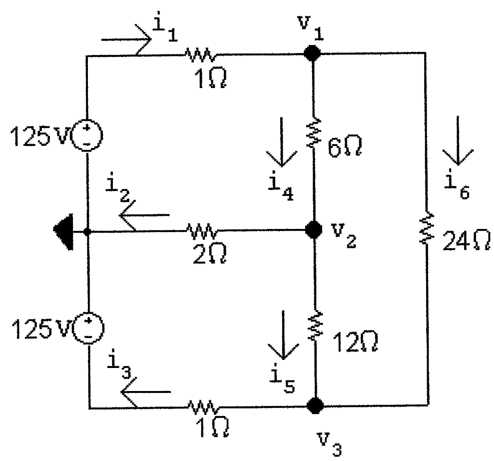
Solving, $v_1 = 25$ V; $\qquad v_2 = 90$ V

$$i_d = \frac{60}{20} = 3 \text{ A}$$

$$i_e = \frac{60 - 70}{10} = -1 \text{ A}$$

[b] $p_{dev} = 128(4) + 70(1) = 582$ W

P 4.11 **[a]**



$$\frac{v_1 - 125}{1} + \frac{v_1 - v_2}{6} + \frac{v_1 - v_3}{24} = 0$$
$$\frac{v_2 - v_1}{6} + \frac{v_2}{2} + \frac{v_2 - v_3}{12} = 0$$
$$\frac{v_3 + 125}{1} + \frac{v_3 - v_2}{12} + \frac{v_3 - v_1}{24} = 0$$

In standard form:

$$v_1\left(\frac{1}{1} + \frac{1}{6} + \frac{1}{24}\right) + v_2\left(-\frac{1}{6}\right) + v_3\left(-\frac{1}{24}\right) = 125$$
$$v_1\left(-\frac{1}{6}\right) + v_2\left(\frac{1}{6} + \frac{1}{2} + \frac{1}{12}\right) + v_3\left(-\frac{1}{12}\right) = 0$$
$$v_1\left(-\frac{1}{24}\right) + v_2\left(-\frac{1}{12}\right) + v_3\left(\frac{1}{1} + \frac{1}{12} + \frac{1}{24}\right) = -125$$

Solving, $v_1 = 101.24$ V; $v_2 = 10.66$ V; $v_3 = -106.57$ V

Thus, $i_1 = \frac{125 - v_1}{1} = 23.76$ A $\quad i_4 = \frac{v_1 - v_2}{6} = 15$ A

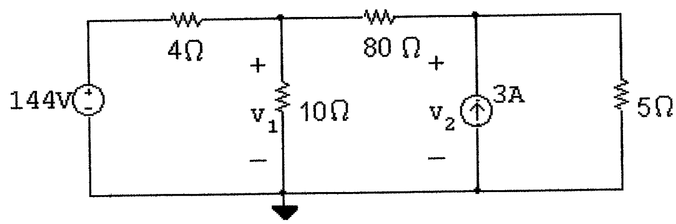
$i_2 = \frac{v_2}{2} = 5.33$ A $\quad i_5 = \frac{v_2 - v_3}{12} = 9.77$ A

$i_3 = \frac{v_3 + 125}{1} = 18.43$ A $\quad i_6 = \frac{v_1 - v_3}{24} = 8.66$ A

[b] $\sum P_{dev} = 125i_1 + 125i_3 = 5273.09$ W

$$\sum P_{dis} = i_1^2(1) + i_2^2(2) + i_3^2(1) + i_4^2(6) + i_5^2(12) + i_6^2(24) = 5273.09 \text{ W}$$

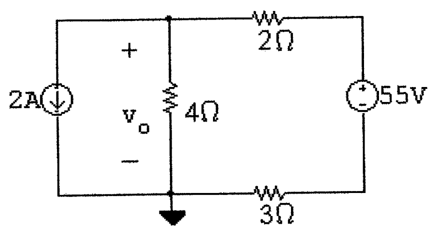
P 4.12



$$\frac{v_1 - 144}{4} + \frac{v_1}{10} + \frac{v_1 - v_2}{80} = 0 \qquad \text{so} \qquad 29v_1 - v_2 = 2880$$
$$-3 + \frac{v_2 - v_1}{80} + \frac{v_2}{5} = 0 \qquad \text{so} \qquad -v_1 + 17v_2 = 240$$

Solving, $v_1 = 100$ V; $v_2 = 20$ V

P 4.13

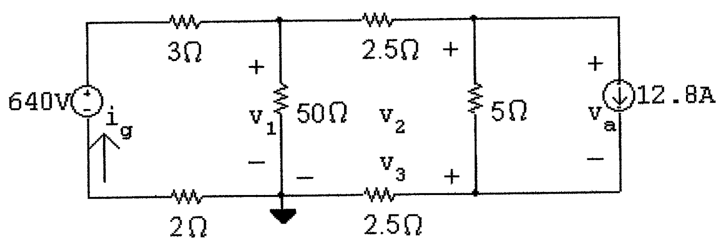


$$2 + \frac{v_o}{4} + \frac{v_o - 55}{5} = 0$$

$v_o = 20$ V

$p_{2A} = (20)(2) = 40$ W (absorbing)

P 4.14 [a]



$$\frac{v_1}{50} + \frac{v_1 - 640}{5} + \frac{v_1 - v_2}{2.5} = 0 \qquad \text{so} \qquad 31v_1 - 20v_2 + 0v_3 = 6400$$
$$\frac{v_2 - v_1}{2.5} + \frac{v_2 - v_3}{5} + 12.8 = 0 \qquad \text{so} \qquad -2v_1 + 3v_2 - v_3 = -64$$
$$\frac{v_3}{2.5} + \frac{v_3 - v_2}{5} - 12.8 = 0 \qquad \text{so} \qquad 0v_1 - v_2 + 3v_3 = 64$$

Solving, $v_1 = 380\ V$; $v_2 = 269$ V; $v_3 = 111$ V,

[b] $i_g = \dfrac{640 - 380}{5} = 52$ A

$p_g(\text{del}) = (640)(52) = 33{,}280$ W

P 4.16 [a] $\frac{v_o - v_1}{R} + \frac{v_o - v_2}{R} + \frac{v_o - v_3}{R} + \cdots + \frac{v_o - v_n}{R} = 0$

$\therefore \quad nv_o = v_1 + v_2 + v_3 + \cdots + v_n$

$\therefore \quad v_o = \frac{1}{n}[v_1 + v_2 + v_3 + \cdots + v_n] = \frac{1}{n}\sum_{k=1}^{n} v_k$

[b] $v_o = \frac{1}{3}(150 + 200 - 50) = 100$ V

P 4.17 $-3 + \frac{v_o}{200} + \frac{v_o + 5i_\Delta}{10} + \frac{v_o - 80}{20} = 0; \quad i_\Delta = \frac{v_o - 80}{20}$

[a] Solving, $v_o = 50$ V

[b] $i_{ds} = \frac{v_o + 5i_\Delta}{10}$

$i_\Delta = (50 - 80)/20 = -1.5$ A

$\therefore \quad i_{ds} = 4.25$ A; $\quad 5i_\Delta = -7.5$ V : $\quad p_{ds} = (-5i_\Delta)(i_{ds}) = 31.875$ W

[c] $p_{3A} = -3v_o = -3(50) = -150$ W (del)

$p_{80V} = 80i_\Delta = 80(-1.5) = -120$ W (del)

$\sum p_{del} = 150 + 120 = 270$ W

CHECK:

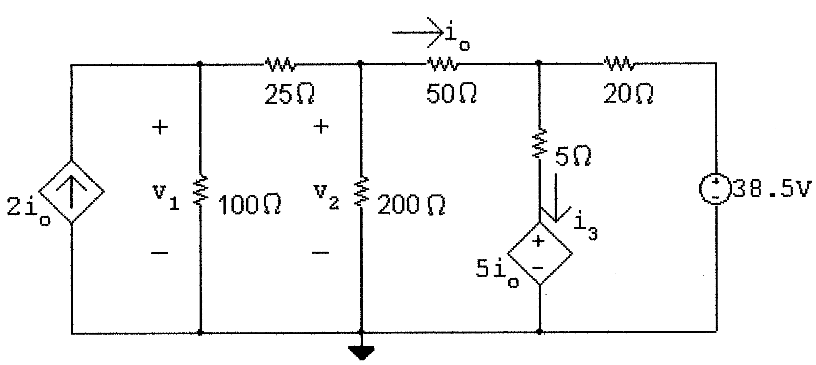
$p_{200\Omega} = 2500/200 = 12.5$ W

$p_{20\Omega} = (80 - 50)^2/20 = 900/20 = 45$ W

$p_{10\Omega} = (4.25)^2(10) = 180.625$ W

$\sum p_{diss} = 31.875 + 180.625 + 12.5 + 45 = 270$ W

P 4.18 [a]



$i_o = \frac{v_2 - v_3}{50}$

$$-2i_o + \frac{v_1}{100} + \frac{v_1 - v_2}{25} = 0 \qquad \text{so} \qquad 5v_1 - 8v_2 + 4v_3 = 0$$

$$\frac{v_2 - v_1}{25} + \frac{v_2}{200} + \frac{v_2 - v_3}{50} \qquad \text{so} \qquad -8v_1 + 13v_2 - 4v_3 = 0$$

$$\frac{v_3 - v_2}{50} + \frac{v_3 - 5i_o}{5} + \frac{v_3 - 38.5}{20} = 0 \qquad \text{so} \qquad 0v_1 - 4v_2 + 29v_3 = 192.5$$

Solving, $v_1 = -50$ V; $v_2 = -30$ V; $v_3 = 2.5$ V

[b] $$i_o = \frac{v_2 - v_3}{50} = \frac{-30 - 2.5}{50} = -0.65 \text{ A}$$

$$i_3 = \frac{v_3 - 5i_o}{5} = \frac{2.5 - 5(-0.65)}{5} = 1.15 \text{ A}$$

$$i_g = \frac{38.5 - 2.5}{20} = 1.8 \text{ A}$$

$$\sum p_{\text{dis}} = \sum p_{\text{dev}}$$

Calculate $\sum p_{\text{dev}}$ because we don't know if the dependent sources are developing or absorbing power. Likewise for the independent source.

$$p_{2i_o} = -2i_o v_1 = -2(-0.65)(-50) = -65 \text{ W(dev)}$$

$$p_{5i_o} = 5i_o i_3 = 5(-0.65)(1.15) = -3.7375 \text{ W(dev)}$$

$$p_g = -38.5(1.8) = -69.30 \text{ W(dev)}$$

$$\sum p_{\text{dev}} = 69.3 + 65 + 3.7375 = 138.0375 \text{ W}$$

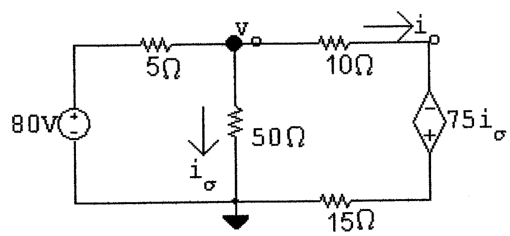
CHECK

$$\sum p_{\text{dis}} = \frac{2500}{100} + \frac{900}{200} + \frac{400}{25} + (0.65)^2(50) + (1.15)^2 5 + (1.8)^2(20)$$

$$= 138.0375 \text{ W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{dis}} = 138.0375 \text{ W}$$

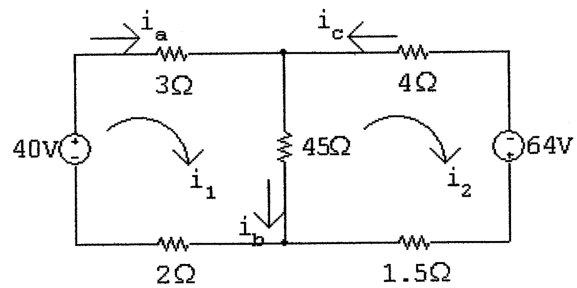
P 4.19



$$\frac{v_o - 80}{5} + \frac{v_o}{50} + \frac{v_o + 75i_\sigma}{25} = 0; \quad i_\sigma = \frac{v_o}{50}$$

Solving, $v_o = 50$ V; $i_\sigma = 1$ A

P 4.31 [a]



$$40 = 50i_1 - 45i_2$$

$$64 = -45i_1 + 50.5i_2$$

Solving, $i_1 = 9.8$ A; $i_2 = 10$ A

$i_a = i_1 = 9.8$ A; $i_b = i_1 - i_2 = -0.2$ A; $i_c = -i_2 = -10$ A

[b] If the polarity of the 64 V source is reversed, we have

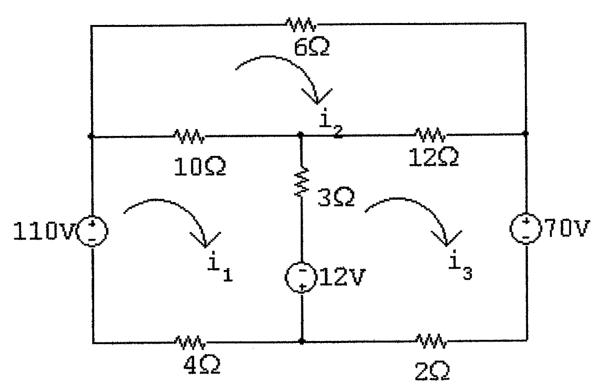
$$40 = 50i_1 - 45i_2$$

$$-64 = -45i_1 + 50.5i_2$$

$i_1 = -1.72$ A and $i_2 = -2.8$ A

$i_a = i_1 = -1.72$ A; $i_b = i_1 - i_2 = 1.08$ A; $i_c = -i_2 = 2.8$ A

P 4.32 [a]



$$110 + 12 = 17i_1 - 10i_2 - 3i_3$$

$$0 = -10i_1 + 28i_2 - 12i_3$$

$$-12 - 70 = -3i_1 - 12i_2 + 17i_3$$

Solving, $i_1 = 8$ A; $i_2 = 2$ A; $i_3 = -2$ A

$$p_{110} = -110i_1 = -880 \text{ W(del)}$$

$$p_{12} = -12(i_1 - i_3) = -120 \text{ W(del)}$$

$$p_{70} = 70i_3 = -140 \text{ W(del)}$$

$$\therefore \sum p_{\text{dev}} = 1140 \text{ W}$$

[b] $p_{4\Omega} = (8)^2(4) = 256$ W

$p_{10\Omega} = (6)^2(10) = 360$ W

$p_{12\Omega} = (-4)^2(12) = 192$ W

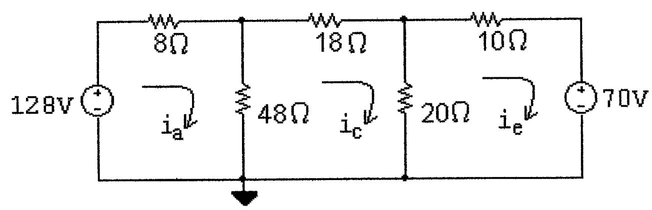
$p_{2\Omega} = (-2)^2(2) = 8$ W

$p_{6\Omega} = (2)^2(6) = 24$ W

$p_{3\Omega} = (10)^2(3) = 300$ W

$\therefore \sum p_{\rm abs} = 1140$ W

P 4.33 [a]



The three mesh current equations are:

$$-128 + 8i_a + 48(i_a - i_c) = 0$$

$$18i_c + 20(i_c - i_e) + 48(i_c - i_a) = 0$$

$$70 + 20(i_e - i_c) + 10i_e = 0$$

Place these equations in standard form:

$$i_a(8+48) + i_c(-48) + i_e(0) = 128$$

$$i_a(-48) + i_c(18+20+48) + i_e(-20) = 0$$

$$i_a(0) + i_c(-20) + i_e(20+10) = -70$$

Solving, $i_a = 4$ A; $i_c = 2$ A; $i_e = -1$ A
Now calculate the remaining branch currents:

$$i_b = i_a - i_c = 2 \text{ A}$$

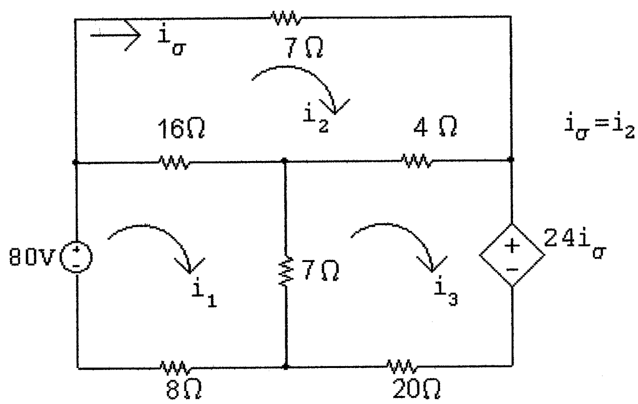
$$i_d = i_c - i_e = 3 \text{ A}$$

[b] $p_{\rm sources} = p_{128V} + p_{70V} = -(128)i_a + (70)i_e$

$= -(128)(4) + (70)(-1) = -512 - 70 = -582$ W

Thus, the power developed in the circuit is 582 W. Note that the resistors cannot develop power!

P 4.37



$-80 + 31i_1 - 16i_2 - 7i_3 = 0$

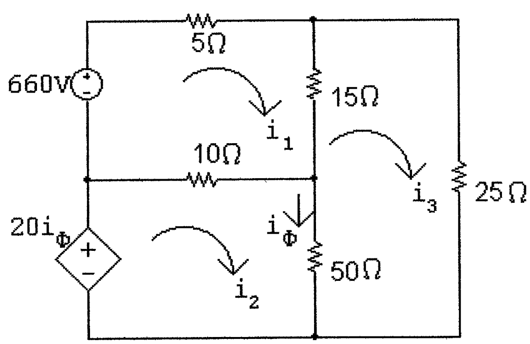
$-16i_1 + 27i_2 - 4i_3 = 0$

$-7i_1 - 4i_2 + 31i_3 + 24i_2 = 0$

Solving, $i_1 = 3.5$ A

$p_{8\Omega} = (3.5)^2(8) = 98$ W

P 4.38



$660 = 30i_1 - 10i_2 - 15i_3$

$20i_\phi = -10i_1 + 60i_2 - 50i_3$

$0 = -15i_1 - 50i_2 + 90i_3$

$i_\phi = i_2 - i_3$

Solving, $i_1 = 42$ A; $i_2 = 27$ A; $i_3 = 22$ A; $i_\phi = 5$ A

$20i_\phi = 100$ V

$p_{20i_\phi} = -100i_2 = -100(27) = -2700$ W

$\therefore\ p_{20i_\phi}$ (developed) $= 2700$ W

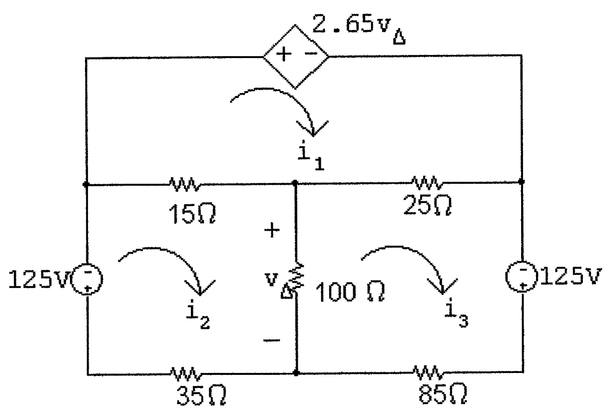
CHECK:

$p_{660\text{V}} = -660(42) = -27{,}720$ W (dev)

$$\therefore \sum P_{\text{dev}} = 27{,}720 + 2700 = 30{,}420 \text{ W}$$

$$\sum P_{\text{dis}} = (42)^2(5) + (22)^2(25) + (20)^2(15) + (5)^2(50) + (15)^2(10) = 30{,}420 \text{ W}$$

P 4.39



Mesh equations:

$2.65v_\Delta + 40i_1 - 15i_2 - 25i_3 = 0$

$-15i_1 + 150i_2 - 100i_3 = -125$

$-25i_1 - 100i_2 + 210i_3 = 125$

Constraint equations:

$v_\Delta = 100(i_2 - i_3)$

Solving, $i_1 = 7$ A; $\quad i_2 = 1.2$ A; $\quad i_3 = 2$ A

$v_\Delta = 100(i_2 - i_3) = 100(1.2 - 2) = -80$ V

$p_{2.65v_\Delta} = 2.65v_\Delta i_1 = -1484$ W