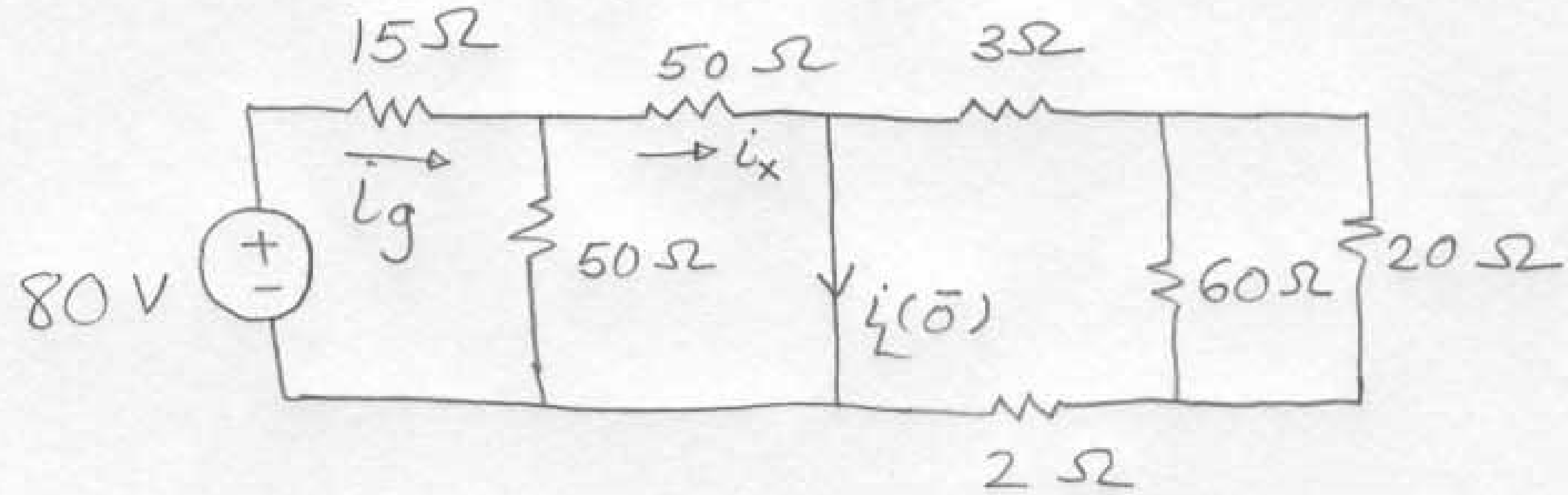
EE 201 HW # 9 Solution

P7.6 a)

For $t < 0$

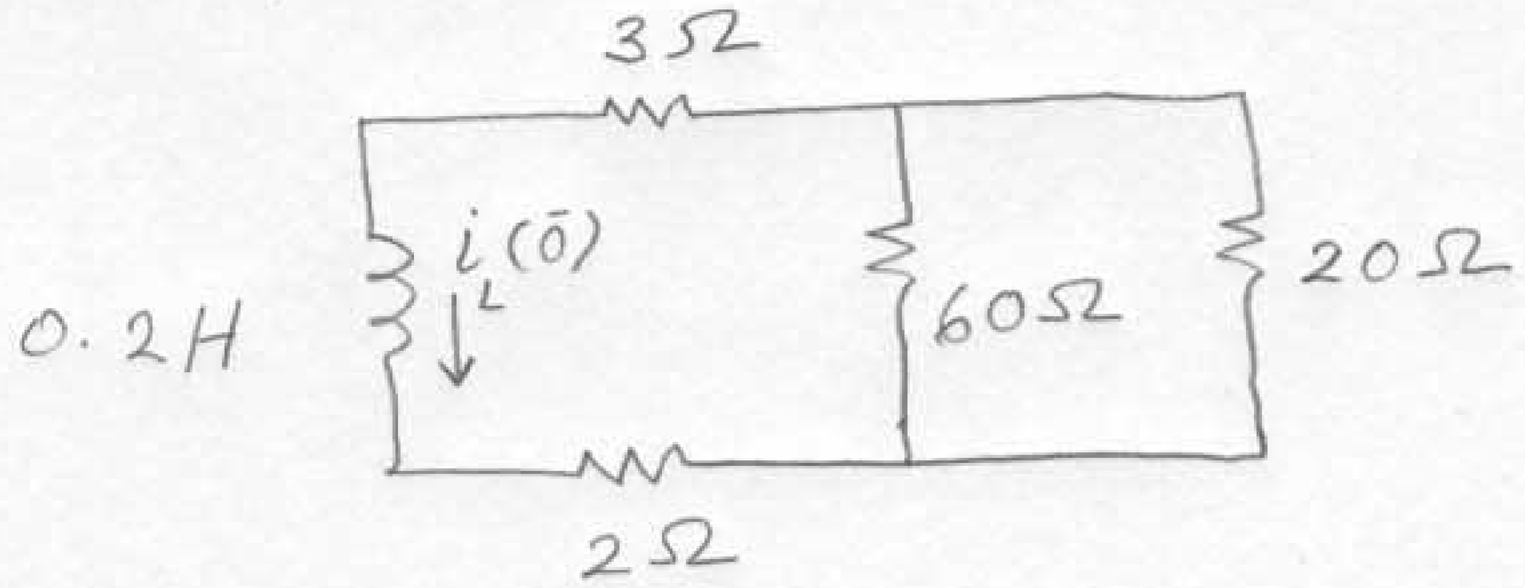


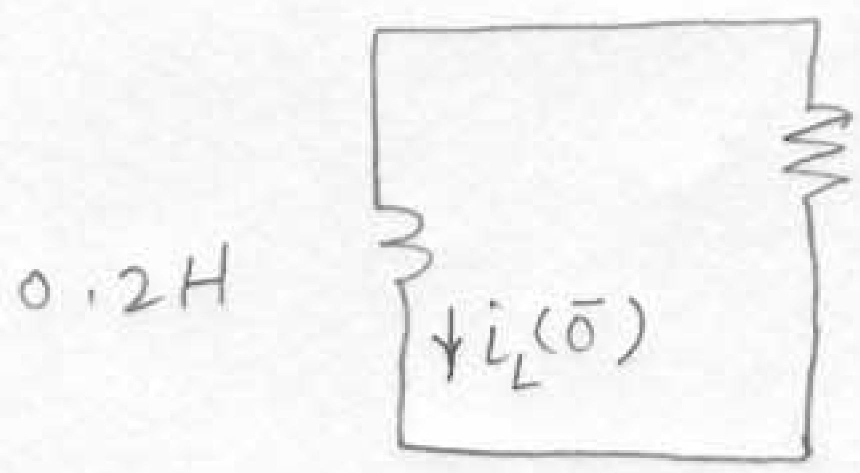
$$i_g = \frac{80}{40} = 2\,A$$

$$i_L(0^-) = i_x = \frac{50}{50+50}(2) = 1\,A = i_L(0^+)$$, because $i_L(t)$ is continuous.

$\therefore i_L(t) = 1\,A,\ t < 0$

$t > 0$





$$3+2+60/\!/20 = 5+\frac{1200}{80} = 5+15 = 20\Omega$$

$$i_L(t) = i_L(0)\,e^{-t/\tau}, \quad i_L(0) = 1\,A, \quad \tau = \frac{L}{R} = \frac{0.2}{20} = \frac{1}{100}$$

$$\therefore i_L(t) = e^{-100t}\ A, \quad t \geq 0$$

$$v_o(t) = 15\,i_L(t) = 15\,e^{-100t}, \quad t \geq 0^+$$, because $v_o(t)$ is discontinuous at $t=0$.

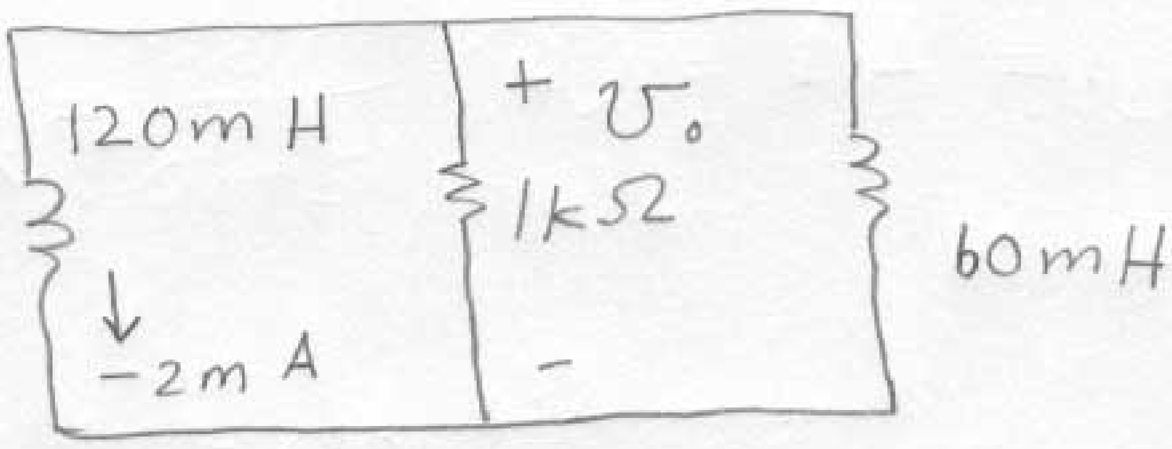
P7.14 a)

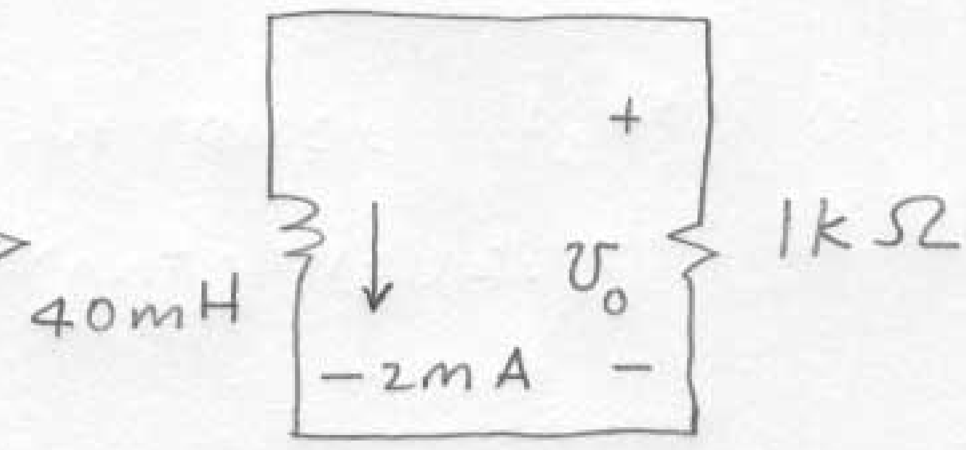
$t < 0$

$$i_L(t) = \frac{-2.5 \times 16}{16+4} = -2\,mA \quad , t < 0$$

$$\therefore i_L(0^-) = -2mA.$$

$t \geq 0$





$$L_{eq} = \frac{120(60)}{120+60} = \frac{120}{3} = 40\,mH$$

$$\tau = \frac{L}{R} = \frac{40mH}{10^3} = 40 \times 10^{-6}, \quad \frac{1}{\tau} = 25 \times 10^3$$

$$i_L(t) = -2 \times 10^{-3} e^{-25,000t}$$

$$\therefore v_o = -1000 i_L(t) = 2e^{-25,000t} \text{ V} , \; t \geq 0^+ .$$

b) $$W_{del.} = \frac{1}{2} L i^2 = \frac{1}{2}(40 \times 10^{-3})(-2 \times 10^{-3})^2 = 80 \times 10^{-9} \text{J}$$

$$= 80\,nJ$$

c) $$0.95 W_{del.} = 76\,nJ$$

$$\therefore 76 \times 10^{-9} = \int_0^{t_o} \underbrace{-i_L(t)\, v_o(t)}_{P(t)}\, dt$$

$$= \int_0^{t_o} 4 \times 10^{-3} e^{-50,000t}\, dt$$

$$= 80 \times 10^{-9} e^{-50,000t} \Big|_0^{t_o}$$

$\therefore\ 76\times10^{-9} = 80\times10^{-9}(1 - e^{-50{,}000t_0})$

$\therefore\ e^{-50{,}000t_0} = 0.05$

$\therefore\ -50{,}000\,t_0 = \ln 0.05$

$\therefore\ t_0 = 59.9\ \mu S$

$\tau = 40\mu s \quad \therefore\ \frac{t_0}{\tau} = \frac{59.9\mu s}{40\mu s} = 1.497\ ,\ \therefore t_0 \simeq 1.5\tau .$

P7.19

$i_L(t) = I_{final} + (I_{initial} - I_{final})e^{-tR/L} = I_F + (I_i - I_F)e^{-tR/L}$

$I_F = 0\ ,\quad I_i = 10mA$

$\therefore\ i_L(t) = 10\,e^{-tR/L}\ mA$

$v_L(t) = L\frac{di_L}{dt} = -10R\,e^{-tR/L}\ mA\ ,\quad R = 50\Omega$

$= -500\,e^{-tR/L}\ mV\ ,\ \therefore v_L(0^+) = -500mV$

$v(5\times10^{-3}) = -500\,e^{-5\times10^{-3}R/L}\ m$

$\therefore\ e^{-5\times10^{-3}\frac{R}{L}} = \frac{1}{4}$

$\therefore\ L = \frac{250\times10^{-3}}{\ln 4} = 180.34\,mH$

b) $i_L(0^-) = 60\left(\frac{1}{6}\right) = 10mA = i_L(0^+)$

$w_{stored} = \frac{1}{2}L\,i_L(0^+)^2 = \frac{1}{2}(180.34m)(10\times10^{-3})^2$

$= 9.02\ \mu J$

or $W_{stored} = \frac{1}{2}L\,i_L(0^+)^2 = \frac{1}{2}(R\tau)(i_L(0^+))^2 = 2500\tau\ \mu J$

$i_L(t) = 10\, e^{-t/\tau}$ mA

$P_{50\Omega} = i_L^2(50) = 5000 \times 10^{-6} e^{-2t/\tau}$ W

$$W_{dis} = \int_0^{5\times10^{-3}} P_{50\Omega}(t)\,dt = \int_0^{5\times10^{-3}} 5000\times10^{-6} e^{-2t/\tau}\,dt$$

$$= 5000\times10^{-6} \frac{e^{-2t/\tau}}{(-2/\tau)}\Bigg|_0^{5\times10^{-3}}$$

$$= 2500\times10^{-6}\tau\left[1 - e^{-10\times10^{-3}/\tau}\right]$$

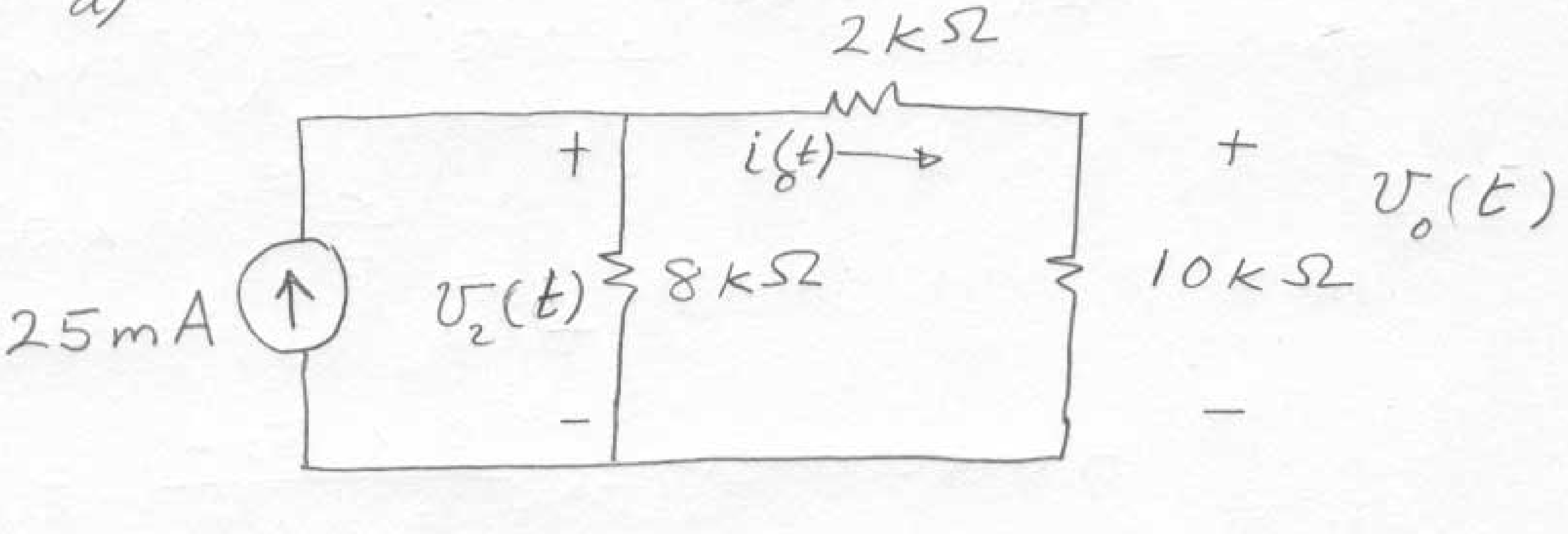
$$e^{-10\times10^{-3}/\tau} = e^{-2\ln 4} = 0.0625$$

$$\therefore\ W_{dis} = 2500\times10^{-6}\tau(0.9375)$$

$$\%\ W_{dis} = \frac{2500\times10^{-6}\tau(0.9375)}{2500\times10^{-6}\tau}\times 100 = 93.75\%$$

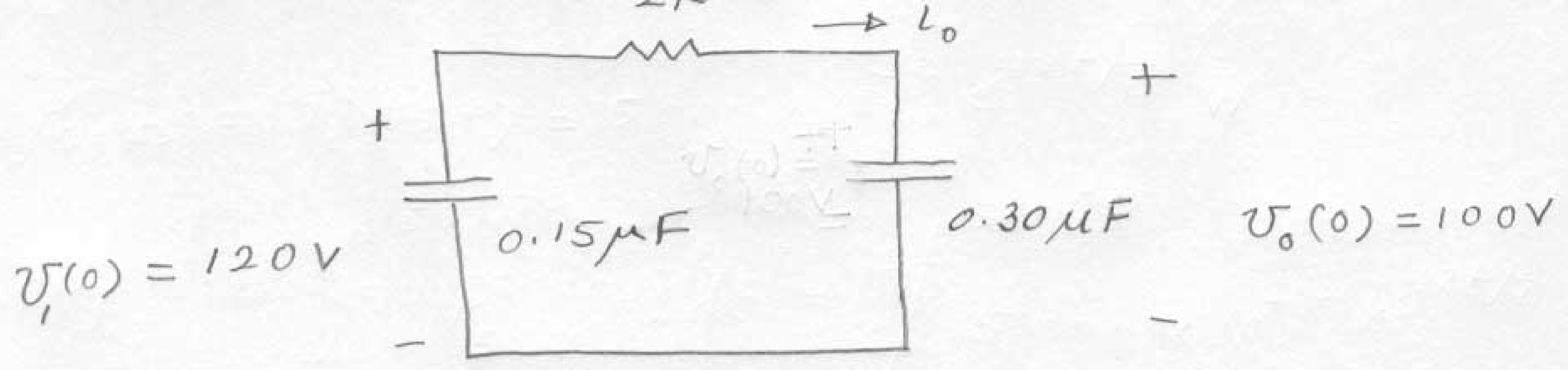
P 7.28 a)

$t < 0$

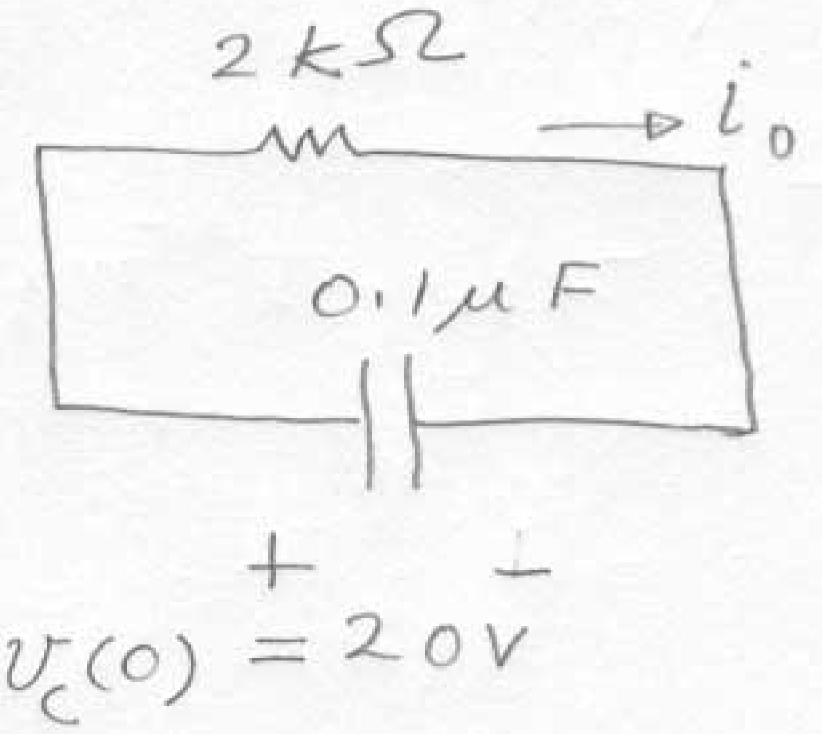


$$\left.\begin{aligned} i_o(t) &= 25\times\frac{8}{20} = 10\text{ mA} \\ v_o(t) &= 10(10) = 100\text{ V} \\ i_2(t) &= 25 - 10 = 15\text{ mA} \\ v_2(t) &= 15(8) = 120\text{ V} \end{aligned}\right\}\quad t < 0$$

$t > 0$



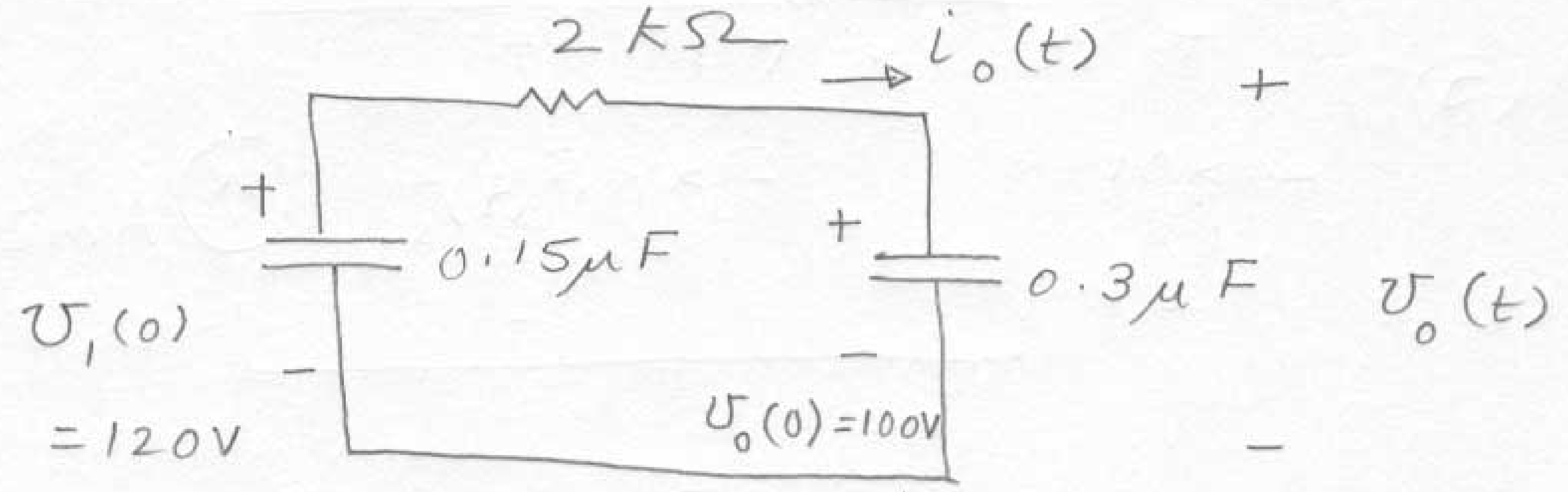
$$\tau = RC = 0.2\,ms = 200\,\mu s, \quad \frac{1}{\tau} = 5000$$



$$i_o(t) = \frac{20}{2 \times 10^3} e^{-t/\tau} = 10 \times 10^{-3} e^{-5000t}, \quad t \geq 0^+$$

$$\left\{ \text{because } i_o(t) = -C \frac{dv_c}{dt} = -C \frac{d}{dt}\left[v_C(0) e^{-t/RC} \right] = \frac{v_C(0)}{R} e^{-t/RC} \right\}$$

b)



$$v_o(t) = \frac{1}{C} \int_0^t i_o(t)\,dt + v_o(0)$$

$$= \frac{10^6}{0.3} \int_0^t 10 \times 10^{-3} e^{-5000t}\,dt + 100$$

$$= \frac{10^5}{3} \left. \frac{e^{-5000t}}{-5000} \right|_0^t + 100$$

$$= -\frac{20}{3} e^{-5000t} + \frac{20}{3} + 100$$

$$\therefore v_o(t) = -\frac{20}{3} e^{-5000t} + \frac{320}{3} \text{, V} \quad \text{for } t \geq 0.$$

c) $$W_{trapped} = \frac{1}{2}(0.15) \times 10^{-6} \left(\frac{320}{3}\right)^2 + \frac{1}{2}(0.3) \times 10^{-6} \left(\frac{320}{3}\right)^2$$

$$= 2560 \, \mu J$$

Check: $$W_{dis.} = \frac{1}{2}(0.1 \times 10^{-6})(20)^2 = 20 \mu J.$$

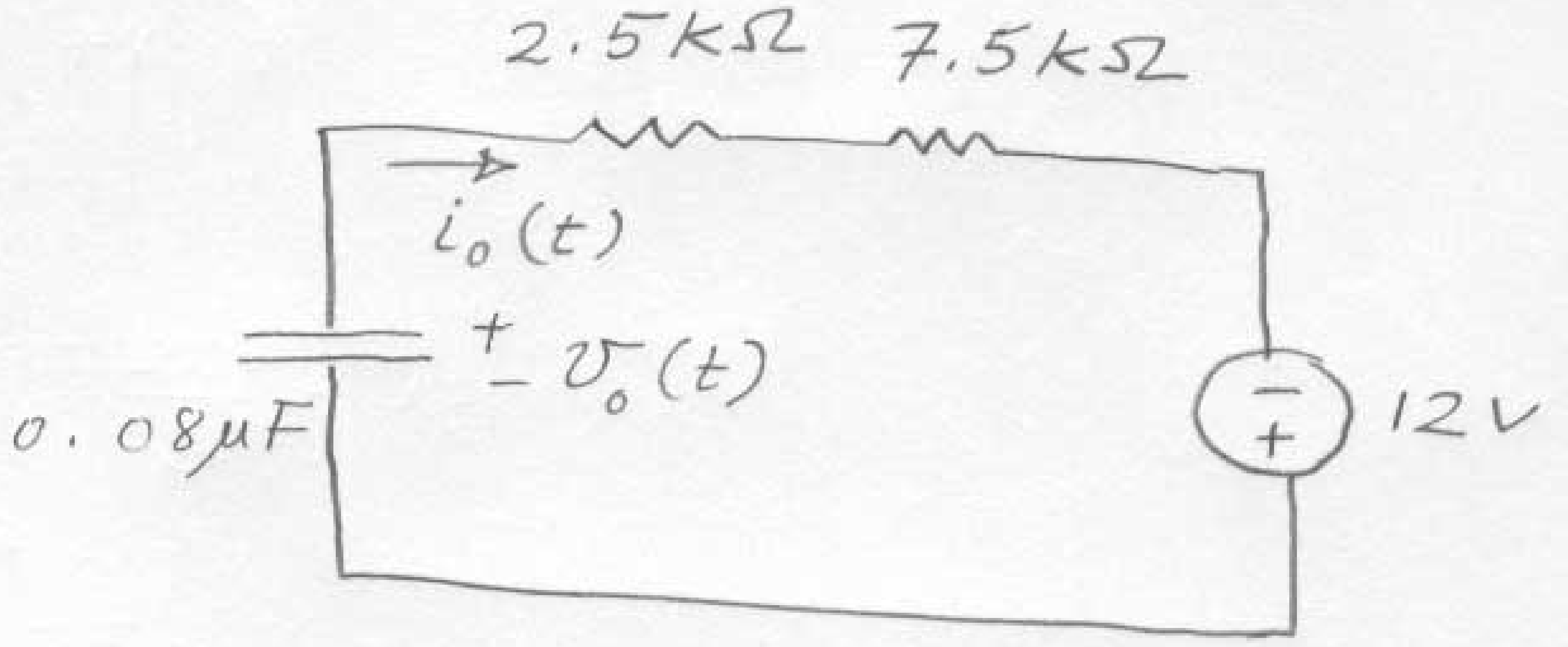
$$W(0) = \frac{1}{2}(0.15) \times 10^{-6}(120)^2 + \frac{1}{2}(0.3 \times 10^{-6})(100)^2$$

$$= 2580 \, \mu J$$

$$W_{trapped} + W_{dis} = W(0)$$

$$2560 + 20 = 2580 \text{ (checks).}$$

P 7.58 a)



$v_o(0^-) = v_o(0^+) = 48\text{ V}$

$v_o(\infty) = -12\text{ V}$

$\tau = 0.8\text{ ms}, \quad \frac{1}{\tau} = 1250\text{ sec.}$

$v_o = -12 + (48 - (-12))e^{-1250t}$

$v_o(t) = -12 + 60e^{-1250t}\text{ V}, \quad t \geq 0.$

b) $i_o(t) = -0.08 \times 10^{-6}\left[-75{,}000e^{-1250t}\right]$

$= 6e^{-1250t}\text{ mA}, \quad t \geq 0^+.$

c) $v_g = v_o - 2.5 \times 10^3 i_o$

$= -12 + 45e^{-1250t}\text{ V}$

d) $v_g(0^+) = -12 + 45 = 33\text{ V}$

check: $v_g(0^+) = i_o(0^+)\, 7.5 \times 10^3 - 12 = 45 - 12 = 33\text{ V}$

$i_{10k} = \frac{v_g}{10k} = -1.2 + 4.5e^{-1250t}\text{ mA}$

$i_{30k} = \frac{v_g}{30k} = -0.4 + 1.5e^{-1250t}\text{ mA}$

$-i_o + i_{10} + i_{30} + 1.6m = -6e^{-1250t}m$

$+(-1.2 + 4.5e^{-1250t})m + (-0.4 + 1.5e^{-1250t})m$

$+1.6m = 0$ (checks).