Example I (Corrected) Find the mutual information different ways?

$$P_C = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}, \quad P_{Y|C} = \begin{bmatrix} 0.95 & 0.25 \\ 0.05 & 0.75 \end{bmatrix} \Rightarrow P_Y = P_{Y|C} \cdot P_C = \begin{bmatrix} 0.67 \\ 0.33 \end{bmatrix}$$

$$I(C;Y) = \sum_C \sum_Y P_{c,y} \log_2\left(\frac{P_{c,y}}{P_c \cdot P_y}\right) \quad \text{——} ①$$

$$= \sum_C P_c \sum_Y P_{y|c} \log_2\left(\frac{P_{y|c}}{P_y}\right) \quad \text{——} ② \quad P_{c,y} = P_{y|c} \cdot P_c$$

$$= H(C) - H(C|Y) \quad \text{——} ③$$

$$= H(Y) - H(Y|C) \quad \text{——} ④$$

Method 1:

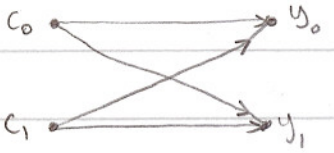
We need to find the joint probabilities:

$P_{c_0,y_0} = P_{y_0|c_0} * P_{c_0} = 0.95 * 0.6 = 0.57$

$P_{c_1,y_0} = P_{y_0|c_1} * P_{c_1} = 0.25 * 0.4 = 0.1$

$P_{c_0,y_1} = P_{y_1|c_0} * P_{c_0} = 0.05 * 0.6 = 0.03$

$P_{c_1,y_1} = P_{y_1|c_1} * P_{c_1} = 0.75 * 0.4 = 0.3$



Therefore,

	c_0	c_1
y_0	0.57	0.1
y_1	0.03	0.3

$$\therefore I(C;Y) = \sum_C \sum_Y P_{c,y} \log_2\left(\frac{P_{c,y}}{P_c \cdot P_y}\right)$$

$$= 0.57 \log_2\left(\frac{0.57}{0.6*0.67}\right) + 0.03 \log_2\left(\frac{0.03}{0.6*0.33}\right)$$

$$+ 0.1 \log_2\left(\frac{0.1}{0.4*0.67}\right) + 0.3 \log_2\left(\frac{0.3}{0.4*0.33}\right)$$

$$= \boxed{0.4186 \text{ bits}}$$

Method 2:

$$I(C;Y) = \sum_C P_C \sum_Y P_{y|c} \log_2\left(\frac{P_{y|c}}{P_y}\right)$$

$$= 0.6\left[0.95\log_2\left(\frac{0.95}{0.67}\right) + 0.05\log_2\left(\frac{0.05}{0.33}\right)\right]$$

$$+ 0.4\left[0.25\log_2\left(\frac{0.25}{0.67}\right) + 0.75\log_2\left(\frac{0.75}{0.33}\right)\right]$$

$$= \boxed{0.4186 \text{ bits}}$$

Method 3 & 4:

$$H(C) = 0.6\log_2\left(\frac{1}{0.6}\right) + 0.4\log_2\left(\frac{1}{0.4}\right) = 0.9710 \text{ bits}$$

$$H(C;Y) = 0.57\log_2\left(\frac{1}{0.57}\right) + 0.1\log_2\left(\frac{1}{0.1}\right) + 0.03\log_2\left(\frac{1}{0.03}\right) + 0.3\log_2\left(\frac{1}{0.3}\right)$$
$$= 1.4673 \text{ bits}$$

$$H(Y) = 0.67\log_2\left(\frac{1}{0.67}\right) + 0.33\log_2\left(\frac{1}{0.33}\right) = 0.9149 \text{ bits}$$

$$H(C|Y) = H(C;Y) - H(Y) = 1.4673 - 0.9149$$
$$= 0.5524 \text{ bits}$$

$$H(Y|C) = H(C;Y) - H(C) = 1.4673 - 0.9710$$
$$= 0.4963 \text{ bits}$$

$$I(C;Y) = H(C) - H(C|Y) = 0.9710 - 0.5524$$
$$= \boxed{0.4186 \text{ bits}}$$

$$I(C;Y) = H(Y) - H(Y|C) = 0.9149 - 0.4963$$
$$= \boxed{0.4186 \text{ bits}}$$