Serial # 0

-1 points for not writing your serial number

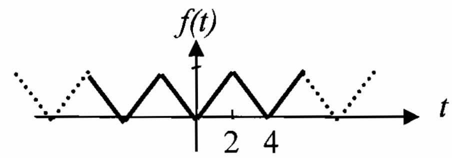
King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE207: Signals and Systems (081)
Quiz 3: Fourier Series

Name: KEY ver. 1

Find the Fourier series of the periodic function $f(t)$. The period= 4.

Simplify your answer.

$$f(t) = |t|, \text{ for } -2 \le t \le 2.$$



$$\int t\cos(bt)dt = \frac{t\sin(bt)}{b} + \frac{\cos(bt)}{b^2}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

. Since the signal is even $\Rightarrow b_n = 0$ for all n.

. The signal has similar to half wave odd symmetry property $\Rightarrow a_n = 0$ for even n.

$$a_0 = \frac{1}{T}\int_{-2}^{2} |t|\, dt = \frac{2}{T}\int_0^2 t\, dt$$

$$= \frac{1}{2}\left[\frac{t^2}{2}\right]_0^2 = \frac{1}{2}\left[\frac{4}{2}\right] = 1$$

Also : can be justified by inspection because the curve varies between 0 & 2 linearly average is 1.

$$a_n = \frac{2}{T_0}\int_{T_0} f(t)\cos n\omega_0 t\, dt \qquad \underline{\underline{n \neq 0}}$$

$$= \frac{1}{2}\int_{-2}^{2} |t| \cos n\omega_0 t\, dt$$

Symmetric problem

$$= \frac{\cancel{2}}{\cancel{2}}\int_0^2 t\cos n\omega_0 t\, dt$$

using the given formula.

$$= \left.\frac{t\sin(n\omega_0 t)}{n\omega_0} + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2}\right|_0^2$$

$$= \frac{2\sin(2n\omega_0)}{n\omega_0} + \frac{\cos 2n\omega_0}{(n\omega_0)^2} - \frac{1}{(n\omega_0)^2}$$

Since $T_0 = 4 \Rightarrow \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$$\Rightarrow a_n = \cancel{\frac{2\sin(n\pi)}{n\pi/2}}^{\,0} + \frac{\cos(n\pi)}{(n\frac{\pi}{2})^2} - \frac{1}{(n\frac{\pi}{2})^2}$$

note that $\sin n\pi = 0$

also note that

$$\cos n\pi = \begin{cases} -1 & \text{odd } n \\ +1 & \text{even } n \end{cases}$$

$\Rightarrow$ for even n

$$a_n = \frac{1}{(n\frac{\pi}{2})^2} - \frac{1}{(n\frac{\pi}{2})^2} = 0 \quad \text{as Expected}$$

for odd n

$$a_n = \frac{-2}{(n\frac{\pi}{2})^2} = \frac{-8}{n^2\pi^2}$$

$$\Rightarrow \boxed{f(t) = 1 - \frac{8}{\pi^2}\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2}\cos\frac{\pi n t}{2}}$$

Plan to learn and learn to plan Good Luck, **Dr. Ali Muqaibel**

Engineering is application of Mathematics to physical problems.