

Name: KEY

ver. 1

1. Is the following system linear or non linear prove your answer?

$$\frac{d^2 y(t)}{dt^2} = x(t) + 100$$

It is not linear.

if you multiply by 2  
if it is linear.

2x(t) should result  
in 2y(t)

$$\frac{2 dy(t)}{dt^2} = 2x(t) + 200 \quad \text{---(1)}$$

$$\frac{2 dy(t)}{dt^2} = 2x(t) + 100 \quad \text{---(2)}$$

Eq. (1) &  
(2)  
cannot be  
both correct.

Also, it is not in the fixed ODE format.

2. Is the following system fixed? justify your answer  $5y(t) = 5x(t) + 5t$

No. The system is not fixed. The time change is excepted.

$$y(t) = x(t) + t$$

at  $t=0$        $y(t) = x(t)$

at  $t=1$        $y(t) = x(t) + 1$

the relation between the input & output varies with time.

3. The Impulse response of an LTI system is given by the following

$$h(t) = 3e^{-10t}u(t)$$

Find the output when the input is  $x(t) = 5\delta(t) + u(t) + 2e^{-2t}u(t)$

Since the system is linear.

We can find the output to the individual terms.

$$5\delta(t) \rightarrow 5h(t) = 15e^{-10t}u(t)$$

$$u(t) \rightarrow a(t) = \int_{-\infty}^t h(\tau) d\tau$$

$$\Rightarrow a(t) = \int_{-\infty}^t 3e^{-10\tau}u(\tau) d\tau = \begin{cases} 0 & t < 0 \\ 3 \int_0^t e^{-10\tau} d\tau & t > 0 \end{cases}$$

$$u(t) \left[ \frac{-3}{10} e^{-10\tau} \right]_0^t = \frac{-3}{10} [e^{-10t} - 1] u(t)$$

$$a(t) = \frac{3}{10} [1 - e^{-10t}] u(t)$$

last term by convolution.

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} 3e^{-10\tau}u(\tau) \cdot 2e^{-2(t-\tau)}u(t-\tau) d\tau$$

$$6 \int_0^t e^{-8\tau-2t} d\tau \quad \begin{matrix} 0 < -t < 0 \\ \dots & t > 0 \end{matrix}$$

$$= \frac{-6}{8} \left[ e^{-8\tau-2t} \right]_0^t$$

$$= -\frac{3}{4} [e^{-10t} - e^{-2t}] u(t)$$

$$\text{Sum } y(t) = \left[ 15e^{-10t} + \frac{3}{10} - \frac{3}{10}e^{-10t} - \frac{3}{4}e^{-10t} + \frac{3}{4}e^{-2t} \right] u(t)$$

$$= \left[ \frac{3}{10} + \frac{3}{4}e^{-2t} + \left( 15 - \frac{3}{10} - \frac{3}{4} \right) e^{-10t} \right] u(t)$$