King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE 207 – Signals and Systems

Serial Number
0

Major Exam 2

December 31, 2008

Time : 7:00-8:30pm (1 ½ Hours)

Student Name : KEY

Student ID Number : 000

Problem	Max Score	Score
Problem 1	10	
Problem 2	10	
Problem 3	10	
Total	30	

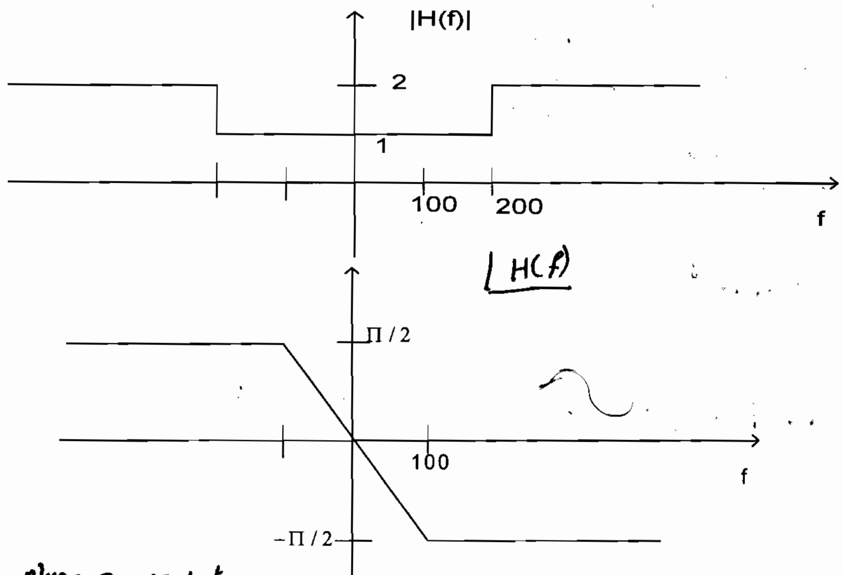
4 Tables attached

La Place
&
Fourier.

Problem 1:

Consider the following input signal: $x(t) = \cos(2\pi 20t) + \cos(2\pi 80t) + \cos(2\pi 140t)$

This signal is input to an LTI system having frequency response H(f) with magnitude and phase spectra below:



2.5 if phase approach

4 a. Determine the output signals $y(t)$ 0.5

2 b. Determine whether there is a distortion or not, if yes, show whether it is a magnitude distortion, a phase distortion, or both. "for the given input" 1 0.5

2 c. Sketch the magnitude spectrum of the signal $x(t)$

2 d. Find the power of the signal $x(t)$. Verify your answer using Parsaval's theorem (*Hint: you need to find the power in two different ways and show that they are equivalent*)

a) The signal has three freq. components at 20, 80, & 140 Hz.

The magnitude is one for all from $|H(f)|$ amp 1

The phase of the output signal.

$\angle H(f) = -\frac{\pi/2}{100} f \qquad f \leq 100$

straight line with slope $\frac{\Delta y}{\Delta x} = \frac{-\pi}{200} f$

$\angle H(f=20) = -\pi/10 \equiv -18^\circ$

$\angle H(f=80) = \frac{-\pi}{200} 80 = \frac{-2\pi}{5} \equiv -72^\circ$

$\angle H(f=140) = -\pi/2 \equiv -90^\circ$

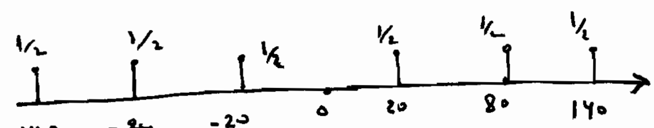
Since

$$\angle H(f) = \begin{cases} +\pi/2 & f \leq -100\ Hz \\ -\frac{\pi}{200} f & |f| \leq 100\ Hz \\ -\pi/2 & f \geq 100\ Hz \end{cases}$$

$\Rightarrow y(t) = \cos\left(2\pi 20t - \frac{\pi}{10}\right) + \cos\left(2\pi 80t - \frac{2\pi}{5}\right) + \cos\left(2\pi 140t - \pi/2\right)$

b) There is phase distortion because the magnitude is constant but the phase is not linear ($f = 140$ Hz !)

c) $x(t) = \frac{1}{2}\left[e^{j(2\pi 20t)} + e^{-j(2\pi 20t)}\right] + \frac{1}{2}\left[e^{j(2\pi 80t)} + e^{-j(2\pi 80t)}\right] + \frac{1}{2}\left[e^{j(2\pi 140t)} + e^{-j(2\pi 140t)}\right]$



d) power of sinusoidal $\frac{(amp)^2}{2}$ different freq.

$P_{x(t)} = \frac{(1)^2}{2} + \frac{(1)^2}{2} + \frac{(1)^2}{2} = \frac{3}{2}$

$P = \sum |X_n|^2 = 6\left(\frac{1}{2}\right)^2 = \frac{6}{4} = \frac{3}{2}$

Same value for power.

Problem 2:

a. Consider the signal $h(t)=10\exp(-at)u(t)$ (for a > 0). Use the FT ***integral definition*** to show that $FT\,[h(t)]=H(f)=\dfrac{10}{a+j2\pi f}$.

b. Assume that a given LTI (Linear Time Invariant) system has impulse response $h(t)$ given above

$$x(t),\ X(f) \longrightarrow \boxed{h(t),\ H(f)} \longrightarrow y(t),\ Y(f)$$

Describe the behavior of system (is it low-pass, high-pass, etc). Justify your answer using ~~plots~~ sketch.

c. Now, suppose that the input is given by $x(t)=2\cos(2\pi t)$, and a=2, find:

- $Y(f)$ [expressed in the *simplest possible* form]. 5
- $y(t)$ [expressed in the *simplest possible* form].

a)

2

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} 10\, \exp(-at)\, u(t)\, \exp(-j2\pi f t)\, dt$$

$$= 10\int_{0}^{\infty} \exp\left((-a-j2\pi f)t\right) dt$$

$$= \frac{-10}{a+j2\pi f} \exp(-a-j2\pi f)t \Big|_0^{\infty}$$

$$= \frac{-10}{a+j2\pi f}\left[0-(1)\right] = \frac{10}{a+j2\pi f}$$

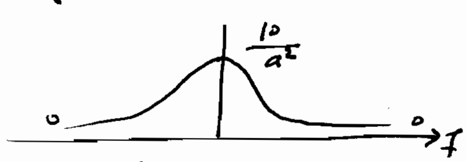
as required.

)

2

$$|H(f)| = \frac{10}{\sqrt{a^2+(2\pi f)^2}}$$

The general shape



It is a lowpass filter (not ideal) because the filter allows low frequency signals to pass. and block higher freq. signals.

c) From the table.

$$\cos(2\pi t f_0) \longleftrightarrow \tfrac{1}{2}\delta(f-f_0)+\tfrac{1}{2}\delta(f+f_0)$$

$$2\cos(2\pi t) \Rightarrow f_0 = 1$$

$$2\cos(2\pi t) \leftrightarrow \delta(f+1)+\delta(f-1)$$

$$Y(f) = X(f)\,H(f)$$

$$= \left(\delta(f+1)+\delta(f-1)\right)\left(\frac{\cancel{10}\ 5}{\cancel{2}\ 1+j\cancel{2}\pi f}\right)$$

By sifting $= \dfrac{5}{1+j\pi}\left[\delta(f+1)+\delta(f-1)\right]$

$$= 1.52\angle -72.34^\circ \left[\delta(f+1)+\delta(f-1)\right]$$

By inverse F.T.

$$y(t) = 3.04 \cos(2\pi t - 72.34^\circ)$$

or simply

$$H(f=1) = \frac{10}{2+j2\pi(1)} = \frac{5}{1+j\pi} = 1.52\angle -72.34^\circ$$

output will be scaled and shifted accordingly

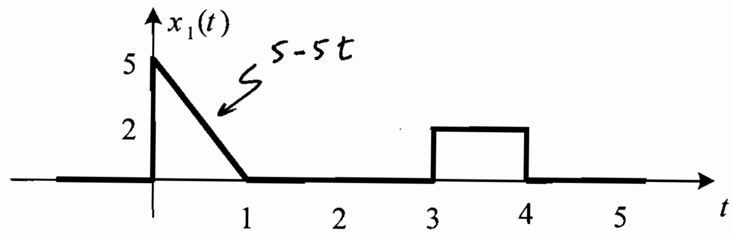
$$y(t) = (1.52)(2)\cos(2\pi t - 72.34^\circ)$$

$$y(t) = 3.04\cos(2\pi t - 72.34^\circ)$$

Same answer.

Problem 3:

a)



The signal $x_1(t)$ shown above can be expressed in terms of singularity functions as:

$x_1(t) = 5u(t) - 5r(t) + 5r(t-1) + 2u(t-3) - 2u(t-4)$

4 i) Find the La Place Transform of $x_1(t)$.

3 ii) Find the La Place Transform of $\frac{dx_1(t)}{dt}$.

3 **b)** Find the inverse La Place Transform (i.e., $x_2(t)$) of the signal $X_2(s) = \frac{1}{3s+12} e^{-7s}$.

i)

using the table $u(t) \leftrightarrow \frac{1}{s}$

" " : $\frac{t^n \exp(-\alpha t)\, u(t)}{n!} \leftrightarrow \frac{1}{(s+\alpha)^{n+1}}$

for $\alpha = 0$

$\frac{t^n\, u(t)}{n!} \leftrightarrow \frac{1}{s^{n+1}}$

$n = 1$ $\quad t\, u(t) = r(t) \leftrightarrow \frac{1}{s^2}$

$$\boxed{X_1(s) = \frac{5}{s} - \frac{5}{s^2} + \frac{5}{s^2} e^{-s} + \frac{2}{s} e^{-3s} - \frac{2}{s} e^{-4s}}$$

ii)

$\frac{d}{dt} x_1(t) = 5\delta(t) - 5u(t) + 5u(t-1) + 2\delta(t-3) - 2\delta(t-4)$

$\delta^{(n)}(t) \leftrightarrow s^n$

$\delta(t) \leftrightarrow s^0 (1)$

$\Rightarrow \mathcal{L}\left[\frac{d}{dt} x_1(t)\right] = 5 - \frac{5}{s} + \frac{5}{s} e^{-s} + 2e^{-3s} - 2e^{-4s}$

or simpler

$\mathcal{L}\left[\frac{dx_1(t)}{dt}\right] = s\, \mathcal{L}\left[x_1(t)\right] - x_1(0^-)$

$= s(X_1(s))$

$$\boxed{= 5 - \frac{5}{s} + \frac{5}{s} e^{-s} + 2e^{-3s} - 2e^{-4s}}$$

b) $\mathcal{L}^{-1}\left[\frac{1}{3s+12} e^{-7s}\right]$

$= \frac{1}{3} \mathcal{L}\left[\frac{1}{s+4} e^{-7s}\right]$ time delay

$\exp(-4t)\, u(t)$

$= \frac{1}{3} e^{-4(t-7)} u(t-7)$