

electric field intensity is the largest. Determine the electric field intensity at these points if the intensity of the current through the grounding sphere is I . — *Hint: make use of the method of images, Fig. P10.19.* (a) $x_{\max} = d/2$, $E_{\text{total max}} = I/(3\pi\sigma d^2)$. (b) $x_{\max} = d/\sqrt{2}$, $E_{\text{total max}} = I/(3\sqrt{3}\pi\sigma d^2)$. (c) $x_{\max} = d/\sqrt{3}$, $E_{\text{total max}} = I/(3\sqrt{2}\pi\sigma d^2)$.

11. Some Applications of Electrostatics

- Thunderstorms are the most obvious manifestations of electrical phenomena on our planet. A typical storm cloud carries about 10-20 coulombs of each type of charge, at an average height of 5 km above the earth's surface. The beginning of a cloud-to-ground lightning is an invisible discharge, which is called the "stepped leader." The stepped-leader air breakdown is initiated at the bottom of the cloud. It moves in discrete steps, each about 50 m long and lasting for about $1\mu\text{s}$. When the leader is about 100 m above ground, a spark (the visible "return stroke" of only about $100\mu\text{s}$ duration) moves up from the ground to meet it and a conducting channel from the cloud to the ground lights up. The currents in the return stroke range from few kA to as much as 200 kA.

- Electric charges in a vacuum or in gases are propelled by the electric field of stationary charges. The point form of Ohm's law does not hold. In rarefied gases the paths of accelerated ions between two successive collisions are relatively long, so that they can acquire a considerable kinetic energy. As a consequence, various new effects can be produced. The best known is probably a chain production of new pairs of ions by collisions of high-velocity ions with neutral molecules, which may result either in a corona (ionized layer around charged bodies), or in breakdown of the gas (discharge), depending on the structure geometry and voltage.

- For a charge Q of mass m moving in an electrostatic field in a vacuum the equation of motion has the form

$$m \frac{d^2 \mathbf{r}(t)}{dt^2} = Q \mathbf{E}(\mathbf{r}), \quad (11.1)$$

where $\mathbf{r}(t)$ is the position vector (variable in time) of the charge, and $\mathbf{E}$ is the electric field strength, a function of coordinates (i.e., of $\mathbf{r}$). In a uniform electric field along the x axis, for a charge starting to move at $t = 0$ from $x = 0$, this yields

$$x = \frac{QE}{2m} t^2. \quad (11.2)$$

- Let a particle of charge Q and mass m leave a point 1 at a potential V_1 with a velocity of magnitude v_1 . The magnitude of its velocity at a point 2 at potential V_2 is

$$v_2 = \sqrt{v_1^2 + \frac{2Q(V_1 - V_2)}{m}}. \quad (11.3)$$

- Electrostatic filters are used for removing fine particles from exhaust gases. The particles are charged, separated from the rest of the gas by a strong electric field, and finally attracted to a pollutant-collecting electrode.
- The modern copier machines use the process known as *xerography* (from the Greek words *xeros* — dry and *graphos* — writing). Xerography uses a photosensitive material such as selenium. Selenium is normally a dielectric, but when illuminated, it becomes conductive. A photosensitive plate is first charged over its surface. It is then illuminated by an image of the document. A charge image of the document is thus obtained, with dark places (e.g., letters) charged, the rest of the plate being discharged by light. This is followed by a process of obtaining a copy of the image on a sheet of paper. In order that this be possible, the charge image, i.e., the surface charge of density σ , must remain on the plate for a sufficiently long time. This time is determined by the equation

$$\frac{d\sigma}{dt} + \frac{1}{\epsilon\rho}\sigma = 0, \quad (11.4)$$

where ρ and ϵ are resistivity and permittivity of dark selenium. Assuming that at $t = 0$ the surface charge density is σ_0 , the solution of this equation is $\sigma = \sigma_0 e^{-t/(\epsilon\rho)}$. The quantity $(\epsilon\rho)$ is called the *charge transfer time constant*, or *dielectric relaxation constant* of dark selenium, and is similar to the *RC* time constant of a *RC* circuit.

- An important application of electrostatic fields is separation, used in industry for purification of food, purification of ores, sorting of reusable wastes and sizing (sorting according to size and weight). Basically, in these processes particles are charged, and then separated by an electric force, or by a combination of an electric force and some other force.
- Four-point probes are commonly used instruments in every semiconductor lab for measuring resistivity of a material, although they are used for other purposes as well. The basic idea is to create an electric field over the surface of the specimen by two current probes, and then to measure the voltage between two convenient points by two other (voltage) probes. In this way the contact between the current probes and the material, which is very difficult to control, practically does not influence the results. Two-point probes have only two probes, which are used both for injecting the current in the material and for measuring the voltage between these two points. As explained, the contact between the probes and the material is not well defined, and therefore two-point probes cannot yield accurate values of the material resistivity. (See also explanations in Lab 3.)
- There are many other applications of electrostatic fields: coating with paint or other material, highly sensitive charge-coupled device (CCD) cameras, non-impact printing (e.g., in ink-jet printers), electrostatic motors, electrostatic generators and electrophoresis (separation of charged colloidal particles by the electric field) used in biology, etc.

QUESTIONS

Q11.1. Describe the formation of a lightning stroke. — (a) *Thunderstorm clouds have large negative charge, and a spark is obtained when the clouds approach the ground.* (b) *The clouds have a large positive charge, and a spark is obtained when it is initiated somewhere on the ground.* (c) *A cloud is charged like a vertical dipole, its lower part being a large negative*

charge. The conducting channel starts there, and when it approaches the ground, there is a spark from the ground to that channel.

Q11.2. How large are currents in a lightning stroke? — (a) Between about 100 A and 1000 A. (b) Between about 5000 A and 200,000 A. (c) Between about 50,000 A and 1,000,000 A.

- S **Q11.3.** According to which physical law does thunder occur? — (a) It is produced by the light of the lightning. (b) It is produced by the pressure of air heated up in the return stroke channel. (c) It is produced when the lightning hits the ground.

Answer. Thunder is the result of rapid heating of the lightning channel, more precisely of rapid increase in air pressure in the channel resulting from this heating.

Q11.4. A spherical cloud of positive charges is allowed to disperse under the influence of its own repulsive forces. Will charges follow the lines of the electric field strength vector? — (a) No (explain). (b) Yes (explain). (c) Depends on the initial conditions.

- S **Q11.5.** A cloud of identical, charged particles is situated in a vacuum in the gravitational field of the earth. Is there an impressed electric field in addition to the electric field of the charges themselves? Explain. — (a) Yes, because there are nonelectric forces acting on the particles. (b) No, because there are no nonelectric forces acting on the particles. (c) Depends on the initial velocities of the particles.

Answer. There is the gravitational force acting on the particles, which is not of electric origin. So, there is an impressed electric field acting on the particles. If the mass of the particles is m and their charge Q , the downward force acting on the particles is $F_i = mg$. The impressed electric field is $E_i = mg/Q$.

Q11.6. Is Eq. (11.1) valid if the charge Q from time to time collides with another particle? — (a) It is valid at all times. (b) It is not valid at all. (c) It is valid for all time intervals between two collisions.

Q11.7. Explain why the equation $mv_1^2/2 - mv_2^2/2 = Q(V_2 - V_1)$ is correct, and why $m(v_2 - v_1)^2/2 = Q(V_2 - V_1)$ is not. — *Hint: have in mind the definition of kinetic energy.*

Q11.8. Discuss the validity of Eqs. (11.3) if the charge Q is negative. — (a) It is valid only for positive charges. (b) For negative charges, Q should be replaced by $|Q|$. (c) It is valid, because the potential difference ($V_1 - V_2$) is then also negative.

- S **Q11.9.** An electron is emitted parallel to a large uncharged conducting flat plate. Describe qualitatively the motion of the electron. — (a) It moves parallel to the plate, because the plate is uncharged. (b) It is repelled by the plate (why?). (c) The electron is attracted towards the plate, and eventually hits the plate (why?).

Answer. Due to the induced charges of opposite sign on the plate, the electron is attracted towards the plate. The attraction increases as the distance between the electron and the plate decreases, and eventually the electron hits the plate.

Q11.10. If the voltage between the electrodes of an air-filled parallel-plate capacitor is increased so that corona starts on the plates, what will eventually happen without increasing the voltage further? — (a) The corona will remain around the plates. (b) Breakdown of air in the entire capacitor will occur. (c) After a while the corona will disappear.

Q11.11. Electric charge is continually brought on the inner surface of an isolated hollow metal sphere situated in air. Explain what will happen outside the sphere. — (a) *Nothing, since the sphere is a Faraday cage.* (b) *Depends on the thickness of the sphere wall (explain).* (c) *The electric field on the outer surface will increase, and corona will be formed for sufficiently large charge.*

- S Q11.12.** Give a few examples of desirable and undesirable (1) corona and (2) spark discharges. — *Hint: think of lightning rods, sharp spikes on airplanes, high-voltage transmission lines, and sparks between electrodes of spark plugs of internal combustion engines and electrostatic gas lighters, or between bodies in any environment containing explosive gas mixtures.*

Answer. (1) Corona is desirable on airplane pointed parts, to reduce the airplane charge, and at the tip of the lighting rod, to initiate the lightning. It is undesirable along high-voltage transmission lines, because it causes a loss of charge, i.e., of energy. (2) A spark discharge is desirable between the electrodes of spark plugs of internal combustion engines and between the electrodes of electrostatic gas lighters. It is undesirable between charged bodies in any environment containing explosive gas mixtures.

Q11.13. Describe how an electrostatic pollution-control filter works. — (a) *Dust and other particles are charged, so that they remain in filters like those of a vacuum cleaner.* (b) *Dust and other particles are charged, and are attracted towards the ground, which is always negatively charged.* (c) *Dust and other particles are charged by convenient means, and then attracted to an oppositely charged electrode.*

Q11.14. Sketch the field that results when an uncharged spherical conductive particle is brought into an originally uniform electric field. — *Hint: correct the sketch in Fig. Q11.14 if you think it needs to be corrected.*

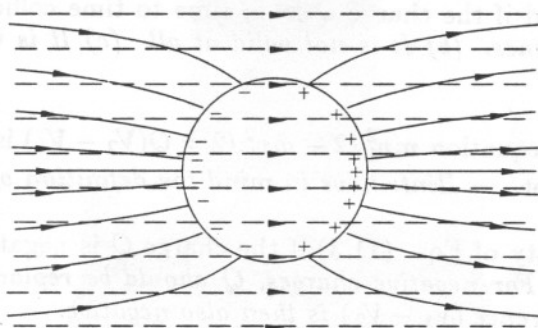


Fig. Q11.14. Lines of originally uniform electric field (dashed) and of the resultant field around an uncharged conducting sphere introduced in the uniform field (solid).

- S Q11.15.** Describe the process of making a xerographic copy. — (a) *Charge a selenium plate, and press the document over it to leave an image.* (b) *Charge a selenium plate, and expose the reverse image of the document onto the plate, to obtain dark places where the document is white, and conversely, which will produce a charge image on the plate.* (c) *Charge a selenium plate, expose the image of the document onto the plate, to discharge bright areas, and reproduce this charge image by convenient means.*

Answer. (1) Charge a selenium plate evenly over its surface. (2) Expose the image of the document onto the plate, to discharge bright areas. (3) Attract charged toner particles to the charged image of

the document on the selenium plate. (4) Charge paper sheet with opposite charge, and place it over selenium plate to attract toner particles. (5) Bake toner on paper to fix it.

Q11.16. Explain the physical meaning of the charge transfer time constant, or dielectric relaxation constant. — (a) *The time it takes that the charge density at a point goes to zero.* (b) *The time it takes that the charge density at a point decreases to $1/e$ of its initial value.* (c) *The time it takes that the charge density at a point decreases to 0.1 of its initial value.*

Q11.17. Consider a thin film of resistivity ρ and permittivity ϵ residing over a good conductor. Assume there is a surface charge of density $\sigma(t)$ over the free surface of the film. Prove that $\sigma(t)$ satisfies the equation $d\sigma(t)/dt + \sigma(t)/(\epsilon\rho) = 0$. Find the solution of the equation. — *Hint: write the equation of continuity and the generalized Gauss' law to a coinlike surface with one base inside the film, and the other in air, and combine these two equations. Note that in the continuity equation you can write $\mathbf{J} = \mathbf{E}/\rho$. The solution of the equation is: (a) $\sigma(t) = \sigma_0 e^{-t/\epsilon\rho}$, (b) $\sigma(t) = \sigma_0$, (c) $\sigma(t) = \sigma_0 e^{t/\epsilon\rho}$.*

Q11.18. Describe the difference in the xerox image with and without the developer plate. — (a) *There is no difference.* (b) *With the developer plate, contours of images are better defined.* (c) *With the plate, the entire areas of images are covered with toner.*

Q11.19. Derive the equation of particle trajectory in a forming chute electrostatic separation process. — *Hint: the equation is that of the motion of a charged particle.*

- S **Q11.20.** Why is a four-point probe measurement more precise than a two-point probe measurement? — (a) *It is not more precise.* (b) *It is more precise because it has four, instead of two, probes.* (c) *A two-point probe has inadequately defined radius of electrodes embedded in the substance. A four-point probe measures the potential difference between two points away from these undefined regions.*

Answer. A two-point probe has inadequately defined radius of electrodes touching the substance the resistivity of which we are measuring. A four-point probe measures the potential difference between two points away from these undefined regions, and is therefore much more accurate.

- S **Q11.21.** Describe how a CCD camera works.

Answer. The receiving screen consists of a large number of tiny MOS capacitors. The capacitors consist of small metal patches on thin layer of oxide, and the oxide layer is over a p -type semiconductor (silicon). The metal patches are positively charged, and are not tightly packed.

The light photons incident on the interpatch regions pass through the thin oxide layer and create pairs of free charge carriers. The electrons are attracted to the positively charged capacitor electrodes, their number being proportional to the number of photons incident in the vicinity of the electrode. By measuring the number of electrons, it is possible to reconstruct the image.

PROBLEMS

- S **P11.1.** Calculate the voltage between two feet of a person (0.5 m apart), standing $r = 20$ m away from a 10 kA lightning stroke, if the moderately wet homogeneous soil conductivity is 10^{-3} S/m. Do the calculations for the two cases when the person is standing in positions (A) and (B) as shown in Fig. P11.1. — (a) *In position A, $[V_1 - V_2]_A \simeq 1940$ V, and in position B it is close to zero.* (b) *1940 V, 385 V.* (c) *3750 V, approximately zero.*

Solution. The electric field strength on the surface of the ground due to the current of the lightning stroke is

$$E(r) = \frac{I}{2\pi\sigma r^2}.$$

The potential difference between a point a distance r from the point of the stroke, and a point a distance $r + d$ from it, is

$$V_1 - V_2 = \int_r^{r+d} E(r) dr = \frac{I}{2\pi\sigma} \left(\frac{1}{r} - \frac{1}{r+d} \right).$$

For given data, we thus obtain that, for the person in position A, $[V_1 - V_2]_A \simeq 1940$ V! In position B, the person has feet close together and the voltage is approximately zero.

P11.2. Calculate the electric field strength above a tree that is $d = 1$ km away from the projection of the center of a cloud onto the earth (Fig. P11.2). Assume that because the tree is like a sharp point, the field above the tree is about 100 times that on the flat ground. As earlier, you can assume the cloud is an electric dipole above a perfectly conducting earth, with dimensions as shown in the figure, and with $Q = 4$ C of charge. (Note that the height of any tree is much smaller than the indicated height of the cloud.) — *Let the x axis be directed upward, with the origin at the position of the tree.* (a) $E_{\text{tree}} \simeq 184,000$ V/m. (b) $E_{\text{tree}} \simeq 368,000$ V/m. (c) $E_{\text{tree}} \simeq 265,000$ V/m.

P11.3. Derive the equation $m d^2 x / dt^2 = QE$ for a x -directed motion of a charged particle. — *Hint: start from Eq. (11.1).*

P11.4. Assuming a nonzero, and x -directed initial velocity of a charged particle situated in a x -directed electric field, find the velocity and the position of the charge Q as a function of time. Plot your results. — *Hint: start from the differential equation of the preceding problem.*

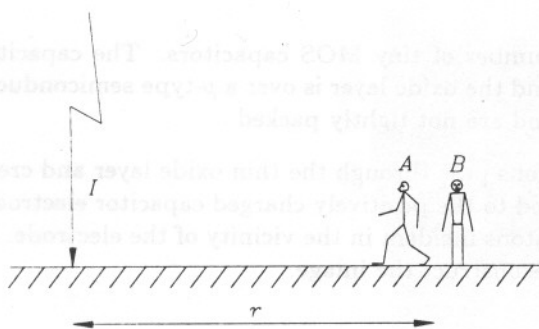


Fig. P11.1. A person near a lightning stroke.

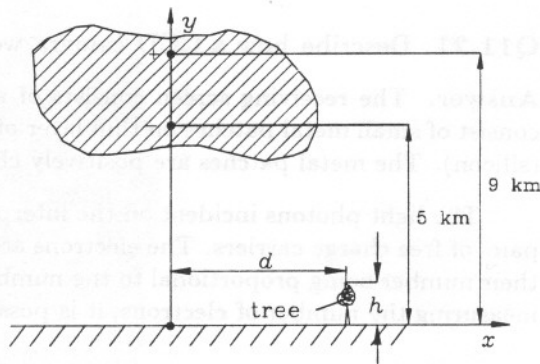


Fig. P11.2. Field above a tree in a storm.

P11.5. Assuming that the initial velocity in problem P11.4 is y -directed, solve for the velocity and the position of the charge Q as a function of time. Plot your results. — *Hint: in this case, Eq. (11.1) needs to be solved for two scalar unknowns, the x and y coordinates of the charge.*

P11.6. A thin electron beam is formed with some convenient electrode system. The electrons in the beam are accelerated by a voltage V_0 . The beam passes between two parallel plates, which electrostatically deflect the beam, and later falls on the screen S (Fig. P11.6). Determine and plot the deflection y_0 of the beam as a function of the voltage V between the plates. (This method is used for electrostatic deflection of the electron beam in some cathode-ray tubes.) — (a) $y_0/V = l(L/2 + l)/(2V_0d)$. (b) $y_0/V = l(L + l/2)/(V_0d)$. (c) $y_0/V = l(L + l/2)/(2V_0d)$.

- S P11.7.** A beam of charged particles of positive charge Q , mass m , and different velocities, enters between two closely spaced curved metal plates. The distance d between the plates is much smaller than the radius R of their curvature (Fig. P11.7). Determine the velocity v_0 of the particles which are deflected by the electric field between the plates so that they leave the plates without hitting any of them. Note that this is a kind of filter for charged particles, resulting in a beam of particles of the same velocity. — (a) $v_0 = \sqrt{QRV/(md)}$. (b) $v_0 = \sqrt{QRV/(md)}$. (c) $v_0 = \sqrt{QRV/(md)}$.

Solution. In the space between the plates, two forces act on the charged particles. The electric force, F_e , equal in magnitude to QV/d , acts towards the grounded plate. The centrifugal force, F_c , equal in magnitude to mv_0^2/R , is perpendicular to the trajectory of the charges. The forces are of equal magnitudes and in opposite directions. Only those charges will follow a circular arc of radius R which have a velocity v_0 determined by

$$\frac{mv_0^2}{R} = \frac{QV}{d}, \quad \text{from which} \quad v_0 = \sqrt{\frac{QRV}{md}}.$$

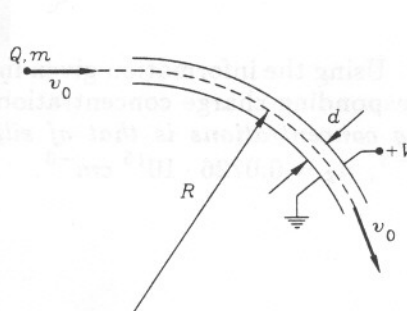
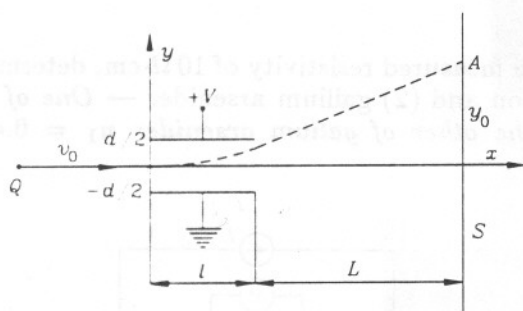


Fig. P11.6. Deflection of an electron beam. **Fig. P11.7.** An electrostatic velocity-filter of charged particles.

P11.8. A metal sphere is placed in a uniform electric field E_0 . What is the maximum value of this field which does not produce air breakdown when the metal ball is brought into it? — *Hint: note that the maximum field on the sphere surface is three times the original uniform field.*

P11.9. Calculate the dielectric relaxation constants for selenium, n -doped silicon with carrier concentration $n = 10^{16} \text{ cm}^{-3}$, and n -doped gallium arsenide with concentration $n = 10^{16} \text{ cm}^{-3}$. For semiconductors, such as silicon and gallium arsenide, the conductivity is given by $\sigma = Q\mu n$, where Q is the electron charge. μ is a property of electrons inside a material, and it is called the mobility (defined as $v = \mu E$, where v is the velocity of charges that are moved by a field).

E). For silicon, $\mu = 0.135 \text{ m}^2/\text{Vs}$ and $\epsilon_r = 12$, and for gallium arsenide, $\mu = 0.86 \text{ m}^2/\text{Vs}$ and $\epsilon_r = 11$. For selenium, $\rho = 10^{12} \Omega\text{-m}$ and $\epsilon_r = 6.1$. — *Hint: use the definition of the relaxation constant following Eq. (11.4). The following three answers are correct, but you need to associate them with the material: $\tau_1 = 54 \text{ s}$, $\tau_2 = 0.5 \text{ ps}$, and $\tau_3 = 0.073 \text{ ps}$.*

P11.10. How far do 1-mm-diameter quartz particles charged with $Q = 1 \text{ pC}$ need to fall in a field $E = 2 \cdot 10^5 \text{ V/m}$ in order to be separated by 0.5 m in a forming chute separation process? The mass density of quartz is $\rho_m = 2.2 \text{ g/cm}^3$. — *Hint: use the equations of motion for a charged particle in the electric field.* (a) $y = -18.34 \text{ m}$. (b) $y = -28.25 \text{ m}$. (c) $y = -7.28 \text{ m}$.

- S P11.11.** Find the expression for determining resistivity from a four point probe measurement, as in Fig. P11.11. — *Assume that the dimension of the contacts is very small when compared with a .* (a) $\rho = 2\pi a(V_1 - V_2)/I$. (b) $\rho = 4\pi a(V_1 - V_2)/I$. (c) $\rho = \pi a(V_1 - V_2)/I$.

Solution. Assume that the dimension of the contacts is very small when compared with a . The potential difference between the two middle contacts, as measured by the voltmeter, is

$$V_1 - V_2 = \int_a^{2a} \left[\frac{\rho I}{2\pi r^2} + \frac{\rho I}{2\pi(3a - r)^2} \right] dr = \frac{\rho I}{2\pi a},$$

from which

$$\rho = \frac{2\pi a(V_1 - V_2)}{I}.$$

P11.12. Using the information given in P11.9, for a measured resistivity of $10 \Omega\text{-cm}$, determine the corresponding charge concentration of (1) silicon and (2) gallium arsenide. — *One of the following concentrations is that of silicon, and the other of gallium arsenide: $n_1 = 0.46 \cdot 10^{15} \text{ cm}^{-3}$, $n_2 = 0.0726 \cdot 10^{15} \text{ cm}^{-3}$.*

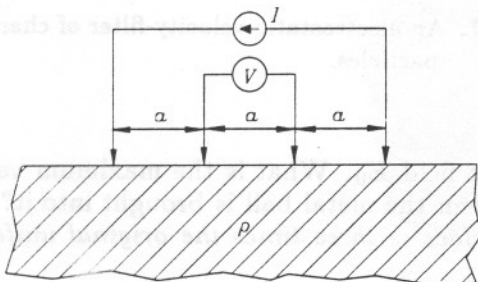


Fig. P11.11. A 4-point probe measurement.

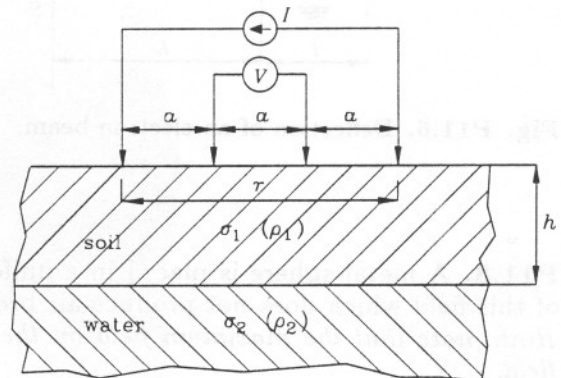


Fig. P11.13. A Wenner array used in geology.

P11.13. A Wenner array used in geology is shown in Fig. P11.13. This instrument is used for determining approximately the depth of a water layer under ground. First the electrodes

are placed close together, and the resistivity of soil is determined. Then the electrodes are moved further and further apart, until the resistivity measurement changes due to the effect of the water layer. Assuming that the top layer of soil has a very different conductivity than the water layer, what is the approximate spacing between the probes, r , that detects a water layer at depth h under the surface? The exact analysis is complicated, so think of an approximate qualitative solution. — *Hint: try to conclude what happens with the measured voltage when r is increased from a value much smaller than h to approximately $2h$.*

P11.14. A thin film of resistive material is deposited on a perfect insulator. Using a four-point probe measurement, determine the expression for surface resistivity ρ_s of the thin film. Assume the film is very thin. — (a) $\rho_s = 2\pi(V_1 - V_2)/(I \ln 2)$. (b) $\rho_s = 2\pi(V_1 - V_2)/(I \ln 4)$. (c) $\rho_s = \pi(V_1 - V_2)/(I \ln 2)$.

- S P11.15.** Consider an approximate circuit equivalent of a thin resistive film as in Fig. P11.15. The mesh is infinite, and all resistors are equal and have a value of $R = 1 \Omega$. Using a two-point probe analogy, determine the resistance between any two adjacent nodes A and B in the mesh. — *Hint: assume two current generators of equal, but opposite, currents at the two adjacent nodes and use superposition.* (a) $R/2$. (b) $R/3$. (c) $R/4$.

Solution. Assume we inject a current of intensity I at point A , and collect it at infinity. Due to symmetry, the currents in the four resistors meeting at A are $I/4$ each (reference direction from point A). If a current $-I$ is inserted at B and collected at infinity, the currents in the four resistors meeting at B are $-I/4$ (reference direction from point B). Thus, if a generator of current I is connected between A and B , the current in the resistor joining the two points is $I/2$, and the voltage across the resistor is $RI/2$. The resistance between A and B is hence $R/2$.

P11.16. Find the resistance between nodes (1) A and C and (2) A and D in Fig. P11.15. — *Hint: follow the suggestion given in the preceding problem and use superposition.* (a) $R/2$ in case 1, $R/4$ in case 2. (b) $R/3$ in case 1, $R/2$ in case 2. (c) R in both cases.

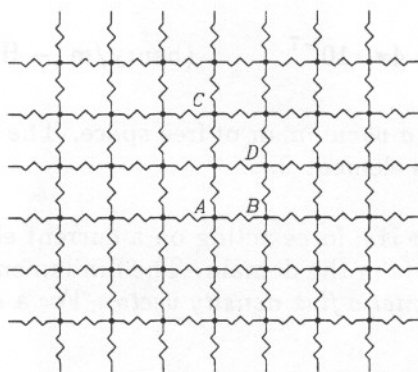


Fig. P11.15. An approximate equivalent circuit of a thin resistive film.

P11.17. Construct an approximate equivalent circuit for a block of homogeneous resistive material. Determine the resistance between two adjacent nodes of the equivalent circuit. — *Hint: assume two current generators of equal, but opposite, currents at the two adjacent nodes and use superposition.* (a) $R/2$. (b) $R/3$. (c) $R/4$.