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A Decentralized, Convergent, Nearest Neighbor, Spatial Consensus, Control Protocol

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Abstract - In this paper a convergent, nearest-neighbor, consensus control protocol is suggested for agents with nontrivial dynamics. The protocol guarantees convergence to a common point in space even if each agent is restricted to communicate with its nearest neighbor. The neighbor, however, is restricted to lie outside an arbitrarily small priority zone surrounding the agent. The control protocol consists of two layers interconnected in a provably-correct manner. The first layer guides the agent to the rendezvous point while the other converts the guidance signal to a control signal that suits realistic agents such as UGVs, UAVs and holonomic agents with second order dynamics.

I. Introduction

Consensus protocols have applications in many areas, e.g. decision making, planning, computer networks and robotics [1,2]. The nearest neighbor consensus protocols are the most important . They involve asynchronous exchange of information on a communication graph whose topology is continuously switching. Nearest neighbor consensus was first examined by Vicsek et al. [3] who modeled the ability of a flock of birds to converge to the same heading by each member averaging the headings of its neighbors. An analysis of this behavior was carried out in [4] by Jadbabaie et al. Cucker & Smale [23] suggested a distance tunable model for velocity consensus. Each member of the flock interacts with all other members. They provided conditions for convergence that depend only on the initial state of the flock. In [19] a decentralized nearest-neighbor multi-agent controller was suggested to de-conflict the use of space. It uses only two behavioral primitives: collision avoidance and moving out of the way on close encounters with others. It was noticed in some of the simulation results that synchronous behavior emerged where the agent platooned moving at the same speed in the same direction.

Design and analysis of protocols that would guarantee convergence to a common value of a desired attribute of operation [5-10] is a major focus of attention in studying consensus. Other aspects such as ability to converge in the presence of noise [11] and placing constraints on the process [12] were also investigated. Unfortunately, despite their simplicity, efficiency and practicality, nearest neighbor protocols may not be able to guarantee convergence of a group.

Most consensus protocols are utilized by agents that have involved dynamics. Even if a protocol is convergent, the interaction between the protocol and the dynamics of the agents may prevent consensus from happening. The interaction between the communication graph and the dynamics of the agents are being studied [13-16] to derive and tune protocols that would guarantee convergence when they are utilized by dynamical agents.

This work has two contributions. First, it offers a variant of the traditional, nearest neighbor consensus protocol. The suggested protocol guarantees convergence of the group to a common rendezvous point. It only requires each agent to be able to communicate with at least one other neighbor. This neighbor is the one closest to the agent provided that it lies outside an arbitrarily small priority zone surrounding the agent. The second contribution has to do with guaranteeing stability when the protocol is used by a group of dynamical agents. The procedure for converting the guidance signal from the consensus protocol to a control signal does not require exchange of velocity information among the agents (i.e. exchange intentions). Along with the consensus guidance signal each agent uses its own velocity information for generating the control signal.

The generation of the consensus control protocol is based on a series of methods suggested by this author. The methods convert the guidance planning signal from a harmonic potential [17-19] to a control signal for holonomic systems with second order dynamics [20], nonholonomic mobile robots [21] and a large class of UAVs [22]. It is demonstrated in this paper that these techniques which are designed for a single agent are fully capable of functioning as control protocols in a multi-agent environment. The close relation between harmonic potential and the consensus problem is the main motivation for examining the use of these techniques. The value of a harmonic potential at a point is arrived at iteratively as the average value of its immediate neighbors.

This paper is organized as follows: section II presents the modified protocol, section III discusses the convergence of the protocol. Section IV examines the communication burden needed for each agent to remain connected to it's closest priority agent. Section V presents the techniques used for converting the protocol signal into a consensus control signal for different types of dynamical agents. Simulation results are in section VI and conclusions are in section VII.

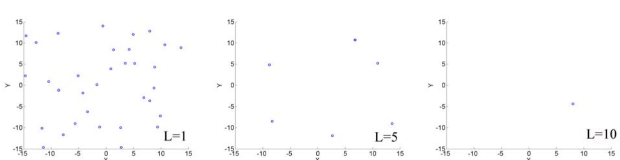

Figure-1: nearest neighbor consensus protocol does not guarantee unconditional convergence

II. The consensus protocol

Nearest neighbor-based consensus protocols are important and practical. Whether the information exchange among the agents is based on sensing or communication, nearest neighbors always have the best chance succeeding in such a regard. Unfortunately, existing protocols do not guarantee convergence to a consensus state using conditions that are both controllable and *a priori* known to the operator. Figure-1 shows the effect of increasing the number of neighbors (L) on the ability of 50, single-integrator agents to rendezvous at one point. As can be seen the less the number of neighbors is the more separate rendezvous points are created.

This author strongly believe that the main cause of the convergence problem has to do with the manner in which the effort to establish consensus is distributed. It does not make sense for an agent to spend an effort establishing consensus with another agent who is already in agreement with it. Such agents may be considered as one agent with multiplicity more than one. Agents with large, but manageable, deviations from the actor agent should have high priority. Others whose state is close to the agent concerned should have low priority as far as dispensing the consensus effort is concerned. The following provides an implementation of the suggested approach.

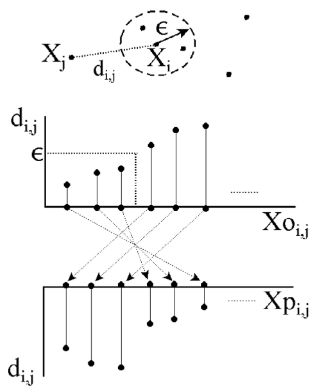

Figure- 2: priority buffer arrangement

Consider an N-dimensional sphere of radius ϵ ($\beta_i(X)$) that is centered around the position of the i'th agent (X_i)

$$\beta_i(X - X_i) = \{X: |X - X_i| \le \varepsilon\}. \qquad (1)$$

If the j'th agent ($i \neq j$) is in β_i ($X_j \in \beta_i$), this agent is considered as a low priority agent. Otherwise, it is a high priority agent. Let d_{ij} be the distance between the i'th and the j'th agents ($d_{i,j} = |X_i - X_j|$). Let $Xo_{i,j}$ be a buffer containing the locations of the agents ordered in an ascending manner based on their distance from X_i ($d_{i,j-1} \le d_{i,j} \le d_{i,j+1}$, i=1,..,N, j=1,...N, $i \neq j$). Existing consensus protocols that use the L closest neighbors generate the velocity vector of the i'th agent as

$$\dot{X}_i = uc_i = \sum_{j=1}^{L} a_{i,j}(Xo_{i,j} - X_i) \qquad (2)$$

where a_{ij} are positive constants.

The modified protocol works as follows: first, the protocol priority orders (figure-2) the agents relative to the i'th agent (Xp_{ij}). Xp_{ij} is constructed as follows

$$\begin{aligned} Xp_{i,j} &= Xo_{i,j+Lo} \quad j = 1,..,N - Lo \\ Xp_{i,j+Lo-1} &= Xo_{i,L0-j+1} \; j = 1,...,Lo \end{aligned} \qquad (3)$$

where $d_{i,Lo} < \epsilon$, N is the total number of agents and Lo is the number of agents inside β_i. In a similar manner to the normal protocol, the velocity the i'th agent is constructed as

$$\dot{X}_i = uc_i = \sum_{j=1}^{L} a_{i,j}(Xp_{i,j} - X_i) \qquad (4)$$

III. Convergence Analysis

The modified protocol is convergent. If the i'th agent maintains connection with at least one agent outside β_i , all agents will converge to β_i. Since, in β_i , the i'th agent guarantees that the most distant agent will converge to its position, it guarantees that all other agents will also converge. An agent's motion is observed relative to X_i. Three distinct hyper-spheres (figure-3) whose center is X_i are used in the proofs: The previously defined priority zone, β_i, in which X_i is guaranteed to communicate with all agents in that zone. A hyper sphere, S_i , containing all the agents of the group. The center of S_i is X_i. Its radius, dx_i, is selected as the distance between X_i and the agent furthest from it, X_j ,

$$S_i(X) = \{X: |X - X_i| \le dx_i\} \quad X_i \in S_i(X) \; \forall \, i \; (5)$$

The last sphere with X_i as a center is σ_i ($\sigma_i \subset S_i(X)$) . This is the largest sphere containing only one agent, X_k, which is the agent closest to X_i that does not belong to β_i

$$X_k \cap \sigma_i = X_k , \quad X_i \cap \sigma_i = \varnothing \quad i \neq k \qquad (6)$$

It ought to be noticed that by construction, for the case of L=1, the consensus protocol will only operate on agents with non-zero distance. Therefore an implicit assumption in the proof is that $X_i \neq X_j$ for any i & j.

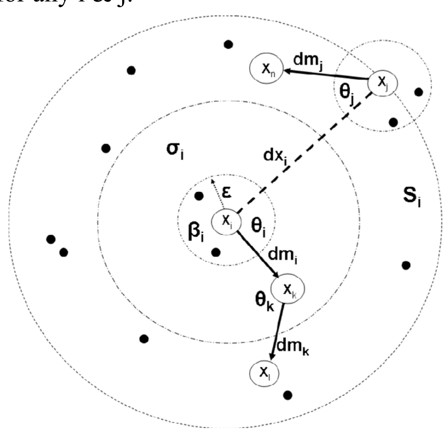

Figure-3: distances relative to agent i

Proposition-1: The distance, dm_i, between X_i and X_k

$$dm_i = |X_k - X_i| \qquad (7)$$

is always decreasing.

Proof: There are two possibilities, either the agent closest to X_k, X_l , is in β_i or it is outside $\sigma_i . \cup \beta_i$. If $X_l \in \beta_i$, then

$$\frac{d(dm_i)}{dt} = -dm_i - dm_k \cdot \cos(\theta_k) \qquad (8)$$

where $-\dfrac{\pi}{2} < \theta_k < \dfrac{\pi}{2}$ & $dm_k = |X_l - X_k|$.

In other words: $\cos(\theta_k) > 0$

and $\dfrac{d(dm_i)}{dt} < 0. \qquad (9)$

In the second case ($X_k \in Si - (\beta_i \cup \sigma_i)$), we have

$$dm_k < dm_i \qquad (10)$$

otherwise X_k will be closer to an agent in β_i . Therefore, regardless of the value of θ_k equation-9 will still hold making the derivative of dm_i strictly negative.

Proposition-2: The modified protocol is globally asymptotically convergent, i.e.

$$\lim_{t\to\infty} X_i = X_c \qquad i=1,...,N \qquad (11)$$

N is the number of agents, X_c is the rendezvous point.

Proof: The proof is carried-out for L=1 connectivity (i.e. each agent moves towards one and only one agent). This proof subsumes the one for L>1 connectivity. The proof is based on showing that the distance, dx_i, from an agent i to the agent furthest from it, agent j, will shrink zero, i.e.

$$\lim_{t\to\infty} dx_i = 0 \qquad i=1,...,N \qquad (12)$$

where $$dx_i = \max_j \left| X_j - X_i \right| \qquad j=1,...,N \qquad (13)$$

The time derivative of dx_i may be written as:

$$\frac{d(dx_i)}{dt} = Fi + Fj \qquad (14)$$

where Fi is the dot product between the velocity vector of agent i ($\dot{X}_i$) and the unit vector from X_i pointing towards X_j. Fj is the dot product between the velocity vector of agent j ($\dot{X}_j$) and the unit vector from X_j pointing towards X_i

$$\frac{d(dx_i)}{dt} = -\frac{(X_i - X_j)^T}{\left| X_i - X_j \right|}(\dot{X}_j - \dot{X}_i) \qquad (15)$$

The derivative may be written as:

$$\frac{d(dx_i)}{dt} = -(dm_i \cdot \cos(\theta_i) + dm_j \cdot \cos(\theta_j)) \qquad (16)$$

where dm_i is the distance between agent X_i and the agent closest to it (X_k) that lies in σ_i, dm_j is the distance between agent X_j and the agent closest to it (X_n) that lies in σ_j, θ_i is the angle between lines dm_i and dx_i and θ_j is the angle between lines dm_j and dx_i.

Let's examine the derivative of dx_i in the two zones: S_i-β_i and β_i. As shown in propositon-1, in the zone S_i-β_i, dm_i is always decreasing. On the other hand, θ_j is restricted to lie between

$$-\frac{\pi}{2} < -\cos^{-1}\left(\frac{\varepsilon}{2 \cdot dx_i}\right) \le \theta_j \le \cos^{-1}\left(\frac{\varepsilon}{2 \cdot dx_i}\right) < \frac{\pi}{2}$$

and $dm_j \ge \epsilon$ (17)

In the limit $$\frac{d(dx_i)}{dt} < 0 \qquad X_j \in S_i - \beta_i \quad j=1,..N,\ j\neq i \qquad (18)$$

which will guarantee that all agents will converge to β_i.

Once X_j enter β_i, the control law moves agent i towards the agent that is furthest from it (agent j). This makes

$$\frac{d(dx_i)}{dt} = -\left| X_j - X_i \right| \qquad (19)$$

$X_i \neq X_j$. This will guarantee that

$$\lim_{t\to\infty} dx_i = 0 \qquad i=1,...,N \qquad (20)$$

The fact that all agents will converge to agent X_i for any i can only hold if all agents converge to the same point Xc,

$$\lim_{t\to\infty} X_i = X_c \qquad i=1,..N \qquad (21)$$

IV. Communication Range limits

The communication limits on the agents is an important factor in determining the practicality of a consensus protocol. In this regard, the suggested protocol has nonastringent requirements. The protocol is guaranteed to converge even if each agent is restricted to communicate with its nearest neighbor outside β. If an agent cannot communicate with the closest agent, this agent is isolated and cannot participate in the consensus effort to begin with. However, the communication limits may still be assessed by examining the behavior of the maximum of the distances connecting each agent in the group to its closest neighbor outside the priority zone (β) corresponding to that agent (d_{xm})

$$d_{xm} = \max_i(\min_j d_{i,j} / d_{i,j} > \varepsilon) \qquad (22)$$

where $d_{i,j}$'s are entries in the distance matrix D, the i'th row has the distances from agent i to all other agents in the group.

A large increase in d_{xm} during operation may jeopardize the ability of the agents to communicate. As shown below, the protocol can inhibit the growth of d_{xm} , hence prevent the communication burden from increasing during the effort to establish consensus. .

$$D=\begin{bmatrix} 0 & d_{12} & d_{13} & . & d_{1N} \\ d_{21} & 0 & d_{23} & . & d_{2N} \\ d_{31} & d_{32} & 0 & . & d_{3N} \\ . & . & d_{ij} & . & . \\ d_{N1} & d_{2N} & d_{3N} & . & 0 \end{bmatrix} \qquad (23)$$

Proposition-3: if an agent j enters into the priority zone of agent i (β_i) it will remain inside β_i .

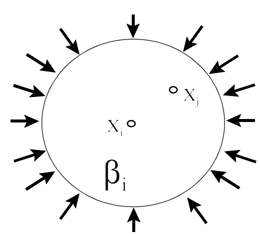

Figure-4: In an agent enters β_i it remains in β_i.

Proof: The proof follows directly from proposition-1. This may also be deduced from the fact that when X_j has just left β_i it becomes the minimum distance agent away from X_i and the protocol will steer it back to β_i.

Proposition-4: If agent-j lies in the intersection of β_i and β_k

$$X_j \in \beta_i \cap \beta_k$$

then $$\left| X_i - X_k \right| < 2 \cdot \varepsilon \qquad \forall t. \qquad (24)$$

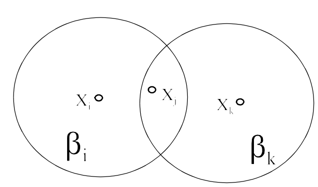

Figure-5: joint sharing of an agent guarantees connectedness

Proof: this follows directly from proposition-3. If X_j is inside β_i and β_k then it will always belong to these two regions. This can only happen if equation-24 holds.

Proposition-5: If $\exists$ an $X_j \in \beta_i \cap \beta_k \quad \forall\, i \neq k$, then

$$d_{xm} < 2\epsilon \qquad \forall t. \qquad (25)$$

Proof: this proposition follows directly from proposition-4.

It ought to be mentioned that attempting to control d_{xm} by increasing the connectivity of the graph maybe ineffective in controlling the growth of this distance. While increasing connectivity will accelerate the consensus process, it will not prevent the creation of drifting clusters each forming a closed group with agents that are temporarily communicating with each other. To reduce the probability of such clusters forming, connectivity has to be increased to an unrealistically high value.

V. The consensus control protocol

Converting the consensus protocol to a consensus control protocol is a challenging task. The challenge is to achieve system stabilization as well as compliance with the guidance signal from the protocol using local information and actions. In other words, each agent must use its own state to synthesize a successful, self-control action. A series of work by this author on the above subject proves to be promising. The control schemes were designed for a single agent to suppress any motion that lies in the space orthogonal to the guidance vector. This paper demonstrates that this approach to design makes the controller a valid control protocol that may be successfully used in a multi-agent environment. Proofs of correctness and extensive simulation to show the robustness of the control protocols to delays, actuator saturation and external drift were omitted from the paper due to space limitations.

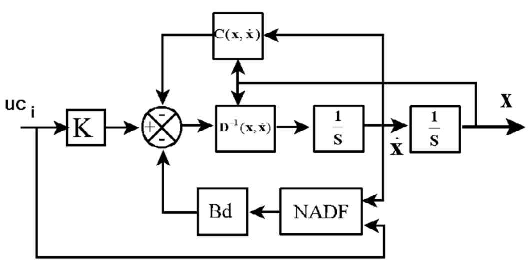

Figure-6: NADF based control protocol

1. Agents with second order dynamics

The simplest holonomic agent with second order dynamics has the form

$$\ddot{X}_i = u_i \qquad i=1,..N \qquad (26)$$

The simplistic way of constructing a control protocol for this case is to augment the consensus protocol with a damping term constructed from the agents' velocities

$$\ddot{X}_i = uc_i - b\dot{X}_i \qquad i=1,..N \qquad (27)$$

If a small b is used, the dynamical interactions among the agents may prevent them from reaching consensus. If an excessively large b is used, motion will be severely impeded and convergence may not be possible. To solve this problem the concept of nonlinear, anisotropic damping forces (NADF) was suggested in [20]. NADFs (Figure-6) selectively apply high motion impedance (ud_i) in the space orthogonal to uc_i

$$ud_i = [n_i^T \dot{X}_i n_i + (\frac{uc_i^T}{|uc_i|} \cdot \dot{X}_i \Phi(uc_i^T \dot{X}_i)) \frac{uc_i}{|uc_i|}] \qquad (28)$$

where n_i is a unit vector orthogonal to uc_i and Φ is the heaviside function. The control protocol in this case is

$$\ddot{X}_i = uc_i - b\dot{X}_i - K_d ud_i \cdot \qquad (29)$$

Excessively high value of K_d may be used without degrading the quality of the control signal.

2. Nonholonomic mobile agents

In [21] a method is suggested for converting uc_i into a control signal for a UGV whose system equation may be written as

$$\dot{P} = F(P)\lambda \qquad (30)$$
$$\lambda = Q(U)$$

where P is the posture of the UGV, λ is the velocity in the local coordinates and U is the control signal.

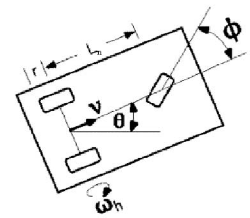

Figure-7: A car-like mobile robot

Many practical robots do fit the above system equation including the car-like, front wheel-steered UGV (figure-7) with system equation and control protocol

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \nu \\ \omega \end{bmatrix}, \qquad (31)$$

$$\begin{bmatrix} \nu \\ \omega \end{bmatrix} = \begin{bmatrix} r \cdot \omega_{\mathbf{h}} \\ r \cdot Ln \cdot \omega_{\mathbf{h}} \cdot \tan(\varphi) \end{bmatrix} \quad \begin{bmatrix} \omega_h \\ \phi \end{bmatrix} = \begin{bmatrix} \nu_r \\ \tan^{-1}(\Delta\theta / (Ln \cdot \nu_r)) \end{bmatrix},$$

$$\nu_r = K1 \cdot |uc_i| \ \& \ \Delta\theta = K2(\arg(uc_i) - \theta)$$

where $P=[x\ y\ \theta]^t$, $\lambda=[\nu\ \omega]^t$, $U=[\omega_h\ \phi]^t$, r is the radius of the robot's wheels,, ν is the tangential velocity of the robot and ω is its angular speed, Ln is the normal distance between the center of the front wheel and the line connecting the rear wheels, ω_h is the angular speed of the rear wheels, and ϕ is the steering angle of the front wheel ($\pi/2>\phi>-\pi/2$).

3. Unmanned Aerial Vehicles.

In [22] a control structure (figure-9) is suggested for converting the guidance signal in a provably-correct manner to a control protocol that suits a dynamical system of the form

$$\begin{aligned} \dot{X} &= G(\lambda) \\ \dot{\lambda} &= F(\lambda, U) \end{aligned} \qquad (32)$$

where U is the control signal, X is a vector containing the location of the center of mass of the UAV in the world coordinates, $X=[x\ y\ z]^t$, λ is the motion vector in the local coordinates of the UAV, $\lambda=[\nu\ \gamma\ \psi]^t$, ν is the radial speed of the UAV, γ and ψ are angles describing its orientation with respect to the world coordinates. This model suits most UAVs. A specific form for equation 32 that describe a fixed-wing (figure-8) aircraft is shown in equation 33,

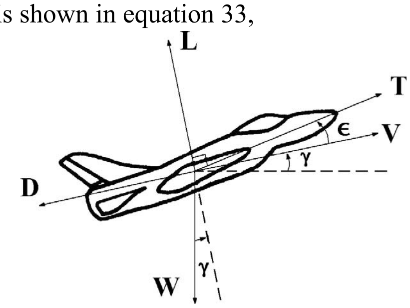

Figure-8: A fixed-wing UAV.

$$
\begin{aligned}
\dot{x} &= \nu \cdot \cos(\gamma)\cos(\psi) \\
\dot{y} &= \nu \cdot \cos(\gamma)\sin(\psi) \\
\dot{z} &= \nu \cdot \sin(\gamma) \\
\dot{\nu} &= \frac{F_T}{m} - g \cdot \sin(\gamma) \\
\dot{\gamma} &= \frac{F_N \cdot \cos(\sigma)}{m \cdot \nu} - g\frac{\cos(\gamma)}{\nu} \\
\dot{\psi} &= \frac{F_N \cdot \sin(\sigma)}{m \cdot \nu \cdot \cos(\gamma)}.
\end{aligned} \tag{33}
$$

where m is the point mass of the UAV, ν radial velocity of the UAV, γ flight path angle, ψ directional angle, σ is the banking angle, F_T the resultant force along the velocity vector:

$$F_T = T \cdot \cos(\epsilon) - D \tag{34}$$

and F_N is the resultant force normal to the velocity vector:

$$F_N = T \cdot \sin(\epsilon) + Lf \tag{35}$$

and g is the constant of gravity, T is the thrust from the engine, D is the aerodynamic drag, ϵ is the angel of attack, Lf is the aerodynamic lift.

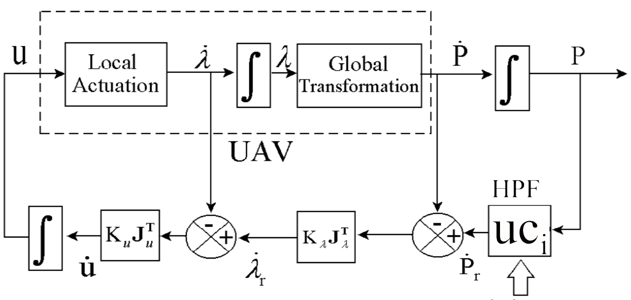

Figure-9: converting guidance into control for a UAV

VI. Simulation Results

In figure-10 the modified consensus protocol is tested for 2400 agents with a uniformly-distributed, random initial configuration. Each agent communicates only with one other agent (the nearest neighbor outside β) in its attempt to establish consensus. The radius of the low priority region (ϵ) is arbitrarily set to 1. As can be seen, consensus was established and the agents converged to a point that is close to the average of the initial configurations.

It is well-known that the more neighbors an agent communicate with (i.e. the more connected the communication graph is) the faster convergence will be. However, the effect of ϵ on convergence need to be examined. The value of ϵ is varied from zero to a high value. The convergence time is measured. The simulation is carried-out for 100 agents each communicate with the closest 5 neighbors (figure-11). All other cases showed a behavior similar to the one obtained for this case. When ϵ is set to zero, i.e. the algorithm reduces to the original nearest neighbor algorithm, the group did not reach consensus and no convergence took place. It is observed that convergence time exponentially drops as a function of ϵ. It settles to a constant value as ϵ increases. It is noticed for this case that small values of ϵ exceeding .02 do not offer any significant improvement as far as the convergence rate is considered.

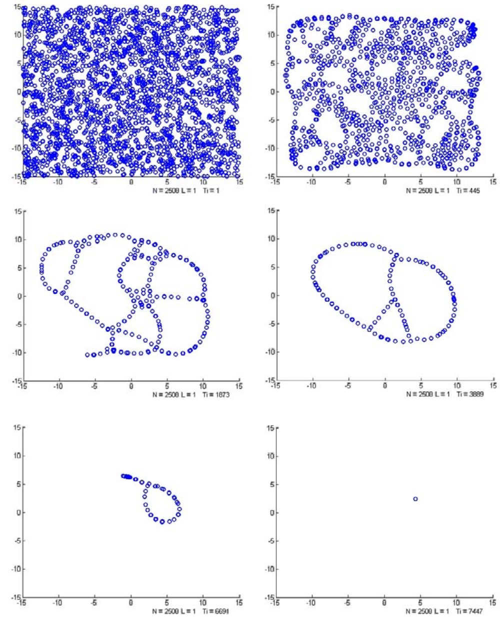
Figure-10: Modified protocol guarantees convergence, L=1.

In table-1, the effect of ϵ on d_{xn} is tested for 200 agents initially located in a 30×30 rectangular region and distributed in space using a uniform PDF. The initial d_{xn}, maximum d_{xn} and time (Tc) to consensus (in time steps) are recorded. The average distance traveled by the agents until consensus is achieved (dL) along with the most distance traveled minus the least distance traveled (Δ) are also recorded. As can be seen there is a considerable growth in d_{xn} for low values of ϵ. At ϵ=3, condition-25 is satisfied. This restricted the maximum value of d_{xn} for ϵ≥3 to less than 2ϵ. The changes in ϵ has minimal effect on the time to reach consensus. Figures 12 & 13 show d_{xn} versus time for ϵ=1 and ϵ=3 respectively. The ability to control the growth of d_{xn} is obvious. Increasing connectivity to control d_{xm} is investigated in table-2. Although the rate of convergence significantly improved, L had practically no effect on d_{xm} until it was set to an unrealistically high value.

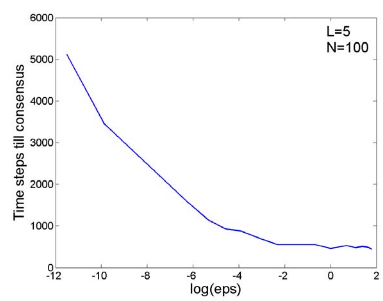

Figure-11: Time to converge versus log(ϵ)

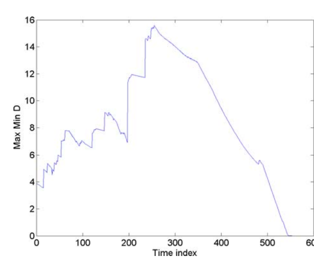

Figure-12: d_{xn} versus time, ϵ=1

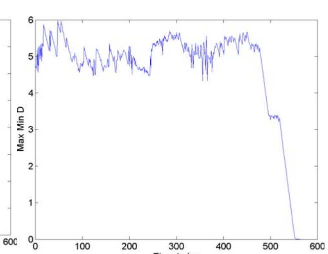

Figure-13: d_{xn} versus time, ϵ

ϵ	d_{xn} initial	d_{xn} maximum	dL	Δ	Tc
.05	4	15.2	26.7	.9	550
.5	3.9	13.3	26.6	.064	560
1	4	15.6	27.9	.074	550
1.5	4	13.9	26.1	.077	510
2.0	4	9.94	35.1	.07	530
2.5	4.8	7.61	32.22	.071	590
3	4.7	5.97	28.77	.066	570
3.5	4.9	6.39	25.9	.073	510
4	5	7.19	24.8	.07	490

Table-1: Maximum d_{xn} versus ϵ, L=1=3

L	d_{xn} initial	d_{xn} maximum	dL	Δ	Tc
1	4	15.6	27.6	.0622	550
2	4	13.1	23.4	7.3	270
3	4	15.9	24.5	4.3	180
4	4	13.4	28.3	6.2	135
5	4	15.3	24.7	5.2	110
6	4	14.1	24.3	5.5	85
7	4	15.2	25.4	6.2	78
8	4	15	24.3	6.1	69
9	4	14.9	30.9	8	60
20	4	12.1	23.1	10.7	27
30	4	9.9	23.2	12.1	18
40	4	4.5	35.1	13.2	13

Table-2: Maximum d_{xn} versus L, ϵ=1.

The ability of the techniques presented in section V to convert the consensus protocol into a decentralized consensus control is tested. The directed communication graph in figure-14 is used for this purpose. The graph has a cycle that contains all the nodes. Therefore, for a single integrator system, convergence is guaranteed.

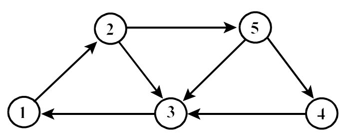

Figure-14: A directed communication graph

Figure-15 shows the response of five single integrator agents attempting to obtain consensus. As can be seen, the group converges to a rendezvous point. In figure-16, the single integrator agents were replaced with double integrator agents. As can be seen, the group failed to converge. In figure-17, the NADF approach is used to generate the consensus control. The following parameters are selected Kd=150 and b=2. As can be seen, the resulting trajectories for the double integrator agents are almost identical to those of the single integrator agents. The control signals for the first agent are shown in figure-18. Despite the use of excessively high NADF, the control signal is well behaved. The convergence rate is also unaffected. The sharp fluctuations in the control signals that occur at the end are caused by interaction forces when the agents are in very close proximity to each other. The problem can be easily solved by requiring convergence to be to a small region instead of a point.

The ability of the control scheme suggested in equation-31 [21] to convert the guidance signal into a consensus control signal for a group of car-like robots is tested in figure-19. A group of five agents that are uniformly distributed on the circumference of a circle with unity radius, all initially oriented along the positive x-axis are used. The agents are using the communication graph in figure-14 to exchange data. The parameters used are K1=0.5 and K2=4. The orientation and control signals of the fifth agent are shown in figures 20,21 respectively. Figure-22 shows the trajectories of the agents for a different set of parameters K1=0.5, K2=10. K2 is responsible for improving the alignment of the robot with the guidance field. Increasing it does improve the response of the group.

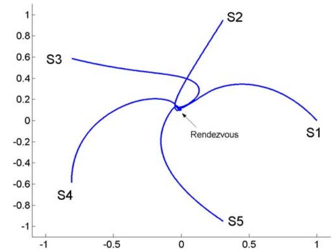

Figure-15: Single integrator agents using the graph in figure-14.

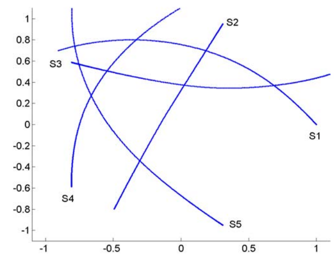

Figure-16: Double integrator agents using the communication graph in figure-14

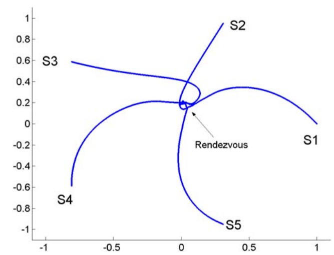

Figure-17: Double integrator agents using the communication graph in figure-14, NADF used.

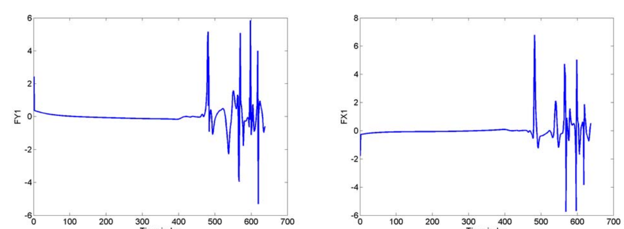

Figure-18: x & y components of the control signal for agent-1.

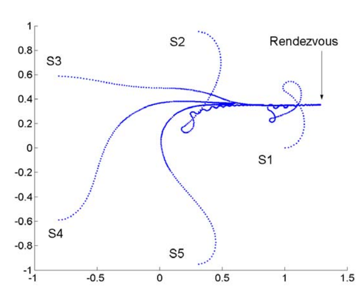

Figure-19: trajectories for car-like agents, K1=.5 K2=4

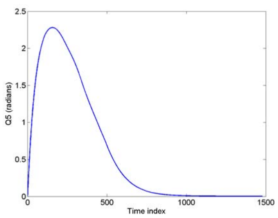

Figure-20: Orientation , agent-5

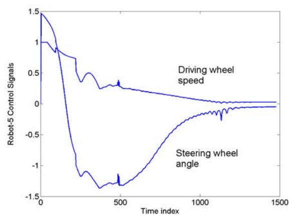

Figure-21: Control signals agent-5

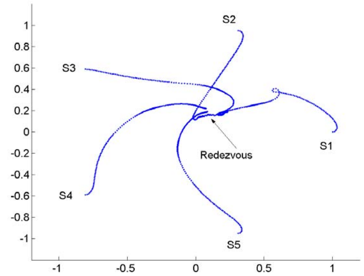

Figure-22: trajectories for car-like agents, K1=.5, K2=10

Figure-23 shows two jets described by the system equation in (33) taking-off and synchronizing their orientation in a decentralized manner by converting the guidance signal into a control signal using the method suggested in [22].

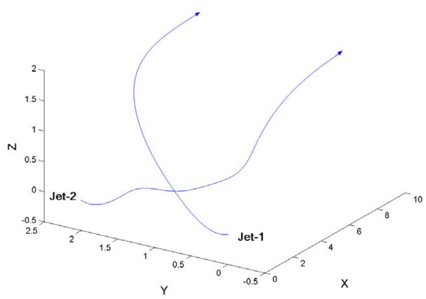

Figure-23: two jets synchronizing their orientations.

VII. Conclusions

A new nearest neighbor consensus control protocol is suggested to solve the convergence problem the traditional protocol suffers from. The protocol is guaranteed to converge regardless of the initial conditions even if each agent is restricted to communicate with its nearest neighbor only. The neighbor is restricted to lie outside an arbitrarily small priority zone surrounding the agent. The protocol has the ability to guarantee that the communication burden during the effort to establish consensus does not increase when compared to the initial burden at the start of the protocol. It is also demonstrated that the consensus guidance signal may be easily converted into a consensus control protocol that may be used by a wide variety of practical dynamical agents.

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